HORIZONTAL -VERTICAL COUPLING CORRECTION AT ALADDIN

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Horizontal-vertical coupling and vertical dispersion can arise from vertical misalignments and tilts of bending magnets, quadrupoles, and sextupoles. Experimental measurements and accelerator code studies suggest that the vertical dispersion from bending magnet errors and quadrupole height errors is largely compensated by optimization of the vertical closed orbit. The majority of the coupling and vertical dispersion remaining after orbit optimization may be the result of quadrupole tilts on the order of 1-2 mrad. A correction scheme is being implemented in which the excitation of skew-quadrupole correctors is determined from measurements of the vertical orbit displacements resulting from horizontal steering bumps and vertical dispersion [1].

I. INTRODUCTION

In an ideal electron storage ring, the horizontal and vertical motions are independent, resulting in an extremely small beam height. In practice, ring imperfections result in a coupling of horizontal and vertical transverse oscillations (betatron coupling) as well as vertical motion associated with energy oscillations (vertical dispersion). The coupling and vertical dispersion then determine the beam height.

The vertical emittance ($\varepsilon_y \equiv \langle y^2 \rangle / \beta_y$) characterizes the excitation of vertical betatron motion. As a result of coupling, both transverse oscillation modes include vertical motion, and both modes are excited by photon emission in bending magnets. With non-zero vertical dispersion in bending magnets, photon emission excites vertical motion even in the absence of coupling. These effects give rise to vertical emittance [2]. In addition, vertical dispersion contributes to the height because of the beam energy spread.

Rotations of bending magnets and quadrupoles and vertical orbit offsets in sextupoles cause betatron coupling and vertical dispersion. Vertical dispersion is also caused by vertical misalignment of bending magnets and quadrupoles.

II. SOURCES

Experiments and modeling have been performed to study the sources of coupling and vertical dispersion. For normal 800 MeV operation ($\nu_x = 7.14$, $\nu_y = 7.23$, optimized orbit, sextupoles on), the rms vertical dispersion is $D_y^{rms} = 0.03$ m, while the emittance ratio ($\kappa = \varepsilon_y/\varepsilon_x$) equals 0.014. The coupling associated with the nearby $\nu_x = \nu_y$ resonance (global coupling) gives rise to a minimum tune separation, $\delta\nu_{min} = 0.009$. The global coupling contribution to the equilibrium emittance ratio is $\kappa_o = 2|\delta\nu_{min}/2(\nu_x\text{-}\nu_y)|^2 = 0.005$, about one-third of the total.

With steering magnets and sextupoles turned off, D_y^{rms} increases to 0.26 m; the minimum tune separation was not measurable because of the broadened tune signals without sextupoles. For normal 1 GeV operation, D_y^{rms} increases by 30%, while the beam height, tilt, and $\delta\nu_{min}$ increase by factors of 2-3, compared to 800 MeV values.

For historical reasons, the bending magnets were installed with measured rolls and pitches, where roll is rotation about the beam axis, and pitch is a height difference between the upstream and downstream ends of the magnet. Typical rolls are 2 mrad, and pitches are about one-tenth as great. We measured the beam properties with these errors removed. For normal 800 MeV operation, D_y^{rms} was reduced by 20%, while little or no change was observed in $\delta\nu_{min}$, the beam height or tilt. $\delta\nu_{min}$ was nearly independent of the horizontally-focusing sextupole strength, but depended upon the horizontally-defocusing sextupoles. Reducing their current from the normal 72 A to 40 A caused $\delta\nu_{min}$ to increase from 0.009 to 0.014, while a larger sextupole current of 85 A resulted in $\delta\nu_{min} = 0.007$. For the case where steering magnets and sextupoles are turned off, D_y^{rms} was 70% smaller than its value before correcting the bending magnet rotations.

We used the MAD computer code [3] to model the measured bending magnet rotations. The effects of pitch errors were negligible, but the roll errors resulted in $\delta\nu_{min} = 0.006$. (This is consistent with $\delta\nu_{min}$ equaling 0.009 both before and after removing the bending magnet rotation errors, because coupling adds as a complex number.)

