Beam Coupling Impedances of Axial Symmetric Structures

W. Bruns

Technische Universität Berlin, EN-2, Einsteinufer 17, D-1000 Berlin 10, Germany

Abstract

The transverse and longitudinal coupling impedances of axial symmetric geometries with an arbitrary number of cross section jumps are calculated. Field matching with renormalized wave amplitudes is applied at planes $z = z_n$. The resulting linear equations are solved straightforward, leading to good numerical stability. The Dirac-like pulses of the impedances below cut-off or of structures with trapped modes are avoided using a complex permittivity. Numerical results are presented for a sample detuned structure with more than 400 different radii.

I. Introduction

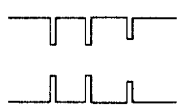
The structure under consideration is a beam pipe with an arbitrary number of different cross section jumps. Parallel to the axis a charge Q travels with a constant velocity $\vec{v} = \vec{e}_z\beta c$. This charge excites a field that is scattered by the inhomogeneous boundary. The scattered field acts on a test charge following behind the exciting charge.

In this paper, the Fourier transformed field is expanded in orthogonal functions, each a solution of Maxwells equations. The continuity and boundary conditions at the planes of the cross section jumps are fulfilled by mode matching. The mode matching is applied at all cross section jumps simultaneously, leading to a single linear system of equations. The effect of the scattered field is expressed in terms of coupling impedances.

In previous studies[2]..[5], similar problems were examined. In [2] no azimuthal dependence was allowed. In [3] the azimuthal dependence $m = 1$ was considered, but with other expansion functions. In [4] a similar problem with azimuthal dependence, but without a current was calculated using scattering matrices. In [5] an even more general problem is considered, where there are jumps also in the φ-direction and with TEM-waves in coaxial segments. Their calculation also uses scattering matrices.

II. Geometry

Consider an infinitely long beam pipe with N cross section jumps. The jumps lie in the planes $z=z_n$, $n=0..N\text{-}1$. The charge travels at a distance $\rho=\rho_q$ off axis, at the azimuthal angle $\varphi=0$.



Inside the cylinder $\rho = \rho_q$ is the area $A_n^{(1)}$, outside the area $A_n^{(2)}$. Left of the plane z_n is the area $A_n^{(i)}$ with radius r_n, right of it the area $A_{n+1}^{(i)}$ with radius r_{n+1}.

III. The excitation

The moving charge represents a current density $\vec{J}[\mathrm{Am}^{-2}]$.

$$\vec{J}(t) = \vec{v}\varrho = \vec{e}_z \int_{-\infty}^{+\infty} J(\omega)e^{+j\omega t}\, d\omega =$$

$$\vec{e}_z \int_{-\infty}^{+\infty} Qe^{-jkz/\beta}\frac{\delta(\rho-\rho_q)}{\rho}\sum_{m=0}^{\infty}\frac{\cos(m\varphi)}{\pi(1+\delta_m^0)}e^{+j\omega t}\, d\omega$$

In the following only the Fourier transformed entities and only the m^{th} component of the Fourier series are considered.

IV. Field representation

The tangential fields in the n^{th} area A_n at the plane $z = z_n$ are represented by

