TREATMENT OF WIGGLER AND UNDULATOR FIELD ERRORS IN TRACKING CODES

W. Decking*, O. Kaul, H. Nesemann, J. Roßbach
Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22603 Hamburg, Germany

Abstract

Wigglers and undulators produce intense synchrotron radiation, used for dedicated experiments or to control beam dimensions. To obtain tolerance criteria for wiggler and undulator field errors, this paper presents a treatment which takes into account the special geometry of wigglers and undulators and the undulating beam trajectory. This is done by creating a nonlinear transfer map of the beam motion through the (measured) magnetic fields. To verify the method, results of tracking calculations using data from magnetic field measurements are compared with dedicated experiments performed at DESY's synchrotron radiation source DORIS III. Measurements of tune shifts with closed orbit amplitudes and of the dynamic acceptance agree with the results of the tracking.

I. INTRODUCTION

The treatment of wigglers[1] in accelerator theory and numerical beam dynamics investigations is focused on the proper description of the combined nonlinear effects of the oscillating fields and particle trajectories. Field errors are commonly described like in 'normal' lattice magnets, i.e by adding appropriate multipole kicks describing the field error of the magnets.

This procedure does not account for the influence of field errors acting along the undulating particle path. We will present in this paper a possible treatment of this problem, based on a sufficient modeling of the wiggler magnets and their field errors by a current sheet model [1]. The tracking calculations are performed using a numerical evaluated generating function [2][3] of the wiggler, which guarantees the simplecticity of the tracking calculations. The combination of these methods allows the treatment of periodicity errors and transversal field gradient errors as well as the investigation of other features of the wiggler fields like the influence of the pole width.

The numerical analysis is exemplarily done for the DORIS III storage ring, DESY's state of the art 1st generation synchrotron light source. It is equipped with 10 wigglers and undulators, covering 10 % of the ring length (for more details see [4][5]). The dynamic acceptance and tune shifts with closed orbit amplitudes in the wiggler have been investigated at DORIS (see also [6]) to verify the numerical results obtained for transversal field gradient errors and the pole width influence.

* e-mail: mpywin@ips102.desy.de

[1] We will in general not distinguish between wigglers and undulators in this text.

II. NUMERICAL METHOD

A. The generating function

The concept of generating functions is used to obtain a symplectic transformation of the particle coordinates through the wiggler. An appropriate generating function $F(x_i, p_{x,f}, y_i, p_{y,f})$ maps the canonical variables $x_i, p_{x,f}, y_i, p_{y,f}$ into $x_f, p_{x,i}, y_f, p_{y,i}$, where x, y are the transversal coordinates, p_x, p_y the canonical impulses and the suffixes i, f symbolize initial respective final coordinates. The generating function is expanded in a Taylor series. By tracking a set of particles distributed in phase space through the magnetic field to be investigated, the coefficients of the expansion can be evaluated numerically from the initial and final coordinates. For more details on this method see [2][3]. This procedure guarantees fast, symplectic tracking and allows in principle the investigation of any magnetic field.

B. The current sheet model

The current sheet model was invented to calculate the magnetic fields of pure permanent magnet wigglers [1]. Due to the special properties of the permanent magnet material - a magnetic permeability of ≈ 1.0 over a large range of the magnetization curve - the field of a permanent magnet block can be calculated by assuming current sheets only on the surfaces of the block. Field contributions of different blocks can be superimposed leading to the total field produced by a permanent magnet array. This model is extended to be used also for hybrid wigglers by scaling the remanent fields and block dimensions to fit the measured magnetic fields of the considered devices. Transversal field gradients, measured as multipole contents of the first field integrals, are thus distributed over the whole device length. The three dimensional field distribution producing this multipole content is now taken into account.

The current sheet model can also be used to investigate the effects of pole width by decreasing the horizontal dimensions of the blocks or to simulate periodicity errors by varying the longitudinal block positions and remanent fields.

III. NUMERICAL ANALYSIS OF FIELD ERRORS

A. Periodicity error

The influence of periodicity errors of a DORIS wiggler and an undulator [7] has been investigated by evaluating the dynamic acceptance in the presence of the wiggler/undulator and the machine sextupoles. The error is parameterized by the rms-variation of the magnetic field maxima $\Delta B_{rms} = \left(\frac{B_{y,max.}}{<B_{y,max.}>}\right)_{rms}$ and the rms-variation of the period length $\Delta\lambda_{rms} = \left(\frac{\Delta\lambda}{\lambda}\right)_{rms}$, measured from maxima to maxima of the magnetic field. The chosen error sets ensure that the first and second field integrals are small compared to those usually measured in wigglers. The results show no decrease of dynamic acceptance for $\Delta B_{rms} = 6.0\%$ and $\Delta\lambda_{rms} = 2.0\%$. Values of $\Delta B_{rms} = 15.0\%$, $\Delta\lambda_{rms} = 3.0\%$ give a slight decrease of $\approx 15\%$. The requirements set by synchrotron radiation users due to the quality of the emitted light are much stronger, so periodicity errors should not be important under beam dynamics aspects.

