TUNE MODULATION DUE TO SYNCHROTRON OSCILLATIONS AND CHROMATICITY, AND THE DYNAMIC APERTURE*

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Abstract

A tracking study was done of the effects of a tune modulations, due to synchrotron oscillations and the tune dependence on momentum (chromaticity), on the dynamic aperture. The studies were done using several RHIC lattices and tracking runs of about 1×10^6 turns. The dynamic aperture was found to decrease roughly linearly with the amplitude of the tune modulation. Lower order non-linear resonances, like the 1/3 and 1/4 resonance are not crossed because of the tune modulation. Three different cases were studied, corresponding to RHIC lattices with different β^*, and with different synchrotron oscillation amplitudes. In each case, the tune modulation amplitude was varied by changing the chromaticity. In each case, roughly the same result, was found. The result found here for the effect of a tune modulation due to chromaticity may be compared with the result found for the effect of a tune modulation due to a gradient ripple in the quadrupoles. The effect of a tune modulation due to a gradient ripple appears to be about 4 times stronger than the effect of a tune modulation due to chromaticity and synchrotron oscillations.

I. INTRODUCTION

A tracking study was done of the effects of a tune modulation, due to synchrotron oscillations and the tune dependence on momentum (chromaticity), on the dynamic aperture. The studies were done using several RHIC lattices and tracking runs of about 1×10^6 turns. The dynamic aperture was found to decrease roughly linearly with the amplitude of the tune modulation and may be represented by

$$A = A_0(1 - 10\ \Delta\nu) \tag{1}$$

where A_0 is the dynamic aperture for $\Delta\nu = 0$, and $\Delta\nu$ is the tune modulation amplitude. In Eq. (1), the range of $\Delta\nu$ is such that lower order non-linear resonances, like the 1/3 and 1/4 resonance are not crossed because of the tune modulation.

Three different cases were studied, corresponding to RHIC lattices with different β^*, and with different synchrotron oscillation amplitudes. In each case, the tune modulation amplitude was varied by changing the chromaticity. In each case, roughly the same result, Eq. (1), was found.

The result found here for the effect of a tune modulation due to chromaticity may be compared with the result found [1] for the effect of a tune modulation due to a gradient ripple in the quadrupoles, which was

$$A = A_0(1 - 42\ \Delta\nu) \tag{2}$$

The effect of a $\Delta\nu$ due to a gradient ripple appears to be about 4 times stronger than the effect of a $\Delta\nu$ due to chromaticity and synchrotron oscillations.

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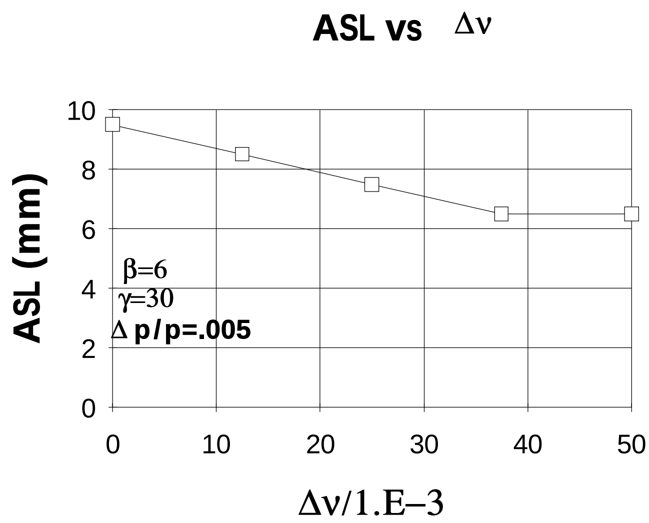

Figure. 1. A plot of the dynamic aperture, A_{SL}, versus the tune modulation amplitude, $\Delta\nu$.

II. TRACKING RESULTS

The tune of the particle is modulated with time due to the chromaticity and the synchrotron oscillations. If the chromaticity is given by C_x, C_y and the amplitude of the synchrotron oscillation in momentum is given by $\pm\Delta p/p$, then the tune is modulated with time with a tune oscillation amplitude given by

$$\begin{aligned} \Delta\nu_x &= C_x\ \Delta p/p \\ \Delta\nu_y &= C_y\ \Delta p/p \end{aligned} \tag{3}$$

The frequency of the tune modulation is that of the synchrotron oscillation. For one case that was studied, the synchrotron oscillation frequency is about 260 hz.

