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Vacuum Tracking

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Abstract

A method to determine longitudinal pressure profiles in the presence of pumps and outgassing elements in conductance limited vacuum systems by a transfer matrix formalism is discussed. The algorithm is capable of dealing with multiple connected vacuum systems. An implementation of this method in the computer codes VAKTRAK and VAKLOOP is briefly described.

I. INTRODUCTION

The longitudinal pressure profile $P(z)$ in a conduction limited vacuum system obeys the following linear differential equation in the presence of pumps and outgassing elements [1]

$$c\frac{d^2P}{dz^2} - sP = -q \tag{1}$$

where the symbols are explained in table 1. We stress that the presented method is only applicable in the molecular flow regime, where the mean free path of the molecules is much larger than the dimensions of the pipes and manifolds such that the viscosity of the gas is negligible.

Table 1: *Definitions of symbols used.*

Quantity	Units	Explanation
P	[Torr]	pressure
Q	[Torr l/s]	gas flow
z	[m]	longitudinal position
c	[m l/s]	specific conductance
s	[l/s m]	linear pumping speed
q	[Torr l/s m]	specific outgassing rate
S	[l/s]	integrated pump speed
L	[m]	element length

Under the assumption that the specific conductance c, the specific pumping speed s, and the specific outgassing rate q are piecewise constant, equation 1 is an ordinary differential equation with (piecewise) constant coefficients. Thus it can be solved by the method of transfer matrices.[1] The general transfer matrix can be easily found from the sum of homogeneous and inhomogeneous solution of eq. 1, namely

$$P(z) = C_1 e^{\alpha z} + C_2 e^{-\alpha z} + \frac{q}{s} \tag{2}$$

with $\alpha = \sqrt{s/c}$. Solving for initial conditions we obtain a transfer matrix R for the pressure P and the gas flow $Q = -c\,dP/dz$, assempled in a state vector $(P, Q, 1)$ which reads

$$\begin{aligned}
R_{11} &= \cosh(\sqrt{s/c}L) \\
R_{12} &= -\frac{L}{c}\frac{\sinh(\sqrt{s/c}L)}{\sqrt{s/c}L} \\
R_{13} &= -\frac{qL^2}{c}\frac{\cosh(\sqrt{s/c}L) - 1}{(s/c)L^2} \\
R_{21} &= -c\sqrt{s/c}\sinh(\sqrt{s/c}L) \\
R_{22} &= \cosh(\sqrt{s/c}L) \\
R_{23} &= qL\frac{\sinh(\sqrt{s/c}L)}{\sqrt{s/c}L} \\
R_{33} &= 1 \quad , R_{31} = R_{32} = 0
\end{aligned} \tag{3}$$

From eq. 3 we can now deduce special cases for elements which do only outgassing or pumping. A piece of beam line which neither pumps or outgasses is given by eq. 3 in the limit $q \to 0$ and $s \to 0$. A long pump is given by $q \to 0$. A very short pump can be described by eq. 3 with $L \to 0$ but constant integrated pump strength $S = sL$. The transfer matrix of a long outgassing element is given by eq. 3 in the limit of $s \to 0$.

Given the transfer matrices it is possible to *track* a given pressure profile through a *vacuum lattice* if the initial values P_0 and Q_0 are known. Another option is to find the initial values under the assumption of periodicity, i.e., if one considers a sequence of equal *vacuum cells*. Given a

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[1] This approach was originally initiated by the author at the Universität Dortmund and led to the development of a PC based program CLOBIVAC [2] by M. Michel.

