Skew Chromaticity in Large Accelerators

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Abstract

The 2-D "skew chromaticity" vector **k** is introduced when the standard on-momentum description of linear coupling is extended to include off-momentum particles. A lattice that is well decoupled on-momentum may be badly decoupled off-momentum, inside the natural momentum spread of the beam. There are two general areas of concern:

1) The free space in the tune plane is decreased.
2) Collective phenomena may be destabilised.

Two strong new criteria for head-tail stability in the presence of off-momentum coupling are derived, which are consistent with experimental and operational observations at the Tevatron, and with tracking data from RHIC.

I. OFF-MOMENTUM COUPLING

A skew quad i is represented by a 2-D vector $\mathbf{q}_i$ with components along the orthogonal axes $\widehat{\mathbf{a}}$ and $\widehat{\mathbf{b}}$,

$$\mathbf{q}_i \;=\; \frac{\sqrt{\beta_x \beta_y}}{2\pi\ f}\left(\cos(\phi_y-\phi_x)\,\widehat{\mathbf{a}} \;+\; \sin(\phi_y-\phi_x)\,\widehat{\mathbf{b}}\right) \tag{1}$$

where f is the focal length, β_x and β_y are the beta functions, and ϕ_y and ϕ_x are the betatron phases. The closest approach of the eigentunes Q_- and Q_+ is given by the length of **q**, the sum of all skew quad vectors[1], [2].

$$\Delta Q_{min} \;=\; |\mathbf{q}| \;=\; \left|\sum_i \mathbf{q}_i\right| \tag{2}$$

The eigentunes depend on the design tunes, Q_x and Q_y,

$$Q_\pm \;=\; \frac{1}{2}(Q_x+Q_y) \;\pm\; \frac{1}{2}\,|\mathbf{r}| \tag{3}$$

$$\mathbf{r} \;=\; \mathbf{q} \;+\; (Q_x-Q_y)\,\widehat{\mathbf{c}} \tag{4}$$

and on a vector **r** with a component along a third axis $\widehat{\mathbf{c}}$. Eqn. 3 also describes the off-momentum eigentunes, $Q_+(\delta)$ and $Q_-(\delta)$, if the tunes and the vector **r** are chromatically expanded in $\delta = \Delta p/p$ (to arbitrary order)[3], [4], [5]

$$Q_x(\delta) \;=\; Q_{x0} + \chi_x\delta \tag{5}$$

$$Q_y(\delta) \;=\; Q_{y0} + \chi_y\delta \tag{6}$$

$$\mathbf{r}(\delta) \;=\; \mathbf{q} + \mathbf{k}\delta + [(Q_{x0}-Q_{y0}) + (\chi_x-\chi_y)\delta]\,\widehat{\mathbf{c}} \tag{7}$$

These equations introduce the "normal chromaticities" χ_x and χ_y, and also the important new "skew chromaticity" vector **k**, which, like **q**, lies in the (a,b) plane.

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The eigenchromaticities χ_- and χ_+ are *defined* as

$$\chi_\pm \;\equiv\; \frac{dQ_\pm}{d\delta} \tag{8}$$

leading to the simple general result,

$$\chi_\pm \;=\; \frac{1}{2}(\chi_x+\chi_y) \;\pm\; \frac{1}{2}\frac{\mathbf{r}.\mathbf{v}}{|\mathbf{r}|} \tag{9}$$

$$\mathbf{v} \;\equiv\; \frac{d\mathbf{r}}{d\delta} \tag{10}$$

where it is convenient to introduce the vector **v**. If the chromatic expansion of **r** is just linear, then **v** is a constant

$$\mathbf{v} \;=\; \mathbf{k} + (\chi_x-\chi_y)\,\widehat{\mathbf{c}} \tag{11}$$

in which case **r** is straight line that advances smoothly

$$\mathbf{r}(\delta) \;=\; \mathbf{r}(0) + \mathbf{v}\,\delta \tag{12}$$

as the off-momentum parameter is scanned.

II. Examples in 2-D

It is pedagogically useful to consider the case when the design tunes and the chromaticities are equal ($Q_{x0} = Q_{y0} \equiv Q_0,\;\; \chi_x = \chi_y \equiv \chi_0$), since then

$$\mathbf{r} \;=\; \mathbf{q} \;+\; \mathbf{k}\,\delta \tag{13}$$

$$\mathbf{v} \;=\; \mathbf{k} \tag{14}$$

and all vectors are confined to the (a,b) plane.

