Resolution Improvement in Beam Profile Measurements with Synchrotron Light

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Abstract

Numerical method for improving optical resolution in electron beam profile measurements with visible synchrotron radiation (SR) is proposed. Image formation of electron beam profile is described by integral convolution-type equation of the first kind – the diagnostic equation. Precise procedure of computing the equation kernel in terms of classical electrodynamics is presented. With this procedure special features of the SR emission and diffraction scheme peculiarities may be taken into account. Numerical regularized solution of the diagnostic equation is shown to lead to the resolution improvement. The technique is supposed to be beneficial in high-energy storage rings.

I. INTRODUCTION

Visible SR is much used as a tool for beam profile measurements in synchrotrons and storage rings [1] - [4]. Fig.1 shows traditional layout of the measurements. Also, an extracting mirror absorbing short-wavelength SR is optionally used.

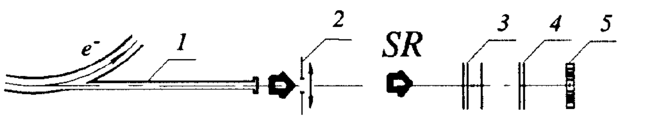

Figure 1. Scheme of measurements.
1- beamline; 2- focusing lens and limiting diaphragm; 3- neutral light filters; 4- monochromatic filter; 5-position-sensitive detector.

Unfortunately, in high-energy electron storage rings the diffraction-limited spatial resolution may be comparable with transversal beam dimensions [3], [4]. Beam profile image may also be distorted by aberrations, for example those resulting from thermal deformation of the extracting mirror.

If one can precisely describe the formation of beam profile image, then one can improve the optical resolution mathematically, by numerical processing of the measurement results. In this paper a diagnostic equation describing the image formation is treated. Technique for precise computation of the equation kernel is proposed. Results of simulation illustrating the numerical solution of the diagnostic equation are exhibited.

II. DIAGNOSTIC EQUATION

Detector response is proportional to the intensity of incident radiation. The visible SR emitted by different electrons is known to be dominantly incoherent in electron storage rings, if longitudinal bunch length is larger than micrometers. The beam profile image formed by focusing lens is insensitive to angular divergences of the emitting beam. Let x and z to be transversal Cartesian coordinates in the object plane (i.e., in the plane the lens is focused on), and x^*, z^* to be coordinates of a point in the detector screen. In line with the assumptions made, if $n(x,z)$ is transversal distribution of particle density in the object plane, then the intensity distribution in the detector screen $I(x^*,z^*)$ is related with $n(x,z)$ as

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} n(x,z)K(x,z,x^*,z^*)\,dxdz = I(x^*,z^*)\ , \qquad (1)$$

where $K(x,z,x^*,z^*)$ is radiation intensity at observation point (x^*,z^*) in the detector screen, which results from passage of a single particle along trajectory intersecting the object plane at (x, z).

Relations similar to (1) are commonly used in optics to describe image formation of extended incoherent source [5]. Function $K(x,z,x^*,z^*)$, being known from physical consideration, and $I(x^*,z^*)$, being determined by the detector, allow to treat relation (1) as the integral equation of the first kind with respect to $n(x,z)$ – the diagnostic equation. In practice $I(x^*,z^*)$ is averaged within detector exposure time, so $n(x,z)$ should be considered correspondingly.

To define the kernel $K(x,z,x^*,z^*)$, let us start from the Fourier transformation of electric field emitted by single electron in its motion along the trajectory $\vec{r}(\tau)$, the relation one can easily obtain from delayed potentials [6] for observation point $P'(\vec{r}\,')$ in space before lens,

$$\vec{E}_\omega = \frac{ie\omega}{c}\int_{-\infty}^{+\infty}\frac{\vec{\beta}-\vec{n}}{R}\exp\left[i\omega(\tau+R/c)\right]d\tau, \qquad (2)$$

where $\vec{\beta}=(d\vec{r}/d\tau)/c$ is relative velocity of electron, $\vec{n}=\vec{R}/R$, $\vec{R}=\vec{r}\,'-\vec{r}$, $R=|\vec{R}|$, ω is radiation frequency, e is the charge of electron, c is the speed of light, i is unit imaginary number. Eq. (2) is valid at $[c/(\omega R)]<<1$.