$$\begin{aligned}
\mu_n\vec{e}_z \times \vec{H}_{tn} &= \sum_{s=1}^{\infty}\left[k^2\vec{F}_{ns}^{TM}Z_{ns}^{TM} - \vec{F}_{ns}^{TE}\frac{\partial}{\partial z}Z_{ns}^{TE}\right]_{z=z_n} + \\
&\quad + \vec{e}_z \times \vec{B}_{tn}^{TMS(i)}(z_n) \\
\frac{\vec{E}_{tn}}{-j\omega} &= \sum_{s=1}^{\infty}\left[\vec{F}_{ns}^{TM}\frac{\partial}{\partial z}Z_{ns}^{TM} + \vec{F}_{ns}^{TE}Z_{ns}^{TE}\right]_{z=z_n} + \\
&\quad + \frac{1}{-j\omega}\vec{E}_{tn}^{TMS(i)}(z_n) \\
\vec{F}_{ns}^{TM} &= \vec{e}_\rho\frac{d}{d\rho}J_m(j_{m,s}\rho/r_n)\cos(m\varphi) + \\
&\quad + \vec{e}_\varphi\frac{1}{\rho}J_m(j_{m,s}\rho/r_n)\frac{d}{d\varphi}\cos(m\varphi) \\
\vec{F}_{ns}^{TE} &= \vec{e}_\rho\frac{1}{\rho}J_m(j_{m,s}'\rho/r_n)\frac{d}{d\varphi}\sin(m\varphi) - \\
&\quad - \vec{e}_\varphi\frac{d}{d\rho}J_m(j_{m,s}'\rho/r_n)\sin(m\varphi) \\
&\quad TX \in \{TM, TE\} \\
Z_{ns}^{TX}(z) &= C_{ns}^{TX}e^{+jq_{ns}^{TX}(z-z_{n-1})} + D_{ns}^{TX}e^{-jq_{ns}^{TX}(z-z_n)} \\
q_{ns}^{TX} &= \sqrt{\omega^2\mu_n\varepsilon_n - {p_{ns}^{TX}}^2}
\end{aligned}$$

$$p_{ns}^{TM} = \frac{j_{m,s}}{r_n};\ p_{ns}^{TE} = \frac{j_{m,s}'}{r_n};\ J_m(j_{m,s}) = J_m'(j_{m,s}') = 0$$

The exponential factors are chosen to be one at the planes $z = z_n$ and in magnitude less or equal one in the areas A_n.

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The source fields are represented by:

$$\vec{e}_z \times \vec{B}_{tn}^{TMS(i)}(z_n) = k^2 e^{-jkz_n/\beta} \times$$
$$\left\{ \vec{e}_\rho \frac{d}{d\rho} R_n^{(i)}(\rho) \cos(m\varphi) + \vec{e}_\varphi \frac{1}{\rho} R_n^{(i)}(\rho) \frac{d}{d\varphi} \cos(m\varphi) \right\}$$
$$\frac{1}{-j\omega} \vec{E}_{tn}^{TMS(i)}(z_n) = \left[\frac{d}{dz} e^{-jkz/\beta} \right]_{z=z_n} \times$$
$$\left\{ \vec{e}_\rho \frac{d}{d\rho} R_n^{(i)}(\rho) \cos(m\varphi) + \vec{e}_\varphi \frac{1}{\rho} R_n^{(i)}(\rho) \frac{d}{d\varphi} \cos(m\varphi) \right\}$$

(abbreviations: $\gamma = 1/\sqrt{1-1/\beta^2}$; $\zeta_q = \frac{\rho_q k}{\gamma}$; $\zeta_n = \frac{\rho_n k}{\gamma}$):

$$R_n^{(1)}(\rho) = A_n^{(1)} J_m(\frac{\rho k}{\gamma})$$
$$R_n^{(2)}(\rho) = A_n^{(2)} J_m(\frac{\rho k}{\gamma}) + B_n^{(2)} Y_m(\frac{\rho k}{\gamma})$$

The amplitudes A, B of the source fields are given trough

$$\begin{pmatrix} -J_m'(\zeta_q) & J_m'(\zeta_q) & Y_m'(\zeta_q) \\ -J_m(\zeta_q) & J_m(\zeta_q) & Y_m(\zeta_q) \\ 0 & J_m(\zeta_n) & Y_m(\zeta_n) \end{pmatrix} \begin{pmatrix} A_n^{(1)} \\ A_n^{(2)} \\ B_n^{(2)} \end{pmatrix} = \left(\frac{\gamma \mu_n Q}{\rho_q k^3 \pi (1+\delta_m^0)}, 0, 0 \right)^T$$

