

# FREE ELECTRON LASER - FEL - STUDY IN INSTITUTE OF NUCLEAR PHYSICS OF MSU

V.K.Grishin, B.S.Ishkhanov, T.A.Novikova, V.I.Shvedunov  
Institute of Nuclear Physics, Moscow State University

## I. FEL BASED ON THE MSU RACE-TRACK MICROTRON

The possibility of creation of a source of coherent shortwave radiation on the basis of the RM of Moscow State University (MSU) facility was considered. The RM is designed to obtain accelerated electrons with a variable energy of 7-180 MeV at high quality of beam: the homogeneity of energy is 0.0001, the beam emittance is not higher than 0.01 mm.mrad. The beam current is a continuous sequence of electron pulses with the duration of 3-4 ps following at a frequency of 2.45 GHz and the average current of 100 mA [1]. A special operation mode of the acceleration is provided for duty cycle decreasing by 100 - 300 times and the amplitude of current pulses 1-10 A.

Two operation regimes of the FEL are possible. At the standard undulator parameters ( length and number of periods are 2.5 cm and 100,  $B=5$  kGz, undulator factor is  $K=0.5-1$ , optical length is 3-5 m) positive gain in the single-particle regime is realised in the 5-170 mkm band [2].

Low emittance permits to provide the collective amplification regime of FEL with channelling radiation [3]. In this case the positive gain may be achieved on the wavelength region of  $\lambda_s < 1$  mkm.

## II. ESTIMATION OF HIGH-CURRENT FEL TOP EFFICIENCY

High-current FEL are used for the exciting of shortwave electromagnetic radiation (EMR). It operates in collective stimulated EMR regime. Due to the bunching process the efficiency of output radiation is significantly increased. Top efficiency evaluation is obviously acquires remarkable interest.

FEL limiting efficiency is defined by the nonlinear saturation of EMR exciting process. The basis stabilisation process in this case is the capture of beam particles by slow wave of electromagnetic system named as the wave of ponderomotive potential. In a result velocities of electron beam and ponderomotive wave are synchronised. Then energy exchange between beam particles and EMR has only oscillating character. Therefore, analysis of equilibrium condition of particles-wave system allows to estimate the magnitude of FEL top efficiency.

Let us consider one of the perspective FEL schemes with helical undulator. Magnetic field of undulator is described by vector potential  $\mathbf{A}=\mathbf{A}_0[\mathbf{e}_x \cos(k_0 z)+\mathbf{e}_y \sin(k_0 z)]$ , where

$(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$  are coordinate orts,  $k_0=2\pi/\lambda_0$ ,  $\lambda_0$  - undulator period. Injected particles with initial velocity  $v_0=\beta_0 c$  acquires a transverse speed  $v_{0\perp}=v_0 \mathbf{A}_0/A_0$  in the undulator field. Triggering circularly polarised light wave with the electric field  $\mathbf{E}_{si}=\mathbf{E}_{0i}[\mathbf{e}_x \cos(\omega_{si} t - k_{si} z) - \mathbf{e}_y \sin(\omega_{si} t - k_{si} z)]$  where  $\omega_{si}$  is the frequency,  $k_{si}$  is initial wave number ( $\omega_{si}=ck_{si}$ ) travels through the laser cavity in the same direction as electrons. Electromagnetic wave is amplified by the oscillating beam of particles. Effective energy exchange requires that the longitudinal velocity of electrons should be equalled to phase speed of the ponderomotive wave:  $v_{0z}=v_{ph}=\omega_s/(k_{si}+k_0)$ . Particles-wave interaction leads to the bunching of beam and then to the capturing of electrons to "traps" of the ponderomotive potential  $U=A_0 A_s/(mc^2)$ , where  $A_s$  is vector potential of EMR. Naturally, the trapped electrons have a speed  $\beta_{zc}=[v^2-v_{\perp}^2]^{1/2}<\beta_{0z}c$ , because of  $\gamma_z<\gamma_{0z}$ . Interaction of electrons with EMR usually takes place on the beam travelling length  $L_{und}=10\div 20/Im(k_s)$ , that is 1÷1.5 m with beam current  $I_b\sim 100$  A÷1 kA.

