# EXPECTED RADIATION SPECTRA OF A 30-M LONG UNDULATOR IN SPRING-8

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## I. INTRODUCTION

In addition to 44 straight sections of 5 m long, SPring-8 storage ring comprises four long straight sections of length 30 m. These sections are prepared for insertion devices and highly brilliant quasi-monochromatic photon source is available for experiments in hard X-ray region. In a simple minded expectation, the brilliance of the undulator radiation is proportional to the square of the number of the undulator period. So that we expect a 60 times higher brilliance from a 30 m long undulator compared with a 4 m long undulator. According to a numerical calculation, however, it turn out that the brilliance is not so high as expected. This is because the brilliance is closely related to the emittance of the electron beam.

In this note we study the spectral properties of the synchrotron radiation from a 30 m long undulator. The decreasing rate of the peak intensity of 30 m long undulator in terms of broadening of spectral band width by the electron beam emittance is larger than that of 4 m. To see this fact, we investigate the dependence of radiation intensity on the electron bean emittance. In addition, the dependence on the transverse emittance coupling, i.e. the shape of the electron beam, is also studied.

The spectral formula used in the above investigation is derived by means of the far field approximation. Since in the present case the undulator is relatively long compared to the observing distance, we estimate the near field effect in the radiation intensity.

## II. SPECTRAL PROPERTIES OF RADIATION FROM 30 M LONG UNDULATOR

### *A. Spectrum of 30 m long undulator*

In calculating the radiation from 30 m long undulator, electron beam parameters are assumed to be the design values of the storage ring, which are listed Table I.

Table I
Storage ring parameters

| | |
|---|---|
| Beam energy | 8 GeV |
| Beam current | 100 mA |
| Total emittance | 6.89 nm rad |
| Transverse emittance coupling | 10 % |
| Horizontal beta function $\beta_{x0}$ | 17.5 m |
| Vertical beta function $\beta_{y0}$ | 19.5 m |

We also assume the long planer undulator to be a idealized one with the reasonable parameters given in Table II.

The spectral formula of the radiation from a single electron in a planer undulator is well-known [1], [2], [3]. In order to incorporate the effect of finite size and divergence of electron beam into the spectral formula, we make convolution of spectral formula with electron distribution [4].

Table II
Undulator parameters

| | |
|---|---|
| Period length | 3 cm |
| Number of periods | 1000 |
| $K$-parameter | 1 |

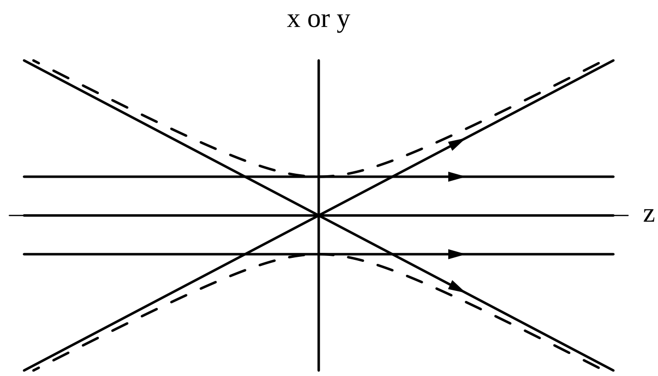


Figure 1. Each straight line corresponds to a mean trajectory of an electron in an undulator. The outer dotted curves indicate the envelope.

Since a mean trajectory of an electron in an undulator is a straight line, an electron beam is described by a gathering of straight lines (see Fig. 1). Giving a transverse position $(x_0, y_0)$ and a divergence $(x'_0, y'_0)$ of an electron at some location in an undulator, we can designate the mean trajectory. The distributions of electron positions and divergences are determined by the emittance and the Twiss parameters. In usual we place a waist of a beam envelope on the center of an undulator so that the Twiss parameters in the undulator are described by the beta functions at the center of the undulator $\beta_{x0}$ and $\beta_{y0}$. Hence the distributions of electron trajectories in an undulator are given by the transverse emittances and the beta functions $\beta_{x0}$ and $\beta_{y0}$.

Figure 2 shows the angular flux density of the synchrotron radiation from the 30 m long undulator at an observing distance 50 m from the center of the undulator. To compare the spectrum with that of 4 m long undulator, we show its angular flux density in Fig. 3.

The peak value of the fundamental mode of 30 m undulator is only about ten times larger than that of 4 m one, which is expected to be 56.5 $(= 1000^2/133^2)$ times larger in case of a single electron. This is supposed to be caused by the broadening effect of spectral band width due to transverse spread of electron beam. To see this fact, we investigate the dependence of the intensity spectrum of the undulator radiations on the transverse emittance.