MAD modeling also showed that vertical dispersion resulting from bending magnet rotations, bending magnet height errors, or quadrupole height errors may be reduced by ~ 90% when the vertical orbit is optimized, comparable to the measured reduction from 0.26 m to 0.03 m. However, vertical orbit optimization does not effectively compensate the vertical dispersion from quadrupole rolls or sextupole height errors. This is expected for quadrupole rolls because they do not affect the closed orbit in an otherwise ideal ring. Thus, for a combination of errors, the relative contribution of bending magnet errors to the vertical dispersion is expected to be less on the optimized orbit than the zero-steering orbit, as observed.

The measurements and modeling suggest that misaligned bending magnets, quadrupoles and sextupoles are all important sources of betatron coupling and vertical dispersion during normal operation. First, MAD modeling of the measured bending magnet rotations gives $\delta\nu_{min} = 0.006$, compared to a measured value of 0.009 in the ring. The experimental removal of these errors reduced the vertical dispersion by 20% on the optimized orbit. However, bending magnet rotations do not appear to be the dominant source of coupling and vertical dispersion during normal operation, since the major portion of the vertical dispersion, height, tilt, and $\delta\nu_{min}$ remained after their removal.

Sextupole current variations on the order of their normal operating current change $\delta\nu_{min}$ by a factor of two, indicating that sextupole positioning errors are also a significant cause of coupling. However, if sextupole errors were the dominant source of global coupling, the coupling would increase with sextupole current, in contrast to the measured decrease. Thus, quadrupole errors are most likely the dominant source of coupling and vertical dispersion during normal operation.

The observed beam height, tilt, and $\delta\nu_{min}$ increase dramatically when the ring energy is increased from 800 MeV to 1 GeV, while the vertical dispersion increases only 30%. This suggests that coupling, and not vertical dispersion, is the dominant source of the beam height. This conclusion is confirmed in the next section by estimates of the beam height from the measured values of $\delta\nu_{min}$ and D_y^{rms}.

We estimated the magnet positioning errors required for the observations. Bending magnet roll errors of order 2 mrad and pitch errors of order 0.2 mrad were measured before the storage ring was assembled. Quadrupole roll errors of order 1 mrad and sextupole height errors of order 1 mm are expected to give a contribution to $\delta\nu_{min}$ and D_y^{rms} comparable to the 800 MeV observations. At 1 GeV, the errors must be 2-3 times as large to account for the observed coupling, perhaps a result of current-induced stress in the quadrupoles.

III. EFFECTS

For a beam whose height results primarily from vertical emittance characterized by the emittance ratio $\kappa = \varepsilon_y/\varepsilon_x$, the height is $\sigma_y^{\kappa} = (\beta_y\varepsilon_y)^{1/2} = (\beta_y\kappa\varepsilon_x)^{1/2}$; its rms value is $\sigma_y^{\kappa,rms} = (\sqrt{\beta_y})^{rms}(\kappa\varepsilon_x)^{1/2}$. For a beam whose height results primarily from the energy spread and vertical dispersion, $\sigma_y^{Dy} = D_y\ \sigma_E$, and $\sigma_y^{Dy,rms} = D_y^{rms}\ \sigma_E$. When both effects contribute, $\sigma_y^2 = (\sigma_y^{\kappa})^2 + (\sigma_y^{Dy})^2$; the rms values combine in the same way.

The vertical emittance has contributions from global coupling, local coupling, and vertical dispersion in bending magnets. When the emittance is primarily a result of global coupling characterized by a minimum tune split, the equilibrium emittance ratio is $\kappa_o = 2|\delta\nu_{min}/2(\nu_x-\nu_y)|^2$. Here, $\kappa_o/2$ is the emittance ratio of the almost-horizontal eigenmode. An equal contribution to the equilibrium vertical emittance arises from excitation of the almost-vertical eigenmode by synchrotron radiation emission [2].

