MODELING OF WLS FIELD WITH PIECEWISELY CONSTANT MAGNETS

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I. INTRODUCTION

A WLS, or a WaveLength Shifter, indicates in this note a superconducting wiggler which, installed as an ID (Insertion Device) in a storage ring, can provide short wavelength synchrotron light and will usually impose a strong effect upon the stored beam.

The magnetic field in a WLS varies dramatically with longitudinal positions in such a way that differs by far from either approximately piecewisely-constant fields in most ring magnets, or sine-like periodical fields in long multi-period ID's. WLS effect analysis is hence made difficult. To refer to commonly used accelerator physics computer codes, a 3-rectangular-pole modeling method is used most frequently, with the parameters chosen according to the 0th order effect of the WLS in question. Unfortunately, such a modeling often gives a distorted account of its 1st order effect and fails to properly represent the WLS effect on the beam motion.

During a visit to BESSY, Germany, the authors carries out studies on WLS effects, including: theoretical study on the field and beam motion, field analysis, particle tracking, up-to-3rd order transport (Taylor's expansion) coefficients calculation, modeling of the field with 3 edge-angled piecewisely constant magnets, and error analysis for the modeling. Detailed calculation results for BESSY-1 WLS's and discussions are presented in a BESSY technical note[1]. This paper will focus on 2 creative pieces of the work: estimation of the biggest 1st and 3rd order terms with field integrals; linear effect modeling by parameters fitting. Most methods and ideas herein described, such as "higher order edge effect", apply in principle to other ID's.

Discussions with P.Kuske are very helpful to the work.

II. WLS EFFECTS

In the terminology of machine physics, motion of a particle is described by 6 variables: the 2 transverse displacements and 2 slopes (x, y, x' and y'), the orbit length (L) and the momentum deviation (δ). When going through a magnet, the first 5 of them change so that their final values are determined by the initial values of the first 4 variables and δ. By Taylor's expansion of the functions, the effect of the magnet can be resolved into orders. Of the 1st order terms (linear transport matrix elements), 25 are dependent on and descriptive of the magnet field. Similarly, there are 5 terms to account for the 0th order effect, 15×5 terms to indicate the 2nd order, 35×5 to go to the 3rd, and much more for even higher.

The WLS effects are thus itemized as: 1) 0th order effect, the closed orbit distortions, increase of the orbit length, as well as the orbit deflection and displacement within the WLS. 2) 1st order effect, an inevitable vertical focusing and maybe a weak horizontal defocusing, causing tune shifts and beta function beating, breaking the superperiodicity and the symmetry of lattice functions in the machine and changing the phase advances between ring components; hence often restricting the ring's dynamic aperture. The 1st order effect is always the most important. 3) 2rd and higher order effects, causing chromatic and geometric aberrations, affecting dynamic aperture and energy acceptance, inducing tune spread and maybe high order resonances. 4) Effect due to the radiated energy in the WLS, i.e. changes of the "synchrotron radiation integrals", therefore a bigger energy loss, shorter damping times, a larger energy spread and longer bunch length, some change in the beam emittance and momentum compaction factor, and so on.

In most WLS's, plane $y = 0$ is the magnetic median plane in which the field is purely vertical and the central orbit lies. Then, the field at position (x, y, s), up to the 3rd order, can be found as