Figure 1 shows the dynamic aperture A_{SL} versus $\Delta\nu$ for a RHIC lattice with $\beta^* = 6$ at all 6 insertions. The synchrotron oscillation amplitude is held constant at $\Delta p/p = 0.005$, $\Delta\nu$ is varied by varying the chromaticity $C_x = C_y$ from 0 to 10 producing a maximum $\Delta\nu$ of 50×10^{-3}. The nominal tune is $\nu_x = 28.826$, $\nu_y = 28.821$ and with this range of $\Delta\nu$ low order resonances like the 1/3 or 1/4 resonances, are not crossed. Figure 1 shows a roughly linear decrease of the dynamic aperture with $\Delta\nu$ from $A_{SL} = 9.5$ mm at $\Delta\nu = 0$ to $A_{SL} = 6.5$ mm at $\Delta\nu = 50 \times 10^{-3}$.

Three cases were studied with the same RF system which at the energy corresponding to $\gamma = 30$ has a synchrotron oscillation frequency of about 260 hz. These three cases are listed in Table 1. Case 2 has $\beta^* = 6$ and $\Delta p/p = 0.0025$. Case 3 has $\beta^* = 2$

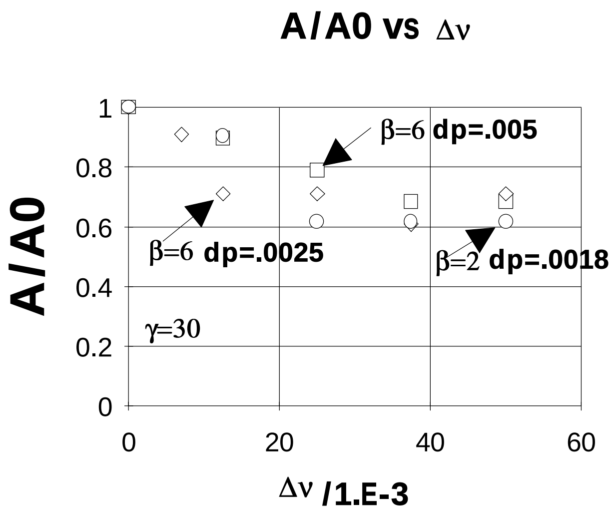

Figure. 2. A plot of A/A_0 versus $\Delta\nu$ for three different cases. A is the dynamic aperture, A_0 is the dynamic apertue for $\Delta\nu = 0$.

and $\Delta p/p = 0.0018$. The height of the RF bucket in $\Delta p/p$ is $\Delta_B = 0.006$ in the 3 cases at $\gamma = 30$.

Table 1: A table of 3 cases studied where the synchrotron oscillation frequency was held constant at $f_s = 260$ Hz.

Case	β^* (m)	$\Delta p/p$ Synch. Osc. Amp.	
1	6	0.005	
2	6	0.0025	
3	2	0.0018	

Figure 2 plots the tracking results for A/A_0 against $\Delta\nu$ for the 3 cases. A is the dynamic aperture, A_0 is the dynamic aperture for $\Delta\nu = 0$, and $\Delta\nu$ is the amplitude of the tune modulation, which is varied by changing the chromaticity $C_x = C_y$. Figure 2 indicates that the data for all 3 cases lie roughly on the straight line

$$A = A_0(1 - 10\ \Delta\nu) \quad (4)$$

In the range of $\Delta\nu$ covered in Fig. 2 there is only one low order resonance, which is below tenth order, that is reached by the tune modulation. This is the 5/6 resonance at $\nu = 28.83333$. The unperturbed tune is at $\nu_x = 28.826$ $\nu_y = 28.821$. The 5/6 resonance is reached at a tune modulation of $\Delta\nu \geq 7 \times 10^{-3}$. The results in Fig. 2 do not clearly show the presence of this resonance. The runs done with $\Delta\nu \geq 7 \times 10^{-3}$ sweep over the 5/6 resonance many times in 10^6 turns, yet the dynamic aperture found for $\Delta\nu \geq 7 \times 10^{-3}$ does not show much effect from the 5/6 resonance.

One may speculate as to under what conditions the result, Eq. (4) may be roughly valid. One may conjecture that Eq. (4) for the dependence of the dynamic aperture on the amplitude of the tune modulation may be roughly valid under the following conditions:

1. The tune modulation does not sweep over the lower order resonances like the 1/3 or 1/4 resonances.
2. The field error multipoles, b_n or a_n, are roughly given by b_0/R^n, where $b_0 \simeq 2 \times 10^{-4}$ and, usually, R is roughly the magnet coil radius.
3. The tune modulation is generated by the presence of a tune dependence on momentum (chromaticity) and synchrotron oscillations, and the frequency of the synchrotron oscillations is small compared to the particle revolution frequency in the accelerator.