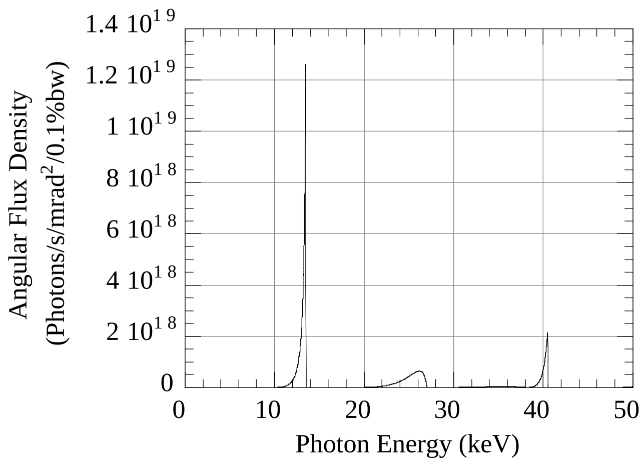


Figure 2. An expected radiation spectrum from the 30 m long undulator up to the third harmonics.

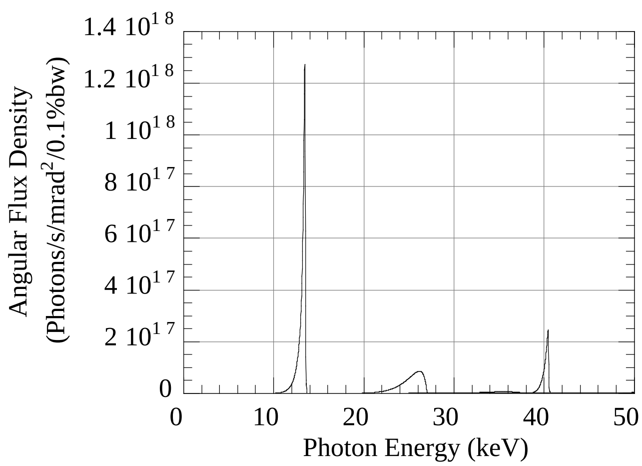


Figure 3. An expected radiation spectrum from the 4 m long undulator up to the third harmonics.

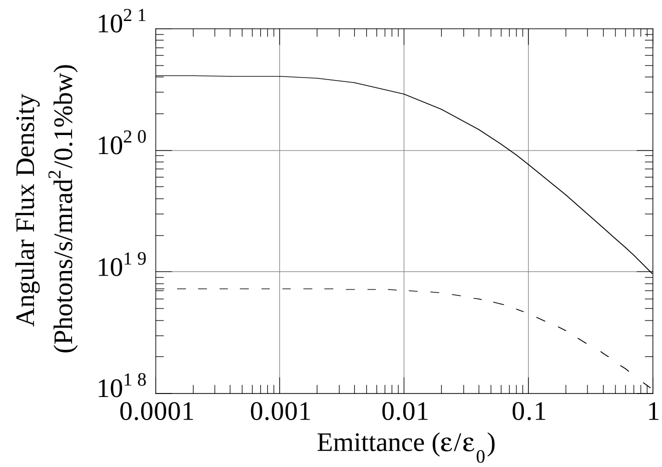


Figure 4. Effect of an electron beam emittance on the peak intensity of fundamental undulator radiation. The emittance is normalized by the design value. The solid line indicates the radiation from 30 m long undulator and the dashed line that from 4 m.

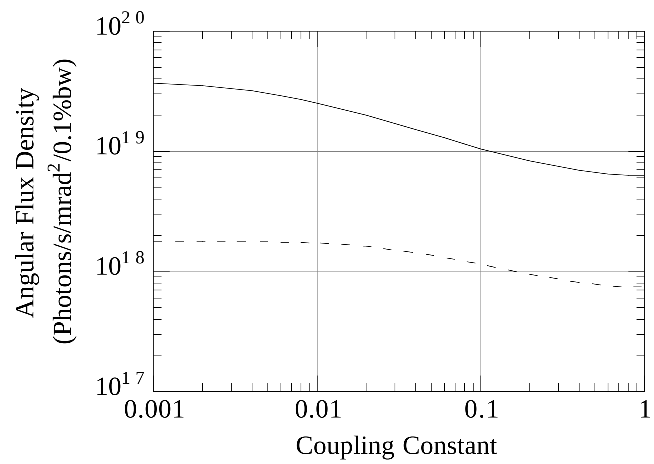


Figure 5. Effect of the transverse emittance coupling on the peak intensity of fundamental undulator radiation. The solid line indicates the radiation from 30 m long undulator and the dashed line that from 4 m.

### *B. Dependence on transverse emittance*

Figure 4 shows the emittance dependence of the peak radiation intensities of 4 and 30 m long undulators. We see that at a low emittance the intensity ratio is 56 as expected and that the intensity of the radiation from 30 m long undulator decreases more rapidly than that from 4 m. This is approximately explained as follows.