FEL action for the sufficiently extending system has a permanent character: electron beam are continuously injected to undulator in  $z=0$  and fluxes of particles and EMR run out from the capture area  $z=z^*$ . Equilibrium equations for the average flows of energy and pulse linear densities of particles ( $\Pi_b$  and  $G_b$ ) and EMR ( $\Pi_s$  and  $G_s$ ) through the transverse section of beam are:

$$\begin{aligned} \Pi_b \Big|_{z=0} &= \{\Pi_b + \Pi_s\} \Big|_{z=z^*} \\ G_b \Big|_{z=0} &= \{G_b + G_s\} \Big|_{z=z^*}. \end{aligned} \quad (1)$$

The contribution of EMR wave to the general energetic equilibrium on the beginning stage is assumed slight. Besides, no flows of beam particles and EMR pass through the sides of system. Taking into consideration the ratio  $\Pi_b \beta_z = c G_b$  the equilibrium equations permit to make an important formula to limiting efficiency [4]:

$$\eta = (\Pi_s \gamma_0) / [\Pi_0 (\gamma_0 - 1)] = \gamma_0 (\beta_{0z} - \beta_z) / [(\gamma_0 - 1)(c G_s / \Pi_s - \beta_z)] \quad (2)$$

where magnitude  $(\gamma_0 - 1)\Pi_0/\gamma_0$  is flow density of start kinetic energy of electron. Value of  $\Pi_s$  is equalled to Pointing vector and  $G_s$  is equalled to field stress tensor  $T_{zz}$  [4]. Depended on the Coulomb interaction of particles longitudinal field components are neglectable. Therefore, only the transverse component of EMR wave is included in the equations for  $\Pi_s$

and  $G_s$ . It is notice that phase velocity of radiation wave is decreased:  $\beta_s = \omega_s/k_s < 1$ . Magnitude of  $\beta_s$  is determined by equality of the velocity of the beam electrons and the ponderomotive wave  $\omega_s = \beta_z c / (k_s + k_0)$ .

After the capturing of the electron the state of system is described by the evolutionary equations. Assuming for the vector potential  $\mathbf{A}_s = \mathbf{A}_{s0}(z) \sin(\int k_s dz - \omega t)$  and for Furrier component of the current density  $\mathbf{j}_s = \beta_{0\perp} j_b \varepsilon \sin(\int k_s dz - \omega t - \tau)$ , where  $\tau$  is slipping phase to relation to equilibrium point of the ponderomotive potential,  $\varepsilon$  is Furrier coefficient characterising a bunching agree,  $j_b$  is the beam current density, we have:

$$2k_s(\delta A_{s0}/\delta z) = (4\pi/c)\varepsilon\beta_{0\perp} j_b \sin(\tau) \quad (3)$$

$$\Delta_{\perp} A_{s0} + q^2 A_{s0} = (4\pi/c)\varepsilon\beta_{0\perp} j_b \cos(\tau)$$

where  $\Delta_{\perp}$  is the transverse part of Laplasian,  $q^2 = k_s^2 - \omega^2/c^2$ . It is important to emphasise that after the trapping the magnitude of  $q^2$  remains greater then zero because phase velocity of wave is decreased.

Equations (3) define the alterations of values  $A_{s0}$ ,  $\varepsilon$ ,  $q^2$  with the oscillating process of phase  $\tau$ . The change of the phase is determined by the equation of particles moving in the field of the ponderomotive wave. Somewhat conclusions are followed from the physical law-governed nature of trapped bunches and EMR interaction.