B. Pole width

Although not a field error, the effects of pole width are mentioned since they can be treated with the same tools as field errors. The half pole width is measured in units of the beam size σ, and the field properties are characterized by the peak field, which decreases with decreasing pole width, and the relative field decrease at a horizontal position corresponding to 10σ. Again the dynamic acceptance was evaluated for a DORIS wiggler, with the results given in table I. The acceptance decreases

Table I
Relative dynamic acceptance for DORIS with a wiggler with different pole width inserted

half pole width [σ]	$B_{y,max.}$ [T]	$\frac{\Delta B_y}{B_{y,max.}}$ at $10\ \sigma$ %	acceptance [arbitrary units] horizontal	vertical
24	1.16	1.7	1.0	1.0
19	1.15	6.1	1.0	1.0
14	1.12	19.6	0.85	0.88
9.5	1.01	46.5	0.71	0.88
5	0.75	72.0	0.71	0.88

at a half pole width between 14σ and 19σ, but even a half pole width of 5σ leads only to a 30% reduction of the acceptance. A half pole width of 20σ ensures no influence on the beam dynamics due to the finite width of the poles.

C. Transversal field gradient errors

We have chosen the asymmetric wiggler (ASYH) as an example for the investigation of field gradient errors. The ASYH has an asymmetric field distribution with weak but long positive field contributions and strong and short negative ones along the wiggler axis. The field integral measurements showed strong sextupole (b_2) and decapole (b_4) multipole components [8], the former being four times stronger then the normal storage ring sextupoles.

The ASYH was modeled by the current sheet model with varying block dimensions, leading to the following multipole coefficients for the first field integral:

name of model	b_2 $[\mathrm{T/m}]$	b_4 $[\mathrm{T/m^3}]$	corresponding multipole
ASYH-0	0.03	80	
ASYH-1	4.6	4000	multipole-1
ASYH-2	20	18000	multipole-2

The ASYH-2 corresponds roughly to the original device before a correction with pole shims, while the multipole contents of the ASYH-1 are similar to the maximum acceptable values for these components.

The tune shift with horizontally displaced closed orbit in the wiggler as well as the dynamic acceptance has been calculated:

- The generating functions for the three models.
- The generating function for the ASYH-0 and a thin lens multipole kick of the same strength as the field integrals (multipole-1 or multipole-2).

The tune shift in dependence on the horizontally displaced closed orbit is shown in figure 1 for the ASYH-0 with multipole-1 and the ASYH-1. It is in good agreement in the inner parts of the wiggler but starts to differ at amplitudes of ≈ 25 mm, which is the border of the good field region for both the numerical investigations and the real wiggler.

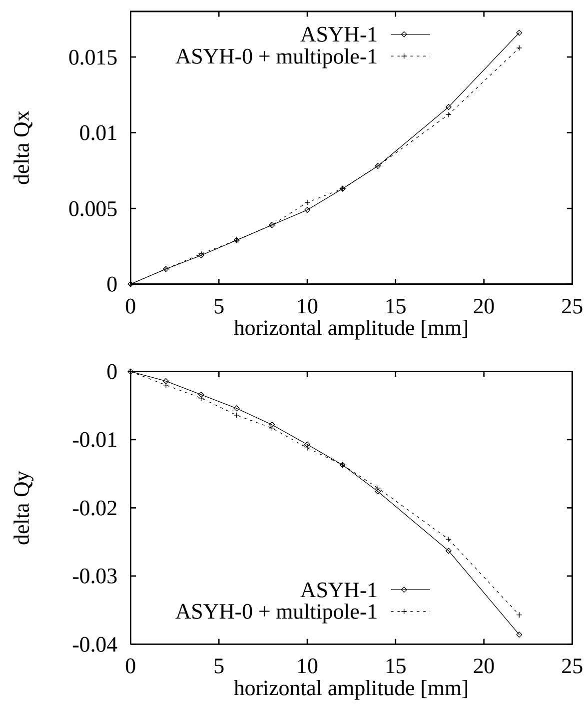

Figure 1. Horizontal (upper) and vertical (lower) tune shift versus horizontal closed orbit amplitude within the wiggler.
solid line: ASYH-0 and multipole-1; dashed line: ASYH-1

The dynamic acceptance, measured as the stable horizontal and vertical amplitudes is displayed in figure 2. The dynamic acceptance remains unchanged for the ASYH-0 and the ASYH-1. The ASYH-2 decreases the dynamic acceptance down to 3 mm, a value which would lead to lifetimes of less than 1 hour. The dynamic acceptance calculated by the generating functions for the ASYH-1 and ASYH-2 does not differ from those obtained from the ASYH-0 and multipole lens.