At a sufficiently low emittance, the broadening effect due to emittance is invisible and then the peak of the radiation intensity scarcely changes. On the other hand, at a large emittance the band width of the radiation spectrum is proportional to the electron emittance. The critical value of the emittance is determined by the band width of radiation spectrum from a single electron. Since the band width of the radiation spectrum of a single electron is inversely proportional to number of undulator periods, the longer the undulator is, the smaller the critical value becomes. Hence, the decreasing rate of the peak intensity is larger for a longer undulator.

For the purpose of utilizing the long straight section efficiently, it is preferable to achieve an electron beam with lower emittance.

### *C. Dependence on the coupling of transverse emittances*

In the previous subsection, we saw that the finite emittance significantly reduces the intensity spectrum of undulator radiation. Now we investigate the effect of the electron beam shape, i.e. the transverse emittance coupling, on the spectrum of undulator radiation. In Fig. 5 we plot the peak intensity of the first harmonic radiation versus the coupling constant of horizontal and vertical betatron oscillations. Figure 5 shows that the peak intensity of flat electron beam is larger than that of round electron beam.

As shown in the previous subsection, the smaller the emittance is, the larger the peak value of the intensity spectrum is. If one makes the electron beam flat with keeping the total emittance, the horizontal emittance increases while the vertical one decreases. The increment of the peak intensity due to lowering the vertical emittance overcomes the decrement owing to height-

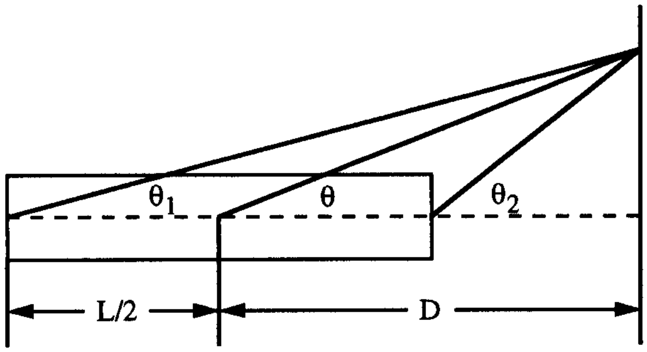


Figure 6. Geometry of near field case.

ening the horizontal emittance. Hence the flatter beam radiates the photon flux with the stronger peak intensity.

*D. Near field effect*

In the above investigation we used the far field approximation, where we assume the observing distance to be sufficiently large compared to the undulator length. If the observer is located at a finite distance from the center of the undulator, the observation angle varies as the electron travels from the entrance of the undulator to the exit (see Fig. 6). The change of the observation angle gives rise to an additional phase difference to the far field approximation. In the case of the observing distance comparable to the undulator length, the near field effect should be included into the radiation with a large angle between the observation direction and the electron mean trajectory.

Since electron beam has a finite divergence, some electron trajectory possesses a large angle to the observation direction. In the case of 30 m long undulator the length is relatively large compared to the observation distance, so that we investigate the near field effect on the undulator radiation of an electron beam with finite emittance. Fig. 7 shows the angular flux density at an observation distance 50 m, where the solid curve indicates the far field approximation and the circles correspond to the intensity with including near field effect. The spectrum shows that at an observation distance 50 m the near field effect is negligible.

Although the near field effect is remarkable at a neighborhood of the undulator exit, we can ignore the effect at a practical observing distance.

## III. CONCLUSIONS

In this note we have studied the spectral properties of 30 m long undulator in SPring-8. The investigation of the dependence of the radiation intensity on the electron beam emittance shows that low emittance is preferable for the long undulator. It is also shown that peak intensity radiated by flat electron beam is stronger than that from round one. Although the the length 30 m of the undulator is relatively long, the near field effect can be ignored for our practical observing distance about 50 m.

## References

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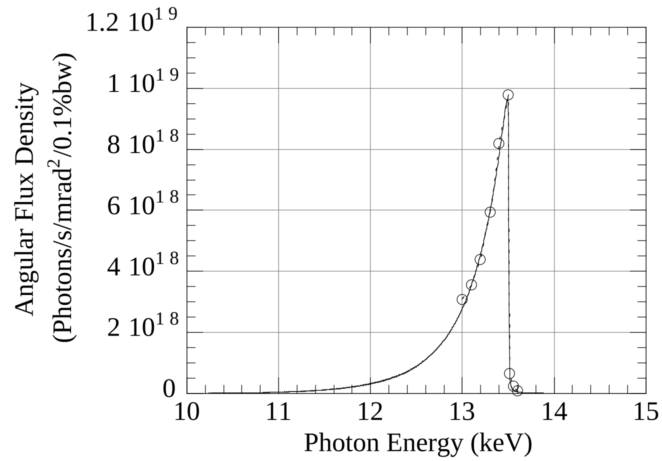


Figure 7. Spectral intensity observed at a distance 50 m. The solid line corresponds to the far field approximation and the circles indicate the intensity including the near field effect.

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