

PERTURBATION OF BEAM ENERGY DUE TO STEERING AND PRETZEL ORBIT*

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Abstract

The energy calibration of the beam directly effects the precision of the mass measurement of particles produced in collision. The beam path length changes due to the pretzel orbit, steering, bumps and quadrupole survey. To provide reasonable extrapolation of energy measurement, this path length change needs to be included in the energy calculation.

I. Introduction

The energy calibration of the beams in CESR directly effect the precision of the mass measurement of particles produced in collision. There are three methods to measure the beam energy (nmr*, Ecleo and B mass) in CESR. It has been seen that there is a 2 MeV shift between nmr*, Ecleo, and B mass measurement from run by run. To provide reasonable extrapolation of energy measurement, there is a need to analyze the effect of machine hardware components (such as steering, bumps, quad survey, pretzels, etc) on the energy.

The energy information could be calculated from the CESR beam trajectory/orbit and the dipole magnetic field throughout the ring. The path length change due to the dipole field error or kicks plays an important roles in energy calculation.

II. The Off-Energy Orbit and Energy Change

The linear transverse particle motion could be described by the differential equations[1]:

$$x'' + [K_1(s) + G^2(s)]x = G(s)\delta - \frac{ec\Delta B_y}{E_0} \quad (1)$$

where: δ is the momentum deviation; e and c are electron charge and speed of light. $ec\Delta B_y/E_0$ is the normalized dipole field imperfection or the quadrupole misalignment; $G(s) = ecB_y(s)/E_0$ is the curvature function; $K_1(s) = ec(\partial B_y/\partial x)/E_0$ is the quadrupole strength.

The coupling terms and high order terms were neglected in Equation.1. The solutions of linear motion of Equation.1 are to be a superposition of periodic forced solutions (energy displacement and closed orbits) with the free oscillation (the betatron oscillation). The total displacement from the ideal orbit could be written as,

$$x(s) = x_\beta(s) + \eta(s)\delta + x_c(s) \quad (2)$$

The betatron oscillation $x_\beta(s)$ is well known and could be represented by

$$x_\beta(s) = a\sqrt{\beta_x}\cos\{\phi_x(s) - \vartheta\} \quad (3)$$

where $\phi_x(s)$ is the horizontal phase, β_x is betatron function, and a and ϑ are constants for a particular trajectory.

The energy displacement term $\eta(s)\delta$ is easy to get by solving the differential equation

$$\eta''(s) + [K_1(s) + G^2(s)]\eta(s) = G(s) \quad (4)$$

$\eta(s)$ is called dispersion function and its solution can be expressed as

$$\eta(s) = \frac{\sqrt{\beta_x(s)}}{2\sin\pi\nu_x} \oint G(s_x)\sqrt{\beta(s_x)}\cos\{|\phi_x(s) - \phi_x(s_x)| - \pi\nu_x\}ds_x \quad (5)$$

If the beam has a slight momentum deviation of δ_0 in relative value, the horizontal closed orbit is displaced to first order:

$$x_{\delta_0}(s) = \eta(s)\delta_0 \quad (6)$$

Since the particles lose a significant fraction of their energy due to the synchrotron radiation in the dipoles while the compensating acceleration only occurs at a few points, the instantaneous momentum therefore varies along the circumference. If the dispersion and its derivative not vanish at the RF cavities, the second order orbit distortion may not be neglected. For simplicity, we neglect all second order terms. There are 98 beam position monitors (BPM) throughout the ring, and be used to take orbits from run to run. By averaging the closed orbit data, the average momentum deviation δ_0 could be extracted.

An important consequence of an energy deviation is associated change in the circumference of the closed orbit. This effect can be expressed by the orbit length dilation factor.

$$\frac{\Delta L_\delta}{L_0} = \alpha_p \delta \quad (7)$$

where $\alpha_p = \frac{1}{L_0} \oint G(s)\eta(s)ds$ is the momentum compaction factor.

The closed orbit derivation term x_c due to the dipole field error or kick could be calculated by solving the equation

$$x'' + [K_1(x) + G^2(x)]x = -\frac{ec\Delta B_y}{E_0} \quad (8)$$

Define the kick $\Delta x'$ by the following equation,

$$\Delta x' = -\frac{ec\Delta B_y \cdot \Delta L}{E_0} \quad (9)$$

By solving Equation.8, the closed orbit displacement due to a kick $\Delta x'(s_x)$ at the location of s_x could be expressed as follows[1],

$$x_c(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu_x} \Delta x'(s_x)\sqrt{\beta(s_x)}\cos(|\phi(s) - \phi(s_x)| - \pi\nu_x) \quad (10)$$

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Obviously the total path length will change due to this closed orbit distortion and can be written as

$$L_{total} = \oint (1 + G(s)\Delta x(s))ds = L_0 + \oint G(s)x_c(s)ds \quad (11)$$

The changed path length δL_β could thus be easily derived,

$$\delta L_\beta = \frac{\Delta x'(s_x)\sqrt{\beta(s_x)}}{2\sin\pi\nu_x} \oint G(s)\sqrt{\beta(s)} \cos(|\phi(s) - \phi(s_x)| - \pi\nu_x)ds \quad (12)$$

Replace the right side of Equation.12 by the definition of dispersion, the δL_β could be very simple,

$$\delta L_\beta = \Delta x'(s_x)\eta(s_x) \quad (13)$$

Since the RF system frequency determines the circumference so that any lengthening of the path due to the betatron closed orbit error must result in an equal and opposite change in path length due to a change in beam energy.

$$\Delta L_\beta = -\Delta L_\delta \quad (14)$$

The energy deviation due to this kick then could be expressed as:

$$\delta = \frac{\Delta L_\delta}{\alpha_p L_0} = -\frac{\Delta L_\beta}{\alpha_p L_0} = -\frac{\eta(s_x)\Delta x'(s_x)}{\alpha_p L_0} \quad (15)$$