Two wave disturbances corresponding to σ- and π-components of $\vec{E}_\omega$, may be sufficiently considered in wave zone. For ultra-relativistic particle, the general contribution to the integral (2) takes place at $|\beta_x|<<1$; the radiation is directed mainly forward ($|n_x|<<1$, $|n_x|<<1$) [6], [7]. Therefore the σ- component wave disturbance may be given by the expression

$$U_\sigma \approx C\int_{-\infty}^{+\infty}\frac{n_x-\beta_x}{R}\exp\left[i\omega(\tau+R/c)\right]d\tau, \qquad (3)$$

where C is constant. Here and later on, the π- component expression is not presented: it may be readily written by direct analogy.

Eq. (3) may be regarded as superposition of disturbances from motionless coherent point sources arranged on the electron trajectory, each disturbance phase and magnitude

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being defined by the position of corresponding hypothetical source. A diffraction integral (see [5]), which allows to calculate the source contribution to the total disturbance value at point P($\vec{r}^*$) on the detector screen, can be written for each source. The total disturbance is given by the expression

$$U_o^* = C^* \int_{-\infty}^{+\infty} d\tau \int_{\Delta\Sigma} \frac{n_x - \beta_x}{R'' S} \exp\left[ig(\tau) + \frac{i\omega}{c}\left(S - R'' + \Psi\right)\right] d\Sigma, \quad (4)$$

where function $g(\tau)$ describes individual initial phase of each source, the value adequate to the image space; R'' is reference sphere radius (see Fig.2); S is distance between the observation point P* and point P' in the reference sphere; Ψ is total wave aberration; C^* is constant. The inner integration in Eq. (4) is on the reference sphere within lens diaphragm.

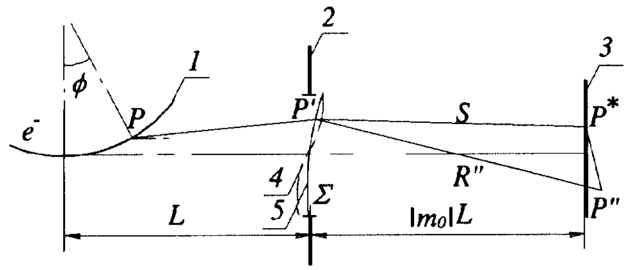

Figure 2. Diffraction scheme.
1- electron trajectory; 2- lens diaphragm plane; 3- detector plane; 4- wave front; 5- reference sphere.

If sufficiently narrow monochromatic filters are used, then the diagnostic equation kernel is defined as

$$K(x, z, x^*, z^*) = \left|U_\sigma^*\right|^2 + \left|U_\pi^*\right|^2. \quad (5)$$

The functions involved in Eq.(4) may by determined from the measurement geometry. Using angular variables of reference sphere integration, horizontal one ξ and vertical one ζ, and applying the expansions of all the phase functions in Eq.(4) in terms of these small values, one can obtain for the radiation from bending magnet

$$U_\sigma^* \approx C^{**} \int_{-\infty}^{+\infty} d\phi \exp(if_0) \int_{-\bar{\xi}}^{\bar{\xi}} (n_x - \beta_x) \exp(if_\xi \xi + if_{\xi\xi}\xi^2) d\xi \times$$
$$\times \int_{-\bar{\zeta}}^{\bar{\zeta}} \exp(if_\zeta \zeta + if_{\zeta\zeta}\zeta^2) d\zeta, \quad (6)$$

where the azimuth ϕ is used as angular integration variable instead of τ ; C^{**} is constant. The phase expansion coefficients f_0, f_ξ, f_ζ, $f_{\xi\xi}$, $f_{\zeta\zeta}$ depend on ϕ.

Numerical estimations of high-order term contributions show that approximation (6) allows to compute the diagnostic equation kernel with a precision of order 0.5%.

According to Eq.(6), variables ξ and ζ are uncoupled under the external integration on ϕ if the limits $\bar{\xi}$ and $\bar{\zeta}$ are constants; that essentially simplifies computation. Eq.(6) was written in assumption of small lens aberrations. Nevertheless, even if a large ξ - independent aberration takes place (for instance, the aberration due to thermal deformation of extracting mirror), the uncoupling structure of Eq.(6) retains.