The coupling which is not associated with the $\nu_x = \nu_y$ resonance is termed local coupling. According to [4], errors of sufficient magnitude to cause a minimum tune split of $\delta\nu_{min}$ are expected to produce local coupling with eigenmode roll angle $|\psi| \sim \pi\ \delta\nu_{min}$. Relating this roll angle to the equilibrium emittance ratio by $\kappa = 2|\psi|^2$, we obtain the estimate $\kappa^{local} \approx 2|\pi\delta\nu_{min}|^2$.

Because this estimate is insensitive to the proximity of coupling resonances ($\nu_x \pm \nu_y$ = integer), it must be regarded as a rough approximation. We tested its validity by using MAD to track betatron oscillations in a ring with skew-quadrupole errors. For both cases modeled, elimination of global coupling with skew-quadrupole correctors (so that $\delta\nu_{min} = 0$) reduced the height of the betatron oscillation envelope by 70% for the normal operating tunes. This suggests that $\sigma_y^{\kappa local} = 0.3\ [(\sigma_y^{\kappa local})^2 + (\sigma_y^{\kappa o})^2\ ]^{1/2}$, which gives $\kappa_{local}/\kappa_o = 0.10$, versus the estimate of 0.32. Thus, the estimate gives the correct order of magnitude.

When vertical dispersion in bending magnets is the primary source of vertical emittance, the emittance ratio is approximately [2] $\kappa_{Dy} = [D_y^{rms}/(\sqrt{\beta_y})^{rms}]^2/[D_x^{rms}/(\sqrt{\beta_x})^{rms}]^2$. When several sources of vertical emittance are important, an approximate calculation may be performed by combining the above formulas as $\kappa = \kappa_o + \kappa_{local} + \kappa_{Dy}$, so that $(\sigma_y^{\kappa})^2 = (\sigma_y^{\kappa o})^2 + (\sigma_y^{\kappa local})^2 + (\sigma_y^{\kappa\ Dy})^2$. Combining the effects of emittance and dispersion gives $(\sigma_y)^2 = (\sigma_y^{\kappa o})^2 + (\sigma_y^{\kappa local})^2 + (\sigma_y^{\kappa\ Dy})^2 + (\sigma_y^{Dy})^2$.

These formulas may be used to estimate the rms beam height before and after global decoupling from measurements of $\delta\nu_{min}$ and D_y^{rms}. At 800 MeV, we measured $\delta\nu_{min} = 0.009$ and $D_y^{rms} = 0.033$ m. The normal operating parameters are $(\nu_x,\nu_y) = (7.14, 7.23)$, $(\sqrt{\beta_y})^{rms} = 3.7\ m^{1/2}$, $(\sqrt{\beta_x})^{rms} = 2.4\ m^{1/2}$, $\varepsilon_x = 0.127 \times 10^{-6}$ m-rad, and $\sigma_E = 4.5 \times 10^{-4}$. Here, the horizontal rms values are taken over horizontal beam position monitor (BPM) positions, while vertical rms values are taken over vertical BPM locations. From the ring parameters and measurements, we estimate: $\kappa_o = 2|\delta\nu_{min}/2(\nu_x-\nu_y)|^2 = 0.005$; $\sigma_y^{\kappa o,\ rms} = (\sqrt{\beta_y})^{rms}(\kappa_o\varepsilon_x)^{1/2} = 93\ \mu m$. $\kappa^{local} \approx 2|\pi\delta\nu_{min}|^2 = 0.0016$; $\sigma_y^{\kappa local,\ rms} = 53\ \mu m$. $\kappa_{Dy} = [D_y^{rms}/(\sqrt{\beta_y})^{rms}]^2/[D_x^{rms}/(\sqrt{\beta_x})^{rms}]^2 = 0.00057$; $\sigma_y^{\kappa Dy,\ rms} = 31\ \mu m$. $\sigma_y^{Dy,\ rms} = D_y^{rms}\ \sigma_E = 15\ \mu m$. Adding the squares of the height contributions gives $\sigma_y^{rms} = 112\ \mu m$. In the bending magnets, $\sqrt{\beta_y}$ is 68% of its rms value, and the FWHM beam height is 2.35 times the standard deviation, so that $\sigma_y^{rms} = 112\ \mu m$ corresponds to a bending magnet FWHM height of 112 x 0.68 x 2.35 = 180 μm. If the global coupling contribution is neglected, an rms height of 63 μm is obtained from the other three contributions, suggesting a 44% reduction may be obtained from global decoupling.