$$
\begin{aligned}
B_y = \quad & B_y(s) + x\frac{\partial B_y}{\partial x} + \frac{1}{2}(x^2 - y^2)\frac{\partial^2 B_y}{\partial x^2} - \frac{1}{2}y^2\frac{\partial^2 B_y}{\partial s^2} \\
& + \frac{1}{6}x(x^2 - 3y^2)\frac{\partial^3 B_y}{\partial x^3} - \frac{1}{2}xy^2\frac{\partial^3 B_y}{\partial x \partial s^2} \cdots \\
B_x = \quad & y\frac{\partial B_y}{\partial x} + xy\frac{\partial^2 B_y}{\partial x^2} + \frac{1}{6}y(3x^2 - y^2)\frac{\partial^3 B_y}{\partial x^3} - \frac{1}{6}y^3\frac{\partial^3 B_y}{\partial x \partial s^2} \cdots \\
B_s = \quad & y\frac{\partial B_y}{\partial s} + xy\frac{\partial^2 B_y}{\partial x \partial s} - \frac{1}{6}y^3\frac{\partial^3 B_y}{\partial s^3} + \frac{1}{6}y(3x^2 - y^2)\frac{\partial^3 B_y}{\partial x^2 \partial s} \cdots
\end{aligned}
$$

where the on-axis field and its on-axis derivatives are s-dependent only, and the 3rd order components in the first 2 equations are much smaller than those in the 3rd.

In the x-y-s Cartesian coordinate system the particle motion equations are (' denotes derivation with respect to s, $B\rho$ is the nominal rigidity of the beam):

$$ L' = \sqrt{1 + x'^2 + y'^2} $$

$$ x'' = -L'((1 + x'^2)B_y - y'B_s - x'y'B_x)/((1 + \delta)B\rho) $$

$$ y'' = L'((1 + y'^2)B_x - x'B_s - x'y'B_y)/((1 + \delta)B\rho) $$

The whole procedure for the authors to deal with WLS effect is summarized as follows. 1) Field mapping. From the WLS field measurement data, it determines the on-axis field components (field itself and its partial derivatives) by a least square fitting, to be used in field reestablishment during tracking. 2) Particle tracking. A lot of particles are tracked through the WLS with Runge-Kutta's method. 3) Taylor's expansion. Another least square fitting, taking the variables' final values of all the particles as functions of their initial values, gives the coefficients up to the wanted order. 4) Modeling. It keeps the above-obtained linear transport matrix of the WLS as intact as possible. 5) Calculation with machine physics codes. The model magnets are put in the place of the WLS for its linear effect and chromatic aberrations. The terms other than 1st order can be used for their corresponding effects.

A few computer codes (in VAX Fortran) are developed by the authors for this purpose. The modeling and tracking/coefficients-fitting codes have been used for BESSY WLS's[1,2].

III. HIGHER ORDER EDGE EFFECTS AND ESTIMATION FORMULAS

Insertion devices always produces vertical focusing because the inside beam orbit does not coincide with the gradient of the field. This is often named as edge effect by analogy to edges in ordinary magnets. The same mechanism gives rise to 3rd and other odd order effects, which the authors refer to as "higher order edge effects".

The WLS field varies slowly in respect to x, but so fast along s-axis that, by Maxwell's law, there must be strong longitudinal field component off the midplane, which gives a vertical focusing to particles on an inclined orbit. This need not be related to x-dependent field component. In general, a beam travelling on a inclined orbit in a varying field will "see" a "one order higher" effect of any transverse field components; here the gradient may translate into a higher order derivation. If the orbit wiggles in a quasi-periodic field, the even orders cancel while the odd ones add up. These edge effects are inevitable in ID's, and often significant, especially when the field is strong, or the ID is of a lot of periods, or there are many ID's in the ring.

The most important edge effects are the 1st and 3rd order ones; of their many terms, the biggest are vertical focusing ones, normal quadrupole-like and octupole-like, respectively. The authors find a way to estimate such biggest terms with on-axis field integrals. The 3rd order field component, which can hardly be extracted convincingly from measurement, need not be explicitly evaluated.

Omit irrelevant terms for simplicity, e.g. suppose the field is not x-dependent. Let $b = B_y(s)/B\rho$. Then the particle motion equations turn into

$$x'' \approx -B_y/B\rho \approx -b + \frac{1}{2}y^2 b''$$

$$y'' \approx -x'B_s/B\rho \approx -x'(yb' - \frac{1}{6}y^3 b''')$$

Here x is the central orbit. Assume y not to change much on the way though y' changes. The equations are combined to yield

$$\Delta y' = -\int y'' ds \approx -y \int b^2 ds + \frac{2}{3}y^3 \int bb'' ds + \cdots$$

with the integrals made over the whole device.