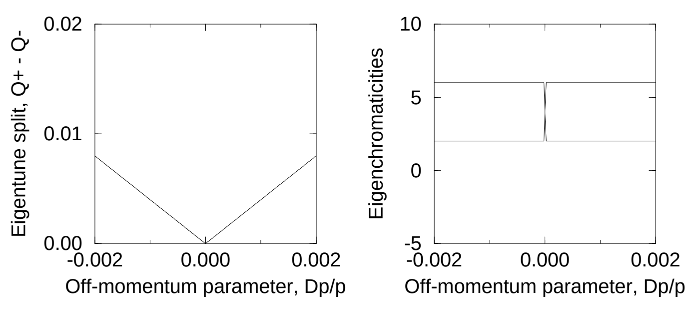

Figure. 1. Eigentune split and eigenchromaticities after perfect global decoupling (**q** = 0), with a typical Tevatron skew chromaticity ($|\mathbf{k}| = 4.0$).

A. Perfect global decoupling.

"Global decoupling" is routinely performed in most contemporary storage rings[2]. Typically, two skew quad families and one erect quadrupole family are adjusted to minimise ΔQ_{min}.

Perfect global decoupling, with $\mathbf{q} = 0$, is the simplest possible case to consider. Substitution into Eqns. 3 and 9 gives

$$Q_\pm \;=\; Q_0 \;\pm\; \frac{1}{2}\,|\mathbf{k}\,\delta| \qquad (15)$$

$$\chi_\pm \;=\; \chi_0 \;\pm\; \frac{1}{2}|\mathbf{k}|\frac{\delta}{|\delta|} \qquad (16)$$

as shown in Fig. 1 with $\chi_0 = |\mathbf{k}| = 4.0$. The eigentune split at a given momentum is just $|\mathbf{k}\,\delta|$, so that, if $|\mathbf{k}| = 4$ and $\sigma_p/p = 10^{-3}$, the eigentune split of a typical particle is 0.004. This is significant when compared to a typical design tune separation of 0.01. Table I summarises **k** measurements made at the Tevatron[4], at CESR[6], and (in tracking) at RHIC[7]. By coincidence, $\sigma_p/p \simeq 0.001$ at all of these machines. The last column of Table I is therefore in good agreement with the approximate prediction

$$\Delta Q_{min}(\text{observed}) \;\approx\; |\mathbf{k}|\,\frac{\sigma_p}{p} \qquad (17)$$

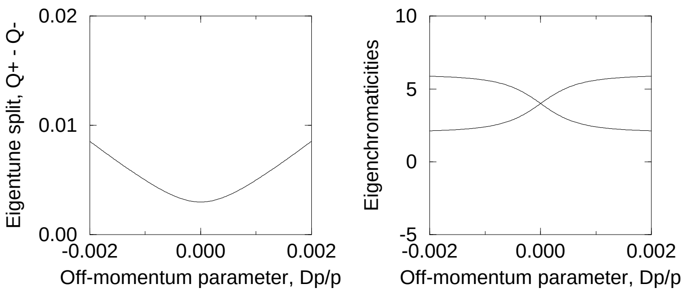

Figure. 2. Eigentune split and eigenchromaticities with $|\mathbf{q}| = 0.003$ and $\widehat{\mathbf{q}}.\widehat{\mathbf{k}} = 0$. Sharp features are broadened.

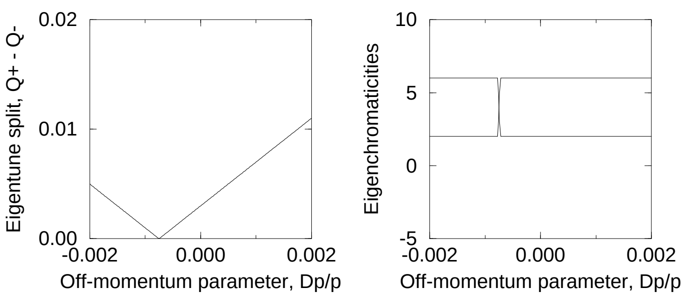

Figure. 3. Eigentune split and eigenchromaticities with the same conditions as Fig. 2, except that $\widehat{\mathbf{q}}.\widehat{\mathbf{k}} = 1$.