In comparison, we measured FWHM beam heights of 184 and 214 μm in the bending magnets, within 20% of the above estimate. When skew-quadrupole correctors were used to eliminate global coupling, the vertical dispersion was increased by 34%, the beam lifetime was reduced by 28%, and the measured heights in the bending magnets were reduced by 5% and 28%. The lifetime and height reductions are less than the 44% predicted if global decoupling did not change the local coupling or vertical dispersion.

For normal 1 GeV operation, we measured $\delta\nu_{min} = 0.026$ and $D_y^{rms} = 0.043$ m with 1 GeV parameters $\varepsilon_x = 0.15 \times 10^{-6}$ m-rad, and $\sigma_E = 6 \times 10^{-4}$. We estimate: $\kappa_o = 2|\delta\nu_{min}/2(\nu_x-\nu_y)|^2 = 0.042$; $\sigma_y^{\kappa o,\ rms} = (\sqrt{\beta_y})^{rms}(\kappa_o\varepsilon_x)^{1/2} = 293\ \mu m$. $\kappa^{local} \approx 2|\pi\delta\nu_{min}|^2 = 0.013$; $\sigma_y^{\kappa local,\ rms} = 166\ \mu m$. $\kappa_{Dy} = [D_y^{rms}/(\sqrt{\beta_y})^{rms}]^2/[D_x^{rms}/(\sqrt{\beta_x})^{rms}]^2 = 0.0009$; $\sigma_y^{\kappa Dy,\ rms} = 43\ \mu m$. $\sigma_y^{Dy,\ rms} = D_y^{rms}\ \sigma_E = 26\ \mu m$. Adding the squares of the height contributions gives $\sigma_y^{rms} = 340\ \mu m$, corresponding to a FWHM beam height of 540 μm in the bending magnets. If the global coupling contribution is neglected, an rms beam height of 173 μm is obtained from the other three contributions, suggesting a 49% height reduction may be obtained from global decoupling.

In comparison, we measured FWHM beam heights of 428 and 439 μm in the bending magnets, within 20% of the above estimate. When skew-quadrupole correctors were used to reduce global coupling by 77%, the vertical dispersion was increased by 16%, the beam lifetime was reduced by 39%, and the measured heights in the bending magnets were reduced by 45% and 54%. The lifetime and height reductions are comparable to the 49% reduction predicted if the global coupling were completely removed with no change in local coupling or vertical dispersion.

Thus, our predictions of the beam height from the measurements of $\delta\nu_{min}$ and D_y^{rms} are in approximate agreement with height measurements in bending magnets and the observed effects of global decoupling. The modeling and experimental results suggest that reducing the height by more than 50% cannot be achieved by global decoupling alone.

IV. CORRECTION

Harmonic correction is a decoupling scheme in which skew-quadrupole correctors are powered to eliminate the driving terms ("harmonics") of important coupling resonances ($\nu_x \pm \nu_y$ = integer) and vertical dispersion resonances (ν_y = integer). For example, global decoupling, in which correctors are used to obtain $\delta\nu_{min} = 0$, is the correction of the $\nu_x - \nu_y = 0$ driving term. The coupling harmonic [5] for $\nu_x \pm \nu_y = p$ is a complex number: $\Delta_p^{\pm} = (4\pi)^{-1}\,\Sigma\, Kl(\beta_x\beta_y)^{1/2}$ exp $i[\varphi_x \pm \varphi_y - (\nu_x \pm \nu_y - p)\theta]$, while the vertical dispersion harmonic [6] for $\nu_y = p$ is $\Delta_p^{Dy} = (\nu_y/4\pi)\ \Sigma\ Kl\ D_x\sqrt{\beta_y}$ exp $(ip\varphi_y/\nu_y)$. Here, the sum is over skew quadrupoles with integrated strength Kl, where $\varphi_{x,y} = \int dz/\beta_{x,y}$ is the betatron phase and $\theta = z/R$ varies between 0 and 2π around the ring. For Aladdin, the five most important harmonics are expected to be Δ_o^-, Δ_{14}^+, Δ_{15}^+, Δ_7^{Dy}, and Δ_8^{Dy}, where $|\Delta_o^-| = \delta\nu_{min}/2$.

