SORTING STRATEGIES FOR THE LHC BASED ON NORMAL FORMS

W. Scandale, M. Giovannozzi, R. Grassi and E. Todesco *
CERN–1211 GENEVA 23

Abstract

Sorting strategies for superconducting dipoles are discussed, in view of maximizing the dynamic aperture in hadron colliders like the CERN LHC. Quality factors based on the perturbative tools of nonlinear maps and normal forms are evaluated with tracking simulations. The best of them is retained and maximized by an appropriate permutation of the magnets. The effectiveness of the proposed ordering is checked again through tracking. This technique is used to sort the sextupolar errors in a lattice that contains a small number of LHC-like cells.

I. INTRODUCTION

One of the main issues in the construction of large hadron colliders like the CERN LHC is the effect of the nonlinear field-shape errors. They fluctuate randomly from magnet to magnet, and considerably reduce the stability domain of the circulating particles. Installing the magnets in ordered sequences can provide a self compensation of the random imperfections i.e. an increase of the dynamic aperture. Due to the huge number of magnet permutations it is not feasible to characterize them by computing the related dynamic aperture. Therefore we propose to use quality factors (QF) based on analytical tools. Intuitive rules based on local compensation and symmetry considerations, which have been shown to be effective in the case of the LHC lattice [1], [2], have also been considered.

In this paper we present strategies based on analytical tools (normal forms [3]) and numerical simulations (element-by-element tracking) [4]. Firstly we introduce three QF: the norm of the nonlinear part of the map, the norm of the amplitude-dependent tuneshift, computed through nonresonant normal forms, and the resonance strength evaluated through resonant normal forms. Secondly we determine numerically which QF has the best correlation with the dynamic aperture for our accelerator model. Thirdly we use the QF to select the best ordering among a limited set of magnet permutations, chosen either randomly or using local compensation rules. As a final check we compute with tracking the dynamic aperture of our selected case. This procedure is repeated for 100 different realizations of the random errors.

II. QUALITY FACTORS

The indicators of nonlinearity, that we use as QF are listed below with the conventions of Ref. [3].

Q_1: **Norm of the map** Terms of different orders are added, neglecting possible compensations; the amplitude A gives the weight for the various contributions.

$$Q_1(A,N) = \sum_{n=2}^{N} A^n \sum_{j_1+j_2+j_3+j_4=n} \sum_{i=1,2} |F_{i;j_1,j_2,j_3,j_4}|. \quad (1)$$

Q_2: **Norm of the tuneshift** A nonresonant normal form truncated at the order N provides the perturbative series at the order $M=(N-1)/2$ for the sum of the squares of the tuneshifts:

$$t_{2M}(\rho_1,\rho_2) = \frac{1}{2}\left[\left(\sum_{i=1}^{M}[\nu_x(\rho_1,\rho_2)]_i\right)^2 + \left(\sum_{i=1}^{M}[\nu_y(\rho_1,\rho_2)]_i\right)^2\right] \quad (2)$$

Q_2 is obtained by averaging this quantity over the invariant amplitude A:

$$Q_2(A;M) = \sqrt{\frac{2}{\pi}\int_0^{\pi/2} t_{2M}(A\cos\phi, A\sin\phi)\, d\phi}. \quad (3)$$

Q_2 is to be used with caution, since in absence of detuning (i.e. $Q_2 \circ 0$) all the resonances are unstable.

Q_3: **Norm of the resonances** The strength of a single resonance $[p,q]$ of order $r=p+q$ can be computed by taking the norm of the resonant part of the interpolating Hamiltonian, truncated at an order L such that $L \geq r$:

$$Q_3([p,q],A;L) = \sum_{l\geq 1} \sum_{2(k_1+k_2)+lr\leq L} |h_{k_1,k_2,l}| A^{2(k_1+k_2)+lr}. \quad (4)$$

In all the three QFs, the amplitude A is fixed to the estimated value of the dynamic aperture.

III. RESULTS

The model Our analysis has been carried out on a simplified lattice made of 8 LHC cells and a phase trombone. Each cell has 6 dipoles and a phase advance of 90^o. The linear tunes are $\nu_x = 2.28$ and $\nu_y = 2.31$. The peak value of the orbit function is $\beta_{max} = 169m$. The chromaticity sextupoles are ignored. Only the random part of the normal sextupolar errors in the dipoles is considered. Its integrated gradient is distributed according to a gaussian truncated at three sigma, where $\sigma = 8.5872 \times 10^{-3} m^{-2}$ is the expected value for the LHC dipoles.

QF correlation We have computed the correlation between the dynamic aperture and the QF's for 100 realizations of the random errors. The dynamic aperture is computed over 1000 turns and it is expressed in meters normalized at $\beta_{max} = 169m$. The norm of the map Q_1 is evaluated up to order $N=6$; the tuneshift Q_2 at the order $M=2$, and Q_3 has been evaluated for resonances up to order seven, with $L \leq 7$. The parameter A is equal

* INFN, Sezione di Bologna, Via Irnerio 46, I-40126 Bologna.