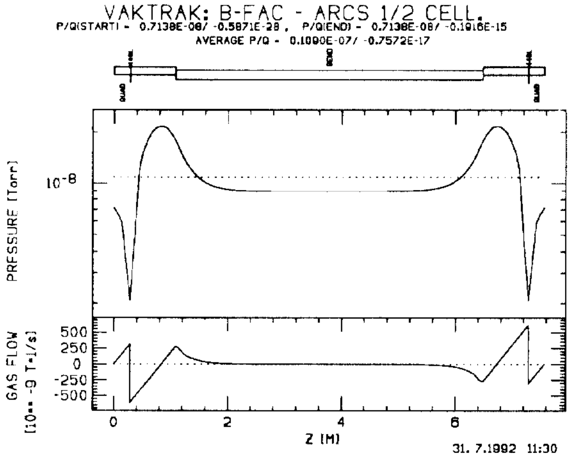

Figure 1: *Longitudinal Pressure Profile*

cumulative transfer matrix $\tilde{M}$ through a vacuum structure

$$\tilde{M} = \begin{pmatrix} M & \vec{v} \\ 0 & 1 \end{pmatrix} , \tag{4}$$

where $M_{ij} = \tilde{M}_{ij}$ for $i, j = 1, 2$ and $v_i = \tilde{M}_{i3}$ for $i = 1, 2$, the periodic solution $\vec{P}_0$ is given by

$$\vec{P}_0 = (1 - M)^{-1} \vec{v} , \tag{5}$$

where $\vec{P}_0$ stands for the vector (P_0, Q_0).

Observe that a transfer matrix represents two equations that relate two components of the input vector $\vec{P}_0$ to two components of the vector at the end of the vacuum beam line. By specifying two of the four values the system can be solved for the other two. In this way more general boundary conditions can be taken into account.

As an example consider a synchrotron radiation beam line. At one end it is linked to the storage ring which can be assumed to be held at constant pressure and at the other end it is closed off, such that the gas flow is zero. If the vacuum components in the beam line are specified the system can be solved for the incoming gas flow from the storage ring and the pressure at the closed off end.

II. VAKTRAK

The code VAKTRAK contains an implementation of the matrices given by eq. 3. It first reads an input file that contains the sequence of elements and their corresponding properties like length, conductance, pump speed and outgassing rate and constructs the transfer matrices. The code then prompts for the boundary conditions and continues to calculate the pressure profile and the gas flow. Finally it writes a TOPDRAW [3] file that displays the vacuum lattice, the pressure profile, and the gas flow as shown in fig. 1. The details of operating VAKTRAK are explained in ref. 4.

One has to keep in mind that this method is numerically touchy and the calculation of the solution for very long vacuum lattices is numerically unstable, so the user must be careful in interpreting the output. This problem is alleviated as far as possible by using `REAL*16` variables in internal calculations. It should be noted that this problem mainly arises in systems with large pumps and small conductance, i.e. systems, which are physically not well designed.

The beam-gas scattering lifetime in a storage ring is determined by the average of the pressure and the beta function over the circumference of the ring [5]. VAKTRAK provides the option to specify a magnet lattice in TRANSPORT style input [6]. This file is read and the periodic solution for the beta functions is determined. Then the averages of β, P, and βP are calculated and displayed. Using this routine it is easy to optimize the placement of pumps in a vacuum system, and taking the detailed behavior of the beta functions into account.

III. MULTIPLE CONNECTED VACUUM SYSTEMS

So far we are dealing with single segments of beam line. Using periodic boundary conditions is equivalent to either an infinite array of such segments or a circular segment. In order to take more complex geometries into account such as pump manifolds connected to beam pipes at various locations, we have to generalize the concept of periodic boundary conditions.

This generalization is based on Kirchhoff-like rules for vacuum systems [7,8]. They are based on the observation that in vacuum systems the pressure plays the same role as the voltage in electric circuits and the gas flow behaves analogously to the current. Using these observations we can state two rules:

1. the sum of gas flows into a node is zero,

2. the sum of pressure differences around a closed loop is zero.

These rules now take the place of periodic boundary conditions.