Ten skew-correctors are required to eliminate these five complex terms. For a given set of ten locations, we constructed a 10x10 matrix containing the real and imaginary contributions to the 5 harmonic terms for unit excitations of the skew correctors. Its inverse gives the skew-corrector strengths associated with unit excitations of the 10 harmonic terms (Re Δ_o^-, Im Δ_o^-, etc.). We selected ten locations for which this inverse had elements with small magnitude.

To model the effectiveness of harmonic correction with these locations, we studied two test lattices with rotated quadrupoles. In one lattice, the roll angles were assigned randomly from a truncated Gaussian distribution with σ = 1 mrad, maximum roll of 3σ; the other lattice was an attempt to fit roll errors to ring data. From the quadrupole roll angles, the 5 harmonic terms were calculated, along with the skew-corrector strengths necessary to cancel them out. To quantify the coupling, we used MAD to track betatron oscillations from an initial horizontal offset, and then measured the envelope height; D_y^{rms} was also calculated.

For the lattice with random quadrupole rolls, the height of the oscillation envelope was reduced by 73% and D_y^{rms} was reduced by 44% with the skew-correctors on. For the lattice fit to data, the envelope height was reduced by 89% while D_y^{rms} was increased by 3%. This indicates that ten skew-correctors can effectively reduce coupling. However, the skew-corrector settings were calculated from known quadrupole roll errors, which is not feasible in practice.

We then modeled the same lattices and corrector locations, setting the correctors to minimize the vertical orbit change resulting from horizontal bumps or an energy offset [1]. Letting K_k equal the k^{th} skew corrector strength, V_{ij} equal the i^{th} vertical BPM deflection from a unit excitation of the j^{th} horizontal steerer (or a unit energy offset for one value of j), and $M_{ijk} = d(V_{ij})/dK_k$, we set the skew correctors according to $\Delta K_k = -(M_{mnk}M_{mnl})^{-1}_{kl}\, M_{ijk}V_{ij}$. Singular value decomposition or regularization may be used to obtain an approximate inverse that prevents "corrector-fighting".

We found that the coupling correction with this technique improved with iteration. The results after 5-10 iterations were better than harmonic correction with the first test lattice and worse with the second. The height of the betatron oscillation envelope was reduced ~ 70% for both cases. For the first lattice, D_y^{rms} was reduced by 60%. With the second lattice, D_y^{rms} was reduced by 25% with the first iteration, but increased with successive iterations to exceed the uncorrected value.

We plan to study the latter correction scheme experimentally as multipoles which can be wired as skew-correctors become available.

V. SUMMARY

The above results may be summarized as follows:

1) Quadrupole rolls appear to be the dominant cause of coupling, but bending magnet rolls and sextupole alignment errors are also important.
2) Global decoupling can reduce the beam height by less than ~ 50%.
3) Using ten skew-quadrupole correctors, a beam height reduction of ~ 70% appears feasible.
4) Coupling correction may be implemented at Aladdin.

VI. REFERENCES

[1] J. Safranek and S. Krinsky, AIP Conference Proc. **315**, p. 163 (1994).
[2] G. Guignard, CERN 87-03, p. 203 (1987).
[3] H. Grote and F. C. Iselin, CERN/SL/90-13 (1990).
[4] S. Peggs, IEEE Trans. Nucl. Sci. **NS-30**, p. 2460 (1983).
[5] G. Guignard, CERN 76-06 (1976).
[6] G. F. Dell, H. Hahn, S. Y. Lee, G. Parzen, S. Tepikian, Proc. 1991 IEEE PAC, p. 1627 (1991).