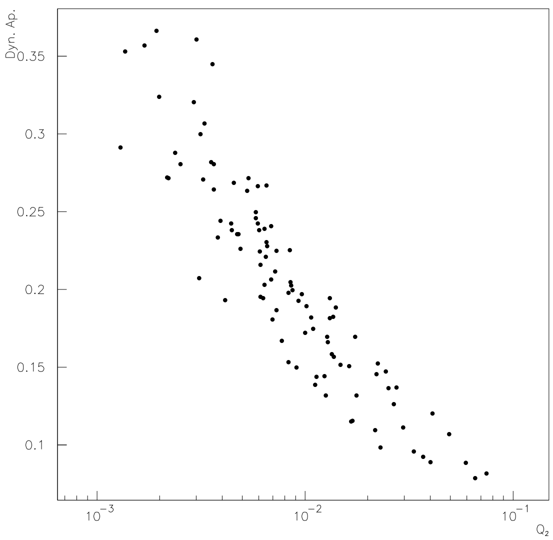

Figure 1. Correlation between the quality factor Q_2 and the dynamic aperture for 100 seeds.

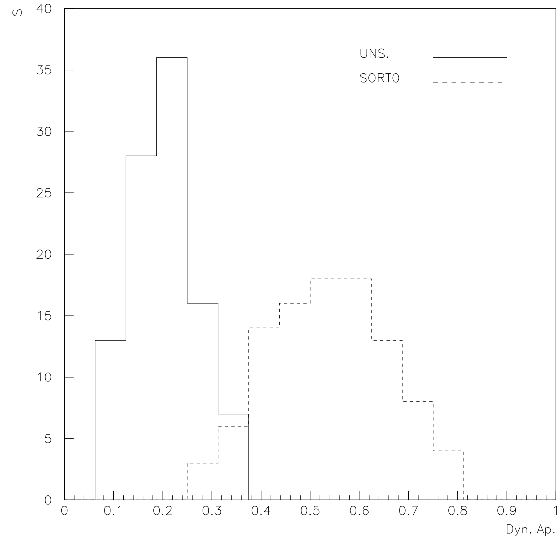

Figure 2. Comparison between the distribution of dynamic aperture for the unsorted machine and SORT0.

to the average dynamic aperture of the 100 unsorted machines, i.e. $0.2 \quad m$.

The tuneshift norm Q_2 shows the best correlation with the dynamic aperture (Fig. 1), while the norm of the map Q_1 has a poor correlation. Among all the resonances the $[3, 0]$, which is the first resonance excited by sextupoles, shows a rather good correlation, and has been used for further tests.

Sorting and tracking check We consider two optimization strategies and for each of them we use two different rules of permutation of the dipoles. The first strategy (SORT1) is based on the norm of the tuneshift Q_2, the second one (SORT2) on the strength of the resonance $[3, 0]$. In the first rule (SORT11 for Q_1, SORT21 for Q_2) we consider 500 random permutations. In the second rule (SORT12 for Q_1, SORT22 for Q_2) we split the 48 dipoles in 24 pairs such that in each pair the sum of the sextupoles errors is minimized, to create a 'first-order local compensation'; then, we consider 500 random permutations of the 24 pairs. The number of permutations P is fixed to 500 as a result of a compromise. For larger values of P one finds better machines, however, the improvement saturates quite rapidly, probably due to the fact that the correlation between the QF and the dynamic aperture is better for bad machines than for good ones (see Fig. 1). This means that the QF easily recognizes bad machines, but it is not very efficient in selecting good machines. The effect of sorting rules based on local compensation as described in [1] has also been computed for comparisons (SORT0).

In Fig. 2 we consider the dynamic aperture for 100 unsorted seeds, to which we apply the sorting rule SORT0. The dynamic aperture increases by a factor 2.7.

In Fig. 3 we show the distributions of the dynamic aperture for 100 seeds for the rule SORT12 and for SORT22. With the rule SORT12 the gain is in average of a factor 3.1 and for the worst machines of a factor two. With SORT22 the gain is lower.

In any case, the permutations of pairs of dipoles has been shown to be more efficient that the random permutations. From this we conclude that, in our lattice model, the tuneshift minimization over pair permutations, is to be considered the most efficient algorithm to improve the beam stability. However, we have indications that this conclusion cannot be simply extrapolated to any lattice model.

IV. EFFECT OF THE SORTING PROCEDURE ON THE GLOBAL DYNAMICS

A check of the effect of the sorting procedure has been carried out through the analysis of the dynamics both in phase space and in frequency space, according to the numerical techniques of frequency analysis originally developed for celestial mechanics [5]. A stability diagram and its related tune footprint gives a description of the stability domain in phase space and in frequency space. This second diagram provides the tune distribution, and brings into evidence the most dangerous resonances and their effect on the stability of motion.

We consider particles with initial conditions given by $(r \cos\alpha, 0, r \sin\alpha, 0)$, with $r \in [0, R]$ and $\alpha \in [0, \pi/2]$, and we track them over 1000 turns. If the motion is stable, we compute the related nonlinear frequencies as the average phase advance per turn.