For many kicks around the ring, the total energy change is simply the sum of the energy change of each kick.

$$\frac{\delta E}{E_0} = -\sum \frac{\Delta x'(s_x)\eta(s_x)}{\alpha_p L_0} \quad (16)$$

CESR is designed to operate over a range of electron energies by arranging all dipole magnetic fields be varied together (scaled in proportion to the desired operating energy). The design orbit is not changed by varying all fields together which only changes the beam energy associated with the design orbit. This could also be verified by Equation.16. Let's assume the fraction of dipole field change $F = \Delta B(s_x)/B_0(s_x)$ is a constant throughout the ring,

$$\Delta x'(s_x) = -\frac{ec\Delta B\Delta L}{E_0} = -\Delta G(s_x)\Delta L = -FG(s_x)\Delta L \quad (17)$$

and it is demonstrated that the beam energy deviation is also equal to F .

$$\delta = \oint \frac{\eta(s_x)FG(s_x)}{\alpha_p L_0} ds_x = \frac{F}{\alpha_p L_0} \oint G(s)\eta(s)ds = F \quad (18)$$

III. Discussion of the Effect on Energy in CESR

Now, let's discuss the effect of steering, bumps, quadrupole survey, pretzels on energy in CESR. We choose the present high energy physics (HEP) lattice with small crossing angle of 2.5 mrad as an example. The main parameter of the lattice is listed in Table.1. The crossing angle optics are designed to accommodate 9 trains of 2 bunches separated by 28 ns.

Steering: There are total 83 horizontal steering coils in CESR. The total magnetic length of all the horizontal steering coils is

Table.1 Some Parameters of HEP Lattice

Lattice Name	N9A18A600.FD92S_4S
Nominal Energy	5.289 GeV
α	0.011256
ϵ_x	0.20×10^{-6}
Horizontal Tune ν_x	10..5235
Vertical Tune ν_y	9.5971
β_y^*	0.18 m
β_x^*	1.09 m
Energy Spread	0.6149×10^{-3}
Circumference L_0	768.426 m
Average Dispersion	1.38 m

450 m. Most single steering coil have magnetic length of 6.504 m, except the steering coils between the IP and Quadrupole Q8. In principal, we could measure the the excitation curve for all the steering coil, and read the the coil current from the CESR save set. The energy change due to the steering coil could be simply calculated according to the Equation.16.

In CESR, two pairs of electrostatic separators are used to separate the two beams (electron and positron beam) at the parasitic points. The orbits of positron and electron are oppositely distorted in pretzels fashion by these two pairs of separators. In the present high energy physics running optics with crossing angle, the horizontal separators are powered antisymmetrically. Since CESR is a symmetric machine, the energy change due to these two pair of antisymmetrically powered separator kicks should be canceled each other and the energy deviation should be zero. If these two pair of separators are powered symmetrically as we did in the head-on collision running before, the energy change could be calculated by Equation.16.

There could be many horizontal bumps throughout the ring. The effect on the energy should be similar as steering. The local bumps are usually closed, but the beta function at each individual bump is not always the same, and the dispersion difference can be large for bumps in areas with varying dispersion (i.e., when bending magnets are in the bump region). The effect on energy could be large. Bumps in long drift spaces, such as the IR, have a smaller effects on the energy.

Quadrupole Survey Misaligned quadrupoles can cause energy changes also. It is important to distinguish the misalignment of a quadrupole relative to the ideal "design" orbit from the passage of the beam off the magnetic center due to a kick somewhere else in the ring. In the second case, the kick given to the beam by the quad is just part of the focusing action of the storage ring optics and, as implied by the treatment above, does not cause any additional energy change since Equation:16 takes into account the kicks from all the quads.

Misaligned quads are properly treated as dipole kicks using Equation:16. The value of the kick must be computed from the displacement of the quad center from the design orbit. In practice this can be determined only from a survey, which puts definite limits on our knowledge of energy changes due to misaligned quads. If we are interested primarily in a reproducible energy measurement, keeping track of quads moved during a down pe-

riod is very important. The unplanned movements of quads must be determined by some other means if we are to understand their effect on the machine energy.

The kick received by the beam from a misaligned quad could be calculated by:

$$\Delta x' = -x_d K_1 L_{quad} \quad (19)$$

where the L_{quad} is the magnetic length of the quad, x_d is the offset of the quad axis away from the ideal design orbit. K_1 is the quad strength. Assume that the beam offset x_d is 0.1 mm at Q2 ($K_1 L \approx 0.5$), the kick received by the beam at Q2 is -0.05 mrad, which lead to the energy change ($\Delta E/E_0$) of 0.43×10^{-7} from Equation.16, since the dispersion at Q2 is very low (0.0075 m). Another example, if we assume that the beam offset x_d is 0.1 mm at Q6 ($K_1 L \approx 0.25$) where the dispersion is 2.35 m, the kick received by the beam could be calculated to be -0.025 mrad and energy change δ to be 0.68×10^{-5} . For the nominal energy of 5.289 GeV, the energy deviation is about 0.036 MeV.

Miscellaneous Orbit Distortion The closed orbit file could be taken from run to run. The miscellaneous orbit distortion could be gotten by subtracting this orbit with the known orbit distortions introduced by steering, pretzels, bumps, etc. The average momentum displacement due to the miscellaneous orbit distortion then could be calculated by averaging this orbit. The average dispersion function of present HEP running optics is 1.38 m and the average displacement of miscellaneous orbit distortion is at the order of 0.02 mm, so the momentum displacement due to this orbit distortion could be at the order of 1.45×10^{-5} .

References

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