V. Mode Matching

At the common boundary of two areas A_n, A_{n+1}, the tangential fields have to be continuous. At the metallic walls the tangential E-field must vanish:

$r_s = \min(r_n, r_{n+1})$; $r_g = \max(r_n, r_{n+1})$

$$\begin{aligned} \vec{H}_{tn}(z_n) &= \vec{H}_{t,n+1}(z_n); & 0 \le \rho \le r_s \\ \vec{E}_{tn}(z_n) &= \vec{E}_{t,n+1}(z_n); & 0 \le \rho \le r_s \\ \vec{E}_{ts}(z_n) &= 0 & r_s \le \rho \le r_g \end{aligned}$$

Expansion of the continuity- and boundary conditions on the tangential components yields a linear system of equations for the unknown C_{ns}^{TX}, D_{ns}^{TX}:

$i \in \{1 \cdots \infty\}$; $TX \in \{TM, TE\}$

$r_s = \min(r_n, r_{n+1})$; $r_g = \max(r_n, r_{n+1})$;

$$\frac{1}{\mu_n} \int_0^{2\pi} \int_0^{r_s} [\vec{e}_z \times \vec{B}_{tn}] \vec{F}_{si}^{TX} \, \rho \, d\rho \, d\varphi = \frac{1}{\mu_{n+1}} \int_0^{2\pi} \int_0^{r_s} [\vec{e}_z \times \vec{B}_{t,n+1}] \vec{F}_{si}^{TX} \, \rho \, d\rho \, d\varphi$$

$$\int_0^{2\pi} \int_0^{r_n} \vec{E}_{tn} \vec{F}_{gi}^{TX} \, \rho \, d\rho \, d\varphi = \int_0^{2\pi} \int_0^{r_{n+1}} \vec{E}_{t,n+1} \vec{F}_{gi}^{TX} \, \rho \, d\rho \, d\varphi$$

The occurring integrals $\int\int \vec{F}_{nj}^{TX} \vec{F}_{li}^{TY} \, \rho d\rho d\varphi$ can be evaluated in closed form. Sorting the equations yields a linear system for the unknown C_{ns}^{TX}, D_{ns}^{TX}:

$$\begin{pmatrix} \mathbf{LC}_n^H & \mathbf{LD}_n^H & \mathbf{RC}_n^H & \mathbf{RD}_n^H \\ \mathbf{LC}_n^E & \mathbf{LD}_n^E & \mathbf{RC}_n^E & \mathbf{RD}_n^E \end{pmatrix} \begin{pmatrix} \vec{C}_n \\ \vec{D}_n \\ \vec{C}_{n+1} \\ \vec{D}_{n+1} \end{pmatrix} = \begin{pmatrix} \vec{HS}_n \\ \vec{ES}_n \end{pmatrix}$$

The matrices $\mathbf{LC}_n^H, \mathbf{LD}_n^H$ come from the expansion of the source free H-field in the n^{th} area at the plane $z = z_n$ (left of it), the matrices $\mathbf{RC}_n^H, \mathbf{RD}_n^H$ from the source free field in the $(n+1)^{\text{th}}$ area (right of z_n). Similar are $\mathbf{LC}_n^E, \mathbf{LD}_n^E, \mathbf{RC}_n^E, \mathbf{RD}_n^E$ matrices containing the coupling integrals for the source free E-field. The vectors $\vec{C}_n, \vec{D}_n$ contain the unknown amplitudes of the waves travelling forward or backward respectively. The vectors $\vec{HS}_n, \vec{ES}_n$ contain the expansion integrals of the source fields.