If the field is x-dependent, the linear term has to be revised. More often than not, the strong field in the central pole goes down with x squared where the beam runs off-axis and sees a negative $\partial B_y/\partial x$, giving a horizontal defocusing and an additional vertical focusing. In building a WLS, attention is usually paid to 2nd order field variation with x, but often only to field integrals along straight lines, on which the central pole term and the side pole terms cancel partially. Along the curved orbit, however, the central pole term is the dominant. Since any additional transverse focusing must be of the same strength but of opposite signs on the 2 planes, the revised estimation formulas are

$$(\Delta y'/y_o) + (\Delta x'/x_o) = -\int b^2 ds$$

$$(\Delta y'/y_o^3) = -\frac{2}{3}\int b'^2 ds$$

A few conclusions are drawn from the formulas: Both terms are focusing vertically, with a definite sign and therefore ineradicable, dependent on on-axis field distribution, going up with device length and with maximum field over particle energy squared. For a WLS, the ideal distribution is made of a narrow positive field peak and flat negative fields in side poles.

Calculation results agree well with the estimations. The accuracy is about 1% for the 1st order and 9% for the 3rd. The 1st order revision is necessary, for the discrepancy in vertical focusing alone may be over 20% in BESSY examples, while in general unknowable without tracking.

IV. MODELING OF WLS 1ST ORDER EFFECT

Modeling of WLS field with commonly used magnet types sets a bridge from the tracking and coefficients-fitting results to machine phicics tools.

The models of the authors are still 3 separate bending magnets. But their parameters, namely, their lengths, bend angles, edge angles and the drift space lengths in between, all are set as variables and chosen by one more fitting so as to reproduce the linear transport matrix, with its elements set as goal functions and a few more relations (total length, final deflection and displacement) as limiting conditions. The fitting is by "Variable Metric Method for Minimization"[3], which minimizes the gradient of the "function of goodness" in steps of accumulative multidimensional space metric variation and has been used successfully in lattice function fitting. The number of variables need not equal to that of goals. The accuracy of fitting is comparable to that in the preceding measurement and matrix calculation. Because of errors in the preceding steps and inner contradictions between a few goals and conditions, the fitting function of goodness cannot and need not be down to zero. One can choose weight factors to make the more significant goals reached sufficiently close.

Modeling calculations are done for the BESSY-1 operational 3.2 T WLS (and a planned 6 T WLS). As sketched in Fig.1, the modeling assumes the bending field is compressed into 3 short magnets, sitting at the turning points of the orbit, with the central field about 10% weaker than the maximum field, and the bend angles about 20% smaller than the maximum deflection angle. The edge angles are so chosen as to produce the focusing strengths more properly. Fig.2 shows the shape of the inclined magnet faces with the angles enlarged 10 times. Because of the horizontal defocusing, the magnet poles are not rectangular, but look like curved edge faces, the "curvatures" of which are convex outward so as to resemble the isomagnetic surfaces of the field. This is in accord with the concept of edge effect, and with existence of 2nd order field variation with x. To follow the tracking results, the parameters of the models are slightly assymetric about the midpoint, and the system is not completely achromatic.