Table I

Skew chromaticity observations at three colliders. High luminosity optics were used in all cases.

machine	$\lvert\mathbf{k}\rvert$	ΔQ_{min} (observed)
CESR	0.5 ± 0.5	0.001
Tevatron	3.8 ± 0.2	0.003
RHIC	2.1	

B. Realistic global decoupling.

Measurements in the Tevatron found that $|\mathbf{q}| = 0.0032 \pm 0.0008$, $|\mathbf{k}| = 3.8 \pm 0.2$, and $\widehat{\mathbf{q}}.\widehat{\mathbf{k}} \approx 1$, after a careful round of global decoupling[4]. Note that the angle between $\widehat{\mathbf{q}}$ and $\widehat{\mathbf{k}}$ *can* be measured. Figs. 2 and 3 shows what happens in the more realistic situation when $|\mathbf{q}| = 0.003$ and when either $\widehat{\mathbf{q}}.\widehat{\mathbf{k}} = 0$ or $\widehat{\mathbf{q}}.\widehat{\mathbf{k}} = 1$. Sharp features in Fig. 1 are broadened when $\widehat{\mathbf{q}}$ and $\widehat{\mathbf{k}}$ are perpendicular, and are shifted when $\widehat{\mathbf{q}}$ and $\widehat{\mathbf{k}}$ are parallel. According to Eqn. 3, the closest approach of eigentunes occurs when **r** is shortest: when

$$\mathbf{r}.\mathbf{v} \;=\; 0 \qquad (18)$$

In the current 2-D context this is solved by

$$\delta_c \;=\; -\frac{|\mathbf{q}|}{|\mathbf{k}|}\;\widehat{\mathbf{q}}.\widehat{\mathbf{k}} \;\sim\; \frac{\sigma_p}{p} \qquad (19)$$

which is consistent with the figures.

III. STABILITY CRITERIA

Extreme values of the eigenchromaticities χ_+ and χ_- (with respect to changes in δ, $\mathbf{q}$, Q_{x0} and Q_{y0}) occur when

$$\frac{\mathbf{r}.\mathbf{v}}{|\mathbf{r}|} \;=\; \pm|\mathbf{v}| \qquad (20)$$

This occurs when **r** and **v** are collinear: for example, when the design tunes are set equal after perfect global decoupling ($\mathbf{q} = 0, Q_{x0} = Q_{y0}$). The extreme values are

$$\chi_{\pm extreme} \;=\; \frac{1}{2}(\chi_x + \chi_y) \;\pm\; \frac{1}{2}\sqrt{\mathbf{k}^2 + (\chi_x - \chi_y)^2} \qquad (21)$$

Insisting that both of the extreme eigenchromaticities are positive leads to the **new and strong criteria**[5], [8]

$$\chi_x + \chi_y \;>\; 0 \qquad (22)$$

$$4\chi_x\chi_y \;>\; \mathbf{k}^2 \qquad (23)$$

If true, neither eigenchromaticity can ever become negative. As such, these criteria are “sufficient but often not necessary”. Both χ_x and χ_y must be positive to meet the criteria, even when $\mathbf{k} = 0$, thereby recovering the standard uncoupled head-tail result (above transition).

IV. EXPLANATION OF TEVATRON OBSERVATIONS

In 1989-1990, high intensity Tevatron bunches ($N_{bunch} > 6 \times 10^{10}$) occasionally became head-tail unstable. Sometimes the beam losses were spontaneous. At other times they were induced when the operators corrected persistent current tune and chromaticity drifts, *often when the tunes were being separated*. Beam studies investigated head-tail stability with different design tunes and chromaticities[4]. The skew quad strengths were held fixed, after a preparatory global decoupling. Entirely different behavior was observed when the chromaticities were equal, and when they were grossly different.

First, the horizontal design tune Q_{x0} was scanned across the diagonal, with equal chromaticities ($\chi_x = \chi_y = \chi_0$). This was repeated for $\chi_0 = 4, 3, 2,$ and 1, with significant beam loss observed *as the diagonal was approached* for the last two values. This is consistent with Eqn. 23, which predicts unequivocal stability when $\chi_0 > 1.9 \pm 0.1$. Figs. 4 and 5 show that, when $\chi_0 = 1.5$, both eigenchromaticities are positive for all momentum offsets when the design tunes are 0.007 apart, but that one is negative for all positive momenta when $Q_{x0} - Q_{y0} \approx 0.001$. Simulation confirms this behaviour [5], [8].