There are N such equations for the N cross section jumps. The submatrices are of order infinity, so are the column vectors. After truncation of the submatrices to a size M, all equations together constitute $N \times 2M$ equations for the $(N+1) \times 2M$ unknown C_{ns}^{TX}, D_{ns}^{TX}. The C_{0s}^{TX} of the leftmost area A_0 have to be zero, as they represent waves coming from a nonexistent source at $z = -\infty$. A similar argument holds for the D_{Ns}^{TX}. Half of the submatrices are diagonal due to the orthogonality of the expansion functions. The equations can be swapped to a structure as (\ means a diagonal matrix, $\mathbf{X}$ means a dense matrix)

$$\begin{pmatrix} \backslash & & \mathbf{X} & \mathbf{X} & & & & & & & & \\ \mathbf{X} & \backslash & \backslash & & & & & & & & & \\ & \backslash & \backslash & \mathbf{X} & \mathbf{X} & & & & & & & \\ & \mathbf{X} & \mathbf{X} & \backslash & \backslash & & & & & & & \\ & & & \backslash & \backslash & \mathbf{X} & \mathbf{X} & & & & & \\ & & & \mathbf{X} & \mathbf{X} & \backslash & \backslash & & & & & \\ & & & & & \backslash & \backslash & \mathbf{X} & \mathbf{X} & & & \\ & & & & & \mathbf{X} & \mathbf{X} & \backslash & \backslash & & & \\ & & & & & & & \backslash & \backslash & \mathbf{X} & & \\ & & & & & & & \mathbf{X} & \mathbf{X} & \backslash & & \end{pmatrix} \vec{x} = \vec{b}$$

This linear system can be transformed by scaling to a very well conditioned one with the diagonal elements being unity and the off-diagonal elements all less then unity.

VI. Coupling Impedances

A test charge Q_p travelling behind the exciting charge at an azimuthal angle φ_p experiences a force by the scattered field. This force changes the impulse of the test charge. Under the assumption, that the velocity of both the exciting charge and the test charge is near the velocity of light, the change in the impulse will not change the velocity but the mass. The integrated impulse deviation (the kick) of the test charge travelling with the same velocity as the exciting charge at a distance $\Delta z = \tau v$ behind is a function of this time τ.

$$\vec{w}(\tau)\,[V] = -\frac{1}{c\beta Q_p} \int_{-\infty}^{+\infty} \frac{d}{dt}(m\vec{v}) \, dt$$
$$= -\frac{1}{c\beta Q_p} \int_{-\infty}^{+\infty} \left[Q_p \vec{E}(\vec{s}(t), t) + Q_p \vec{v} \times \vec{B}(\vec{s}(t), t) \right] dt$$

The Fourier transform of the kick is proportional to the longitudinal- and transverse coupling impedances.

$$Z_L[\mathrm{V/A}] = W_z(\omega) \frac{1}{Q(\rho_q/r_0)^m (\rho_p/r_0)^m \cos(m\varphi_p)}$$

$$= \frac{-\int_{-\infty}^{+\infty} E_z(\rho_p, \varphi_p, z, \omega)\, e^{+jkz/\beta}\, dz}{Q(\rho_q/r_0)^m(\rho_p/r_0)^m \cos(m\varphi_p)}$$

$$Z_T[\mathrm{V/A}] = W_\rho(\omega)\frac{-j}{Q(\rho_q/r_0)^m(\rho_p/r_0)^{m-1}\cos(m\varphi_p)}$$

$$= \frac{-\int_{-\infty}^{+\infty} [E_\rho(\rho_p, \varphi_p, z, \omega) - c\beta B_\varphi(\rho_p, \varphi_p, z, \omega)]\, e^{+jkz/\beta} dz}{jQ(\rho_q/r_0)^m(\rho_p/r_0)^{m-1}\cos(m\varphi_p)}$$

The factors are chosen to make the Fourier transforms impedances and to make the impedances in the ultrarelativistic case independent of the radii ρ_p and ρ_q.