Calculations reveal that the WLS effects on the synchrotron radiation integrals and consequently on the related ring parameters are evaluated in the models in close approximations. The higher order effects are also studied. A key point is to "condense" the matrices into "thin lenses" at the WLS midpoint to get rid of the influence of path length, leaving only a few significant terms outstanding. Of them, the chromatic terms (variance of focusing with energy deviation) of the WLS are well
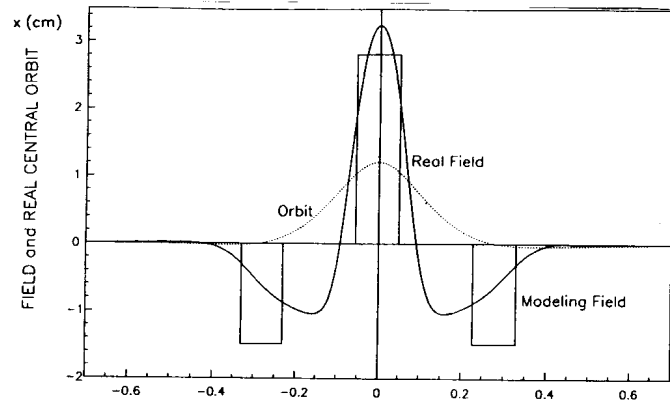
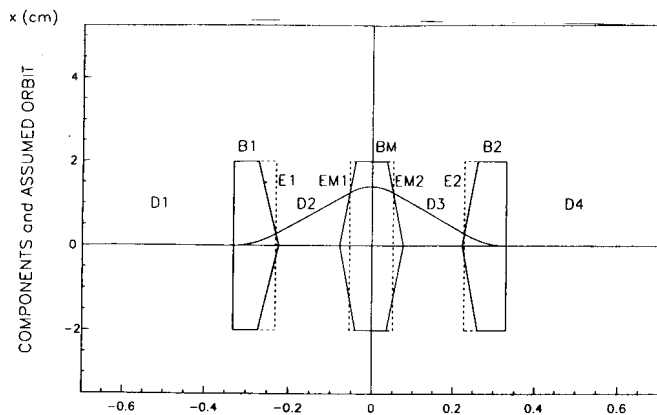Figure. 1. WLS Modeling, Field and Real Orbit

Figure. 2. Fig.2 Modeling Magnets and Assumed Orbit

V. CONCLUSION

Starting with the field data (from measurement or prediction), the methods and computer codes mentioned in this note can do calculations for the WLS effect on the beam and for compensation considerations. Either the simple estimations or the complicated calculations agree with the reality satisfactorily. As for the modeling method, so far as the 1st order effect is concerned in the first place, it can gain its user an access to all the existing accelerator physics computer codes without fear of being misled. There is yet much room for further work in WLS higher order effect study.

VI. REFERENCES

[1] Z.Liu, A.Zhao, Modeling of WLS Field with Piecewisely Constant Magnets, BESSY Technical Note, Nov. 1994.

[2] Z.Liu, A.Zhao, A BESSY-1 6 Tesla WLS Effect Compensation Scheme, BESSY Technical Note, Nov. 1994, also presented at PAC'95.

[3] W.C.Davidson, Variable Metric Method for Minimization, ANL-5990, Rev.2. Also see computer code ⟨COMFORT⟩, developed at SLAC.

represented by the models; while the geometric aberrations held poorly. The latter may be improved by 2nd order parameter fitting with curved edges, or by inserting a high order thin lens at the WLS midpoint. The coupling terms (skew magnet generated terms) are trivial.

There are other methods formerly used in WLS modeling. 1) One is the 3-rectangular-magnet modeling, usually having the field integral of a pole or the maximum bend angle the same as in the real field. In this way the method has the 0th order effect to some extent reproduced, rather than the 1st order. Selection of the field values is still somehow arbitrary. The WLS effect may easily be either overestimated (if at peak field) or underestimated (if at full length). With only rectangular magnets, no way to take into account the horizontal defocusing, which might disturb the lattice quite seriously. Therefore, such a modeling may have the effect distorted. 2) Some computer codes allow transport matrix directly used as a magnet type. It is fine for the tunes and the lattice functions all over the ring. But the functions and beam parameters inside the WLS are not presented, not even their approximations; neither are the influences on the synchrotron radiation integrals; nor the 2nd order chromatic effect of WLS included without special treatment.