After integrating on the transverse section the equations (3) gives the average ratios in which value of  $S_A = 2\pi \int A_{s0} r dr$  appears in left part. The full beam current  $I_b$  and average magnitudes  $\varepsilon$  and  $\tau$  are figured in right part. The top value of  $\tau_m$  corresponds to the capture moment of the beam when electron velocity is equal to phase speed of the ponderomotive wave. Phase value  $\tau=0$  conforms with the beam passing over minimum of the ponderomotive potential. The second ratio in (3) acquires a simple form:

$$q^2 S_A = (4\pi/c)\beta_{0\perp} \varepsilon \cos(\tau) \quad (4)$$

After capturing the electron bunches oscillates with phase  $\tau < \tau_m$ . From the physical point of view it is clear that the process of energy exchange between beam and EMR prolongs to  $\tau \sim 0$ . However, the addition to the EMR energy no very appreciable because the EMR wave on the phase area  $0 < \tau < \tau_m$  passes electron beam. Besides, in the region  $\tau=0$  the electron bunches become blurred due to the heterogeneous velocities. Formally this process leads to decreasing of the magnitude  $\varepsilon$ . Qualitative graphic estimation indicates that  $\varepsilon(\tau \approx \tau_m) = 1.6-1.8$  and  $\varepsilon(\tau=0) = 1.3$ . In the same time the value of  $\cos(\tau)$  increases very small so the value  $\varepsilon_1 = \varepsilon \cos(\tau)$  is oscillates range from 1.5 to 1.3 [5]. Thus, analysis of the trapped state of system allows to evaluate the practical efficiency of FEL.

Interaction between space-heterogeneous channelling EMR and beam electrons leads to the non simultaneous

capturing of the different radial layers of beam. Therefore, the greatest efficiency of particles and EMR interacting are provided by electron beam with the tube configuration. In this case the density of the electron beam is derived from the ratio  $n_b(r) = \varepsilon_1 I_b \delta(r-a) / (2\pi c \varepsilon \beta_z r)$  ( $\delta(r-a)$  is Dirac function). The transverse distribution of the EMR field is determined by the second ratio of (3) with the quasi-constant right part ( $\varepsilon$  and  $\tau$  change very slowly after trapping). Consequently, solutions of wave equation are given by  $A_s(r < a) = A_1 K_0(qa) I_0(qr)$  and  $A_0(r > a) = A_1 I_0(qa) K_0(qr)$ , where  $I_0, K_0$  are Bessel functions of imaginary values. There is strong channelling structure of EMR [6]. In this way the magnitude of the longitudinal velocity  $c\beta_z$  of trapped particles is expressed in terms of the beam current:

$$I_b = [32ak_s I_A \gamma_0 (\beta_{0z} - \beta_z) (1 - \beta_s^2)^{3/2} (\beta_z / \varepsilon_1 \beta_{\perp})^2] \times \\ \times [(1 - \beta_s)^2 + 2\beta_s(1 - \beta_z)]^{-1} \quad (5)$$

here  $I_A = mc^3/e = 17$  kA. Taking into account connection between  $k_s$  and  $k_{si}$  the ratios (2) and (5) give the self-consistent evaluation of top efficiency. In the first approximation on  $\delta\beta = \beta_{0z} - \beta_z$  using the coefficient  $\varepsilon_1 = 1.5$  of the limit bunching, we have:

$$\eta = [(I_b / (2I_A a k_0 \gamma_0^2))^{2/5}] / 2 \quad (6)$$

It is significant that the equation has the same form of the similar ratio of the linear theory but it differs from the later one by the numerical coefficient. The efficiency of the collective FEL is not high and it decreases at more short wavelength. EMR efficiency of FEL in infra-red and optical regions do not exceeds one - two percent for the moderated beam current.

### III. HIGH-CURRENT FEL WITH VARIABLE PARAMETERS. THE DEVICE OPTIMISATION

To increase the device efficiency it is necessary to change the initial beam characteristics or the undulator field parameters in the process of the beam passing through the laser cavity [7]. Efficiency estimations are usually based on the numerical methods. Nevertheless, evaluating procedure for system with tapering parameters may be carried out by means of the method stated above.

Consider some examples. The gain magnitude remains enough considerable if the value of  $\beta_{0z}$  differs from the resonance magnitude  $\beta_r$ . It is clear that the surplus of the velocity  $\delta = \beta_{zi} - \beta_{z0} > 0$  prolongs the process of electron decelerating and delays the particles capturing. In this case the EMR excitation in the same range of frequency is provided by the more high level of the triggering field.