From this analysis follows that the multipole description of the integrated fields is a sufficient treatment of transversal gradient errors in wigglers, if the field integral evolves (more or less) continually along the longitudinal axis.

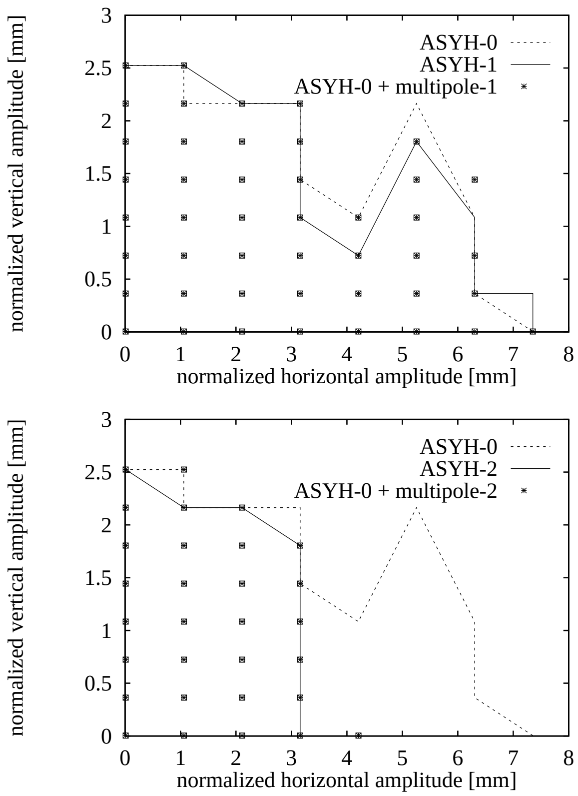

Figure 2. Stable amplitudes (normalized to $\beta = 1m$) for the ASYH-1 (upper) and ASYH-2 (lower). The envelope of the stable amplitudes for the ASYH-0 is plotted as dashed line, for the ASYH-1 and ASYH-2 as solid line. The stable amplitudes for ASYH-0 and appropriate multipole lens (multipole-1 or multipole-2) are plotted as filled boxes.

IV. EXPERIMENTAL VERIFICATIONS

The tune shift with horizontal closed orbit amplitude within the wiggler has been measured for the ASYH. The wiggler was in a different set up during the measurements with respect to the cases investigated above. The measured multipole coefficients for this set up were $b_2 = 0.75\mathrm{T/m}$ and $b_4 = -6500\mathrm{T/m}^3$, which agrees with the ones obtained from a fit on the tune shift measurements (see figure 3).

Although the dynamic acceptance has not been measured in dependence on the gap, i.e. the multipole strength of the ASYH, the observations made on lifetime and free area in the tune space agree with the multipole limits set by the numerical investigations.

The dynamic acceptance calculated with all wigglers including field errors treated as multipole lenses agrees with the measured ones. The vertical acceptance should not decrease due to the closing of the wigglers. It is determined by the vertical geometric aperture. The horizontal acceptance decreases slightly due to the wiggler field errors. Tracking calculations without field errors show no influence of the wigglers on the dynamic acceptance at all.

A dedicated experiment on the pole width influence was performed by mounting a wiggler off axis in the storage ring. The closed orbit in the wiggler was changed with a bump involving four steering magnets. Thus the beam center was moved from the wiggler axis up to 30 mm outward, corresponding to 60% of the pole width. The outermost position corresponds to a half pole width of 10σ. No vertical and only a slight horizontal dynamic acceptance reduction was observed, verifying the small effect expected due to the finite wiggler pole width.

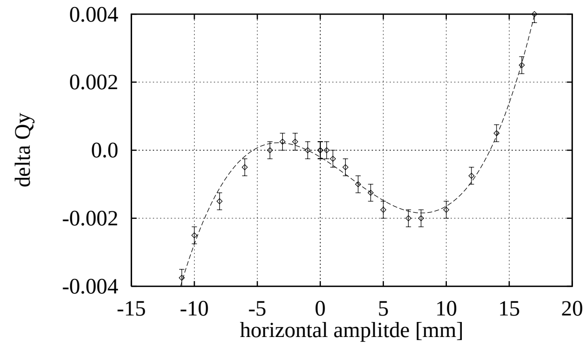

Figure 3. Measured tune shift with horizontal amplitude in the wiggler ASYH. The multipole coefficients obtained from the fit are $b_2 = 1.0 \pm 0.25\mathrm{T/m}$ and $b_4 = -6000 \pm 1000\mathrm{T/m}^3$.

V. CONCLUSION

Effects of wiggler field errors and other field characteristics have been investigated with the current sheet model and numerical evaluated generating functions. Limits for the periodicity error and the wiggler pole width could be obtained. The field gradient error is well enough described by the usual multipole treatment of the field integrals, as has been shown by numerical investigations and is verified by the experimental results.

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