The problem on optical resolution in beam profile measurements was discussed in [3], [4], where diffraction of synchrotron radiation and depth-of-field effect were considered separately. The method under discussion allows to treat these effects as single phenomena closely related with nature of synchrotron radiation.

In the computations discussed below, the coefficients f_0, f_ξ f_ζ $f_{\xi\xi}$ $f_{\zeta\zeta}$ were determined from exact geometrical relations. But it would be well to consider approximate values of the coefficients. It is obtainable,

$$\begin{cases} f_0 \approx \dfrac{\omega}{2\omega_0}\left(\gamma^{-2}\phi + \phi^3/3\right); \quad f_\xi \approx -\dfrac{\omega m_0}{2\omega_0}\phi^2 - \dfrac{\omega}{c}\left(x^* - m_0 x\right); \\ f_\zeta \approx -\dfrac{\omega}{c}\left(z^* - m_0 z\right); \quad f_{\xi\xi} \approx f_{\zeta\zeta} \approx \dfrac{\omega m_0^2}{2\omega_0}\phi, \quad n_x \approx m_0\xi; \quad \beta_x \approx \phi, \end{cases} \quad (7)$$

where ω_0 is cyclotron frequency, γ is the reduced energy of electron (γ >>1); m_0 is transversal optical magnification.

By analogy with unfocused SR, the cubic term in f_0 may be shown to prevail at $\omega << \omega_c$ ($\omega_c = 3\gamma^3\omega_0/2$ is critical SR frequency). Using the corresponding normalization in Eqs. (6), (7), one may see that if an acceptance angle is $|m_0|\bar{\zeta} << (\omega_0/\omega)^{1/3}$, then the vertical diffraction is Fraunhofer's one with the resolution of order $c/(\omega|m_0|\bar{\zeta})$, whereas at $|m_0|\bar{\zeta} >> (\omega_0/\omega)^{1/3}$ the optical resolution has the order of $c/(\omega^2\omega_0)^{1/3}$.

Eqs. (5) - (7) show that

$$K(x,z,x^*,z^*) \cong \mathcal{K}(x^* - m_0 x,\ z^* - m_0 z), \quad (8)$$

i.e., to certain accuracy Eq.(1) may be treated as a convolution type integral equation. Numerical analysis shows the accuracy to be better than 1% for high-energy electron storage rings. Analytically the feature was studied in [8].

Relation (8) allows to apply effective methods based on the convolution theorem for the numerical solution of Eq.(1). The regularized solution of Eq.(1) may be written as follows [9], [10]:

$$n_\alpha(x,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\tilde{\mathcal{K}}(-\omega_x, -\omega_z)\tilde{I}(\omega_x, \omega_z)\exp(-im_0 x\omega_x - im_0 z\omega_z)}{\left|\tilde{\mathcal{K}}(\omega_x, \omega_z)\right|^2 + \alpha M(\omega_x, \omega_z)} d\omega_x d\omega_z, \quad (9)$$

where $\tilde{\mathcal{K}}(\omega_x, \omega_z)$ and $\tilde{I}(\omega_x, \omega_z)$ are Fourier transformations of the kernel and the measured intensity; α is regularization parameter; function $M(\omega_x, \omega_z)$ suppresses high-frequency component of the detector noise. The solution may be found by iteration as well as directly (if certain *a priori* information on its behavior is known).

II. COMPUTATION RESULTS

Computations of the diagnostic equation kernel for bending magnet radiation of 2.5 GeV electron storage ring Siberia-2 were performed according to Eqs. (5), (6). The kernel $\mathcal{K}(x,z)$ computed for radiation wavelength λ=540nm, bending radius r_0=1960cm, distance from object plane to lens L=700cm, the lens diaphragm widths d_x=2cm, d_z=6cm; $m_0=-1$, is shown in Fig.3.