In Fig. 4 we show the frequency distribution for the random machine with an average value of the dynamic aperture, while in Fig. 5 we consider the same machine sorted with the rule SORT12. The analysis of the Figs. 4 and 5 leads to the following observations.

- Tune footprint diagrams show that the ordering procedure not only minimizes the tuneshift, but also changes the low order tuneshift coefficients so that different zones of the frequency space are occupied by the random and sorted ma-

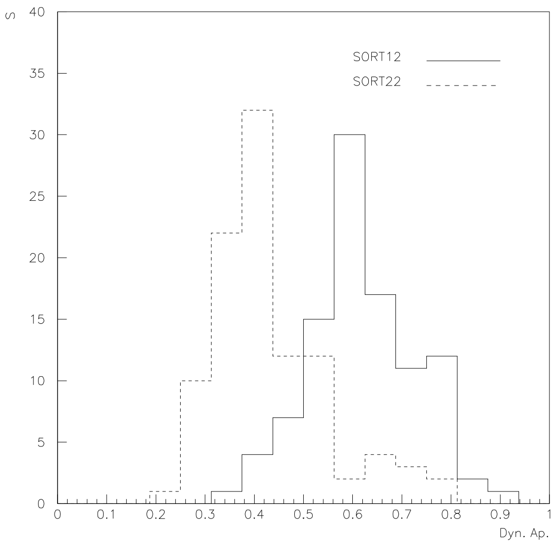

Figure 3. Comparison between the distribution of dynamic aperture for the SORT12 and SORT22.

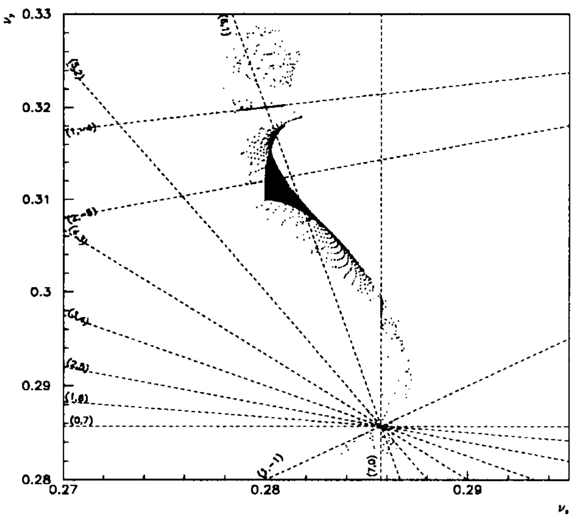

Figure 4. Tune footprint for an average unsorted seed.

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- In the sorted machine the effect of resonances is stronger and more clearly visible: this happens because, having minimized the tuneshift, the islands become larger. Notwithstanding this effect, the stability domain of the sorted machine is three times bigger than the unsorted one. The island width is clearly shown in the tune footprint as an high density of initial conditions on resonance lines, and an empty region around them (e.g. $[1,-4]$ and $[7,0]$ in Fig. 5). On the other hand, some resonances seem to be not excited (e.g. resonances $[2,-5]$ and $[6,1]$ in the same Figure).
- Although the resonance $[3,0]$ is very far from the tune footprint, is shows the best correlation with the dynamic aperture. This fact needs deeper investigations.

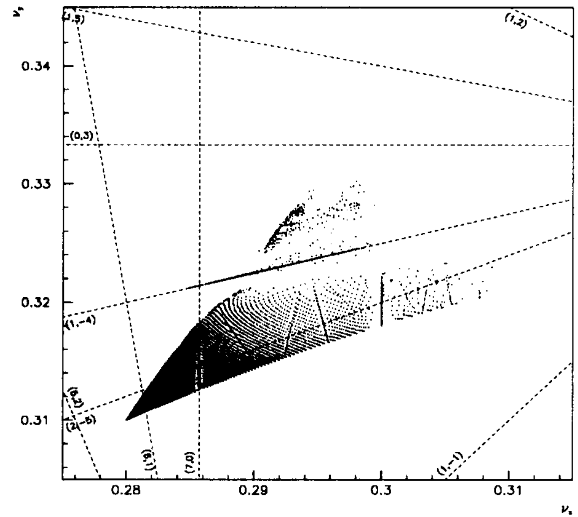

Figure 5. Tune footprint for an average seed sorted according to the rule SORT12.

V. CONCLUSIONS

We have defined a sorting strategy which has the fundamental property of being very flexible. The proposed approach is based on a mixed technique which exploits both tracking simulations for determining the best quality factor, and analytical techniques to evaluate in a fast way the best permutation of the magnets. This technique can be integrated by other intuitive criteria based on the symmetries of the lattice. The outlined approach has been successfully applied to an LHC cell lattice, finding gains which are very good also compared to other known methods. Although we applied our analysis to a simplified cell lattice with only sextupolar errors, there are no restrictions to extend it to a realistic lattice including a set of arbitrary multipolar imperfections.
It remains to be checked if the improvement due to the sorting persists when the operational conditions are changed or when long-term stability is considered.

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