The result found for the decrease in dynamic aperture, Eq. (4), may be compared with the result found for the effect on the dynamic aperture due to a tune modulation generated by a ripple in the gradient of the quadrupoles, which is given by

$$A = A_0(1 - 42\ \Delta\nu), \quad \text{gradient ripple.} \quad (5)$$

One sees that a $\Delta\nu$ due a gradient ripple is more effective in reducing the dynamic aperture than a $\Delta\nu$ due to chromaticity and synchrotron oscillation by about a factor of 4.

III. DEPENDENCE OF DYNAMIC APERTURE ON THE SYNCHROTRON OSCILLATION FREQUENCY

The frequency of the synchrotron oscillation frequency can be varied by varying the voltage, V, and the harmonic number, h, of the RF cavity. If this is done holding V/h constant, then the height of the RF bucket in $\Delta p/p$ is not changed.

The dependence of the dynamic aperture on the synchrotron oscillation frequency f_s was studied for the case $\Delta\nu = 50 \times 10^{-3}$, $\Delta p/p = 0.005$, $C_x = C_y = 10$, $\beta^* = 6$. This case has the largest tune oscillation amplitude studied. The results for the dynamic aperture A_{SL} vs. f_s for this case are shown in Figure 3. f_s was varied from $f_s = 16.25$ hz to 520 hz. Figure 3 does not show much dependence of A_{SL} on f_s over this range in f_s. A_{SL} is given by 5.5 ± 1 mm over the range in f_s. There appears to be a small decrease in A_{SL} for lower f_s. One should note that the tracking results for A_{SL} become more doubtful at lower values f_s. At $f_s = 16.25$, there is time for about 250 synchrotron periods in 1 million turns. This effect would probably reduce the dynamic aperture at lower values of f_s.

IV. COMMENTS ON THE TRACKING

It is important that the tracking be symplectic. To achieve this, the ORBIT program was changed [2] to allow the use of point magnets. The methods used are similar to those used in the TEAPOT [3] program, with some modifications, including the choice of the reference orbit [4].

One 160 MHz RF cavity with an RF voltage of 4.5 MV was used in the tracking. The bucket height is $\Delta p/p = 6 \times 10^{-3}$ at $\gamma = 30$ and $\Delta p/p = 2 \times 10^{-3}$ at $\gamma = 100$.

Random and systematic field errors were present in each magnet at the level given in Ref. 5. Field error multipoles up to order 10 were included. The studies done in this paper were done for a particular set of field errors which gave the smallest dynamic aperture, out of ten different distributions of the random field errors, in the absence of tune modulation.

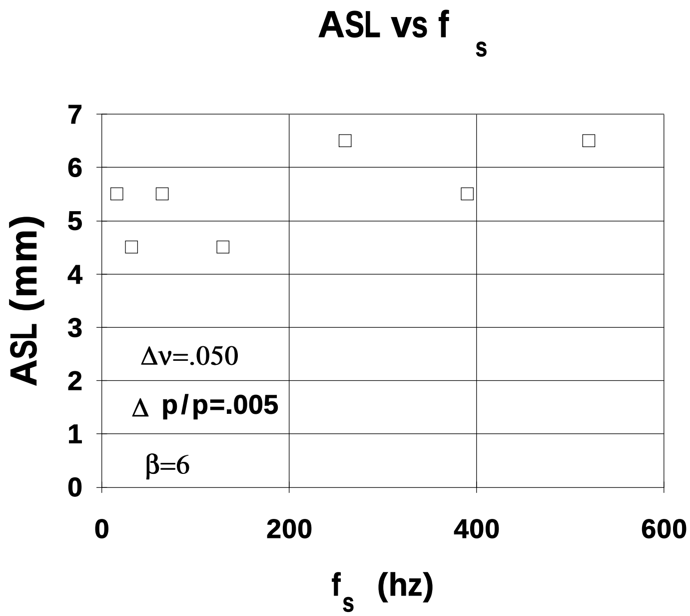

Figure. 3. A plot of A_{SL} versus f_s, the synchrotron oscillation frequency.

The dynamic aperture is computed by doing a series of runs with the starting conditions $\epsilon_x = \epsilon_y$, $x' = y' = 0$, and finding the largest betatron oscillation amplitude that is stable for 800,000 turns.

References

[1] G. Parzen, Tune Modulation due to Gradient Ripple and the Dynamic Aperture, BNL Report AD/RHIC-124 (1993).

[2] G. Parzen, Effect of Synchrobetatron Coupling on the Dynamic Aperture, BNL Report AD/RHIC-119 (1993).

[3] L. Schachinger and R. Talman, TEAPOT, A Thin Element Tracking Program, SSC Report SSC-52 (1985).

[4] G. Parzen, Symplectic Tracking Using Point Magnets and a Reference Orbit Made of Circular Arcs and Straight Lines, BNL Report AD/RHIC-120 (1993).

[5] RHIC Conceptual Design, BNL Report BNL-52195 (1989).