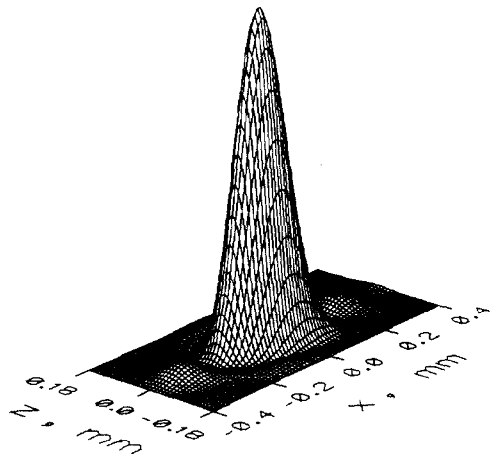

Figure 3. Diagnostic equation kernel.

The computation accounts only for the σ- component of SR. Secondary maxima inherent in Fraunhofer diffraction are recognizable in horizontal direction. Corresponding maxima are absent in vertical direction; the distribution is symmetric with respect to median plane. $\mathcal{K}(x,z)$ is sensitive to d_z/L, d_x/L, but with these values large enough the sensitivity eliminates; it correlates well with qualitative considerations given in previous chapter.

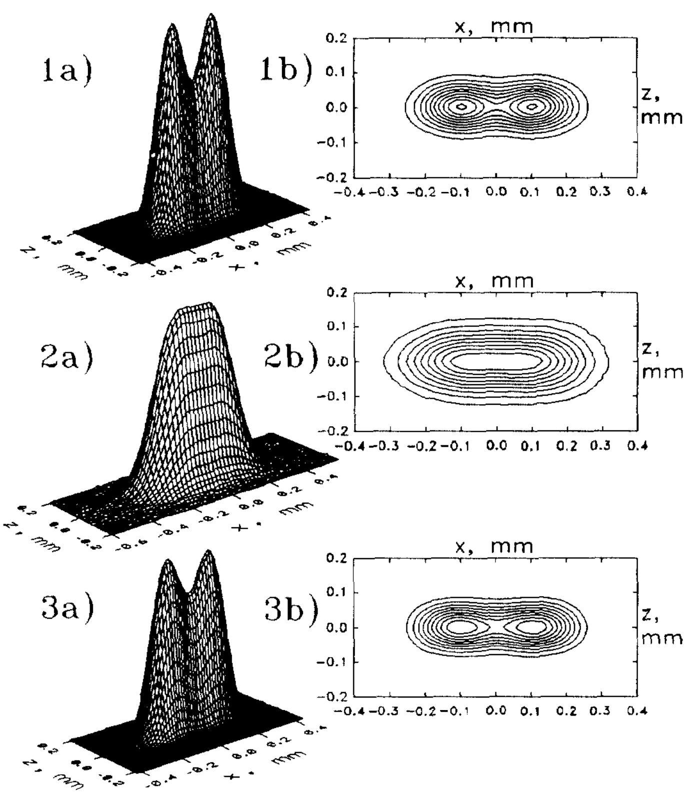

Figure 4. Results of simulation ((a)- surfaces, (b)- level lines): 1- $n(x,z)$; 2- $I(x,z)$ deviated by noise; 3- $n_\alpha(x,z)$.

As an illustration to processing the data on beam profile measurements, Fig.4 gives the results of the corresponding simulation. The simulation was done according to the following traditional algorithm: a function $n(x,z)$ modeling the transversal distribution of particle density was chosen; corresponding SR intensity distribution in the detector plane $I(x^*,z^*)$ was computed by Eqs. (1), (8); $I(x^*,z^*)$ was distorted by random noise assuming the detector dynamic range to be 100; the solution of diagnostic equation, $n_\alpha(x,z)$ was found for the distorted intensity by Eq. (9) in accordance with regularization technique.

The procedure of regularized solution of the diagnostic equation is equivalent to the use of hypothetical measurement system providing higher spatial resolution. Simulations show the possibility of increasing the spatial resolution in beam profile measurements about 1.5 - 2.5 times and even more (the value depends on detector dynamic range, applied regularization algorithm, as well as the solution behavior).

III. SUMMARY

Proposed technique is expected to be efficient when optical resolution in beam profile measurements is comparable with actual beam dimensions, as it takes place in high-energy storage rings. The method proposed for computation of the diagnostic equation kernel allows to take into account practically all main distortion sources in the measurements. The technique may be easily adapted to particular experimental conditions.

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