The condition of electron trapping

$$\beta_z(z^*) = \omega_s / (k_s(z) + k_0) c \quad (7)$$

is not changed and efficiency is estimated by the ratio:

$$\eta = [(\beta_{0z} - \beta_z(z^*))\beta_{0z}\gamma_0] / [(\gamma_0 - 1)(1 - \beta_z(z^*))]. \quad (8)$$

The value of velocities difference  $\delta\beta_z = \beta_{0z} - \beta_z(z^*)$  at the beam current is defined from the nonlinear equation that in case using tube design beam is

$$\delta\beta_z(\delta\beta_z - \delta)^{3/2} = I_b / \Lambda I_A, \quad (9)$$

where  $\Lambda = 2^{7/2} a k_s / \gamma_0^5$ . If the initial beam energy exceeds by the 5% from its resonance value the EMR efficiency is higher on the same magnitude. But raising of the efficiency due to the additional part to the initial particles velocity leads to the significant practical difficulties and may be realised to small  $\gamma_0$  i.e. for the long wave EMR (because  $\delta\beta = \delta\gamma / \gamma^3$ ).

Therefore, other method of the efficiency increasing presents a great interest. We mean the varying of magnetic field parameters of the undulator ( $B_0$  and  $\lambda_0$ ). Let us consider the case of alteration of the undulator period. This situation is enough understandable. The decrease of period that follows to the beam passing supports the resonance condition in system. Suitable estimations are determined by the equations (5) - (9) but in later ratio  $\delta$  is equalled

$$\delta_{\text{und}} = [\lambda_0 / \lambda_0(z^*) - 1] / \gamma_0^2. \quad (10)$$

The magnitude of efficiency is approximated to the corresponding value of  $\delta$  (or  $\delta_{\text{und}}$ ):

$$\eta = \gamma_0^2 [\delta + (I_b / \Lambda I_A \delta)^{2/3}]. \quad (11)$$

For example, variation of the undulator length by the 10% allows to obtain the value of the EMR efficiency up to 11% at  $I_b = 200$  A and 12% at  $I_b = 600$  A ( $\gamma_0 = 30$ ). On the first view the FEL efficiency weakly depends on the beam current. However, the value of  $L_{\text{und}}$  is inversely proportional to  $I_b^{2/5}$ .

Consequently, the high-current FEL may operates with high efficiency in the regime of saturation. This conclusion remains true for the arbitrary beam configurations and different undulator designs. Notice that the significant value of the efficiency may be achieved in the only high-current systems. For example, in [8] experimental value of efficiency was not exceeded 3% when undulator parameters was changed to 12% and beam current was 7 A and  $\gamma_0 = 40$  (electron energy had been 20 MeV). On the more late experiments [8] electron current was raised to 100 A without quantitative beam deterioration. Nevertheless FEL efficiency was not achieved to optimal value. Indeed the FEL was made for the single-particle amplifying only. For example, when the undulator length was  $L_{\text{und}} = 100$  cm, then the FEL efficiency appeared to be  $\sim 1/L_{\text{und}}$  [2]. For this undulator the collective regime with saturating process may be observed in systems with beam current approximately a few kA.

Therefore magnitude of EMR efficiency in these devices may be higher due to using longer undulator or bigger electron current.

Coulomb defocusing of the electron beam is one of problems connecting with the high-current FEL. It may be eliminated by plasma filling of the laser cavity. This method was successfully used at the traditional electronic devices. Plasma at ordinary density  $n_p < 10^{13}$  do not brings any varieties to the resonance conditions of the FEL. Stimulating processes in system are subjected to more complex influence of neutral gas filled the laser cavity. In this case the resonance interaction of the both fast and slow beam modes are possible. So FEL with plasma filling requires the serious theoretical analysis.

Thus the analysis stated above emphasises that the high-current FEL may be both more powerful and more effective shortwave radiating source.

#### IV. REFERENCES

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