The behavior was quite different when $(\chi_x, \chi_y) = (8, -3)$, values that become plausible after persistent current effects have forced the horizontal and vertical chromaticities in opposite directions for several hours. Total beam loss was observed *as the design tunes were separated*. Figures 6 and 7 show that one of the two eigenchromaticities is negative for all positive momentum offsets when the tunes are separated by 0.007, while both are positive when the tunes are only 0.001 apart, at least in a small vicinity around zero momentum offset.

V. Acknowledgements

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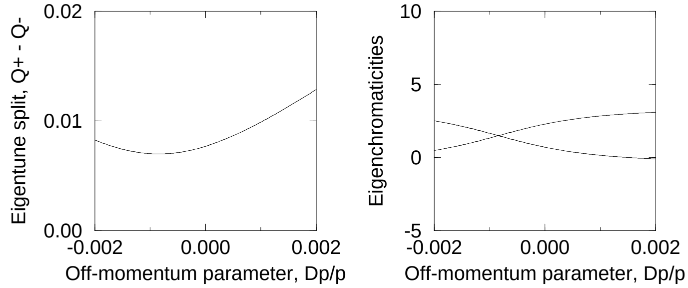

Figure. 4. Tevatron eigentune split and eigenchromaticities with equal chromaticities, and design tunes 0.007 apart ($\chi_x = \chi_y = 1.5, Q_{x0} = .425, Q_{y0} = .418$). The beam was stable.

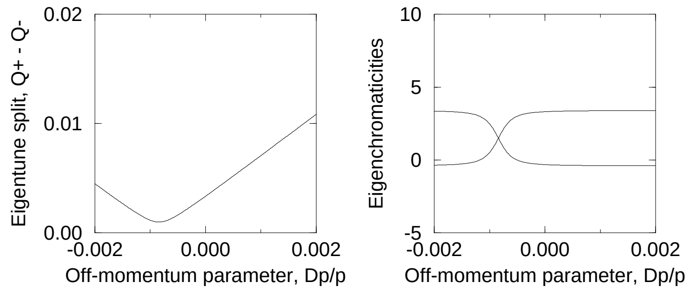

Figure. 5. Eigentune split and eigenchromaticities as in Fig. 4, with design tunes 0.001 apart ($Q_{x0} = .419, Q_{y0} = .418$). Some beam loss was observed. One eigenchromaticity was negative for all positive momentum offsets.

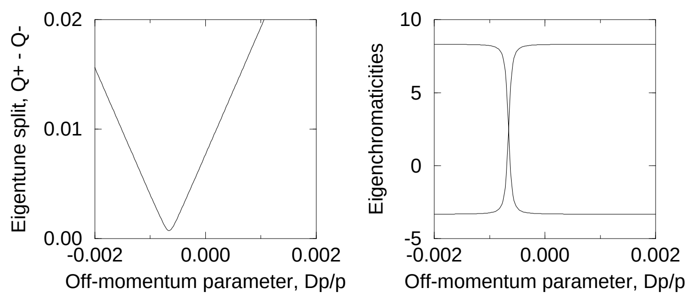

Figure. 6. Eigentune split and eigenchromaticities with very unequal chromaticities and design tunes 0.007 apart ($\chi_x = 8.0, \chi_y = -3.0, Q_{x0} = .425, Q_{y0} = .418$). Total beam loss was observed.

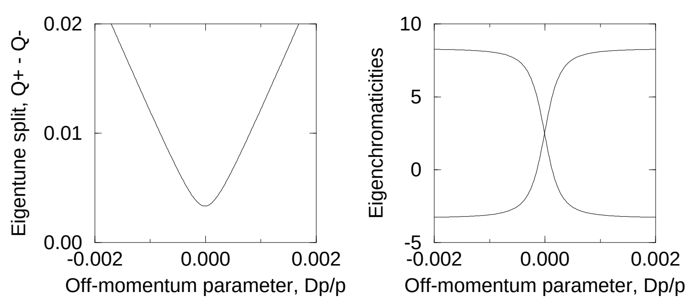

Figure. 7. Eigentune split and eigenchromaticities as in Fig. 6, but with design tunes 0.001 apart ($Q_{x0} = .417, Q_{y0} = .418$). Remarkably, the beam was stable.