A further complication arises from the number of unknowns we have to solve for. For our purposes we choose the pressure at each of the n nodes and the gas flows into and out of a link. In this way we have to deal with $n + 2l$ unknowns, where l is the number of links. Now we have to find the same number of relations among the unknowns in order to find the systems' equilibrium configuration.

The first of the above rules relates the fluxes flowing into and out of each node, thus yielding n equations among the unknowns. The second rule allows us to use the transfer matrices for each of the links j to relate the pressure at the starting node of a link P_s and the gas flow into that link Q_{sj} to the pressure of the ending node of link P_e and the gas flow out of the link Q_{ej} according to

$$\begin{pmatrix} P_e \\ Q_{ej} \end{pmatrix} = \begin{pmatrix} M^j_{11} & M^j_{12} \\ M^j_{21} & M^j_{22} \end{pmatrix} \begin{pmatrix} P_s \\ Q_{sj} \end{pmatrix} + \begin{pmatrix} v^j_1 \\ v^j_2 \end{pmatrix} . \quad (6)$$

In this way we obtain $2l$ more equations for a total of $n+2l$ linear equations among the pressures at the nodes and gas flows into and out of the links. The solution of this system is a simply accomplished by any routine that solves linear equations, e.g., those using a Gauss-Jordan algorithm. One should, however, pay attention to numerical instabilities, because the transfer matrix elements can become very large. The cure is to use the highest available precision on the computer implementation.

The described algorithm is implemented in the code VAKLOOP, which reads an input file that contains the segments (links) element by element just as VAKTRAK does with the exception that the segments are separated by input lines that specify the starting and ending node of the following segment. VAKLOOP then calculates the transfer matrices for each segment and sets up the linear equations as described in the previous section and solves them. Furthermore the code generates plots like those shown in fig. 1 for each of the segments. After the calculation is done a few consistency checks are performed (also needed for accuracy) to test whether the solution actually solves the linear system.

IV. CONCLUSIONS

We presented a method to calculate longitudinal pressure profiles in very general conduction limited vacuum networks. The method is implemented in computer codes which allow fast and simple evaluation of the pressure in such systems.

This method is closely related to a matrix formalism treating charged particle beams in magnetic systems. The pressure takes the role of the transverse position x and the gas flow that of the angle x'. Some of the transfer matrices discussed in the second section of this note can be directly identified with magnetic elements. Pressing this analogy a little further we can identify vacuum gauges with beam position monitors. It needs to be investigated in the future whether this can be used to determine the position of leaks in vacuum systems.

In this note the pumps are assumed to be linear ($S = Q/P$). However, this restriction can easily overcome. The equations in this case will become weakly nonlinear, but can still be solved iteratively [2].

Furthermore, the codes currently deal with one gas at a time and the pressures are partial pressures for that gas. It is easy, at the expense of a lot of bookkeeping, to improve the codes to allow to deal with different pressures at the same time.

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REFERENCES

1. O. Gröbner, *Vacuum Systems,* in CERN 85-19. p. 489.
2. M. Michel, *Überlegungen zum Ultrahoch-Vakuumsystem für DELTA,* Diploma Thesis, Universität Dortmund, 1988.
3. R. Chaffee, *TOPDRAWER,* SLAC-CGTM 178, 1989 rev.
4. V. Ziemann, *Vacuum Tracking,* SLAC-PUB 5962, 1992.
5. J. LeDuff, *Current and Density Limitations in Existing Electron Storage Rings,* Nucl. Instr. Meth. **A239**, 83, 1985.
6. K. Brown, et. al., *TRANSPORT Manual,* SLAC-91, 1977.
7. G. Horikoshi, Y. Saito, K. Kakihara, *An analysis of complex network of vacuum components and its application,* KEK-Preprint-89-138, 1989.
8. H. Hirano, Y. Kondo, N. Yoshimura, *Matrix calculation of pressures in high-vacuum systems,* J. Vac. Sci. Technology **A6**, 2865, 1988.