VII. Numerical results

In [1] a design algorithm for a detuned accelerator is proposed. This procedure was used to generate the geometry data of a detuned accelerator with 204 cells. The cells are designed to have the same resonance frequency for the monopole mode ($m = 0$) and a Gaussian frequency distribution for the dipole resonances ($m = 1$) with a mean of 15,39 GHz and a variance of 0,39 GHz. This expected resonance density is shown in Fig. 3. The iris radii vary from $0,39\text{cm} \leq a_n \leq 0,54\text{cm}$, the caviti radii are between $1,05\text{cm} \leq b_n \leq 1,12\text{cm}$. The iris thickness and gap width are held constant at $t = 0,146\text{cm}$ and $g = 0,729\text{cm}$. In [2] the longitudinal impedance ($m = 0$) of the same geometry was computed. They had designed coupling cavities at the left and right of the structure. These coupling cavities also were used here.

The resulting structure has 413 cross section jumps. The calculation was performed with 10 TM and 10 TE modes per area, thus, the order of the linear system was $2 \times 20 \times 413$. The normalization radius was r_0=1cm.

The structure has trapped modes. The impedance at their resonances has a characteristic like Dirac pulses. This behaviour is circumvented by using a complex permittivity $\varepsilon_r = 1+j10^{-4}$. The impedance gets a Gaussian shape. The resulting transverse impedance (m=1) is shown in Fig. 2.

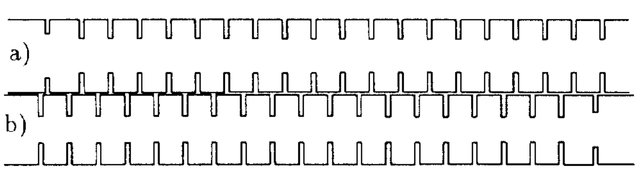

Figure 1: a) The left coupler and the first 18 cells of the detuned structure; b) The last 18 cells and the right coupler.

VIII. Acknowledgement

I like to thank Manfred Filtz for giving me first insights in mode matching.

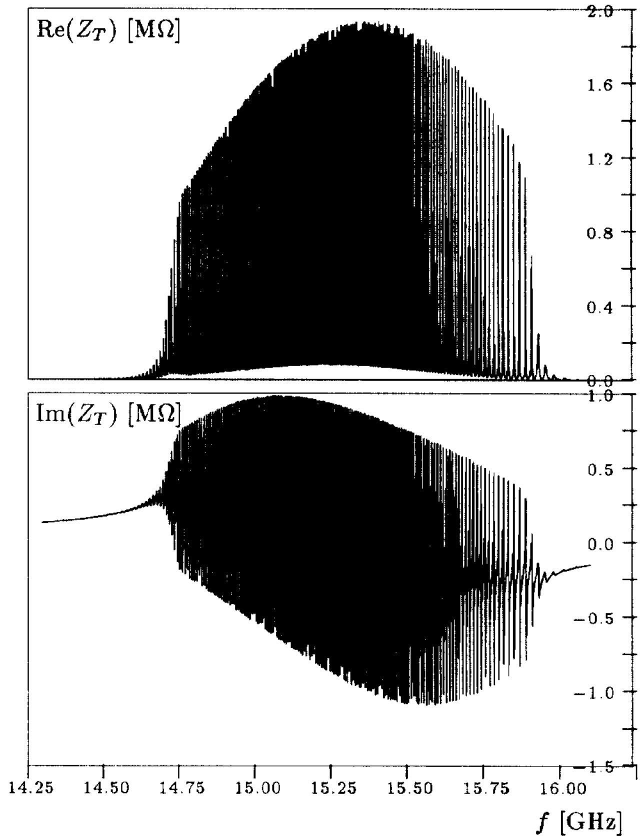

Figure 2: Transverse impedance in the detuned structure

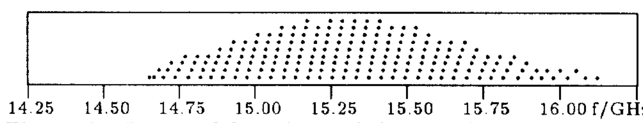

Figure 3: Expected locations of the 204 dipol resonances in the detuned structure

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