THE PHYSICAL MECHANISM OF ULTRA-FAST AUTOMATIC COOLING FOR BEAMS IN THE SIX-DIMENSIONAL EMITTANCE SPACE①

Gao Shuyang, Qian Guangyu, Liang Dongqi and Sun Hancheng
China Institute of Atomic Energy
P. O. Box 275(3), P. R. C. 102413 ,Beijing, P. R. China

Abstract

This paper is to illustrate that the four-dimensional emittance is automatically shrinked in the particular static magnetic field, while the vertical emittance is still a constant. It is the physical mechanism that the transverse velocity of any particles is not only decreased but also makes the longitudinal velocity approximate to the velocity of the equilibrium particle.

1. INTRODUCTION

The ultra-fast automatic cooling for beams (U-FACB) was found out [1]. In the work followed [2] , the damping phase diagrams in the emittance space were obtianed. Such show that UFACB of the emittance spase in the transverse direction is true indeed. In this paper the result of UFACB in six-dimesional emittance space will be given, so we are sure that the phase density of the emittance space is increased indeed. By means of the simply and explicitly physical picture of the transfer between both of the transverse and longitudinal energy, the mechanism of UFACB is illustrated. The transverse cooling is due to the energy transfering to the longitudinal direction, which also results in the longitudinal cooling, so the six-dimesional cooling rate is much faster than the transverse cooling rate.

2. THE TRANS FORMATION OF THE PHASE VOLUME

The relativistic Hamiltonian for a charged particle moving under the influence of an electromagnetic field [3] is

$$H=c\left[\frac{1}{(1+kx)^2}(p_s-eA_s)^2+(p_x-eA_x)^2+(p_y-eA_y)^2+m^2c^2\right]^{\frac{1}{2}}+eV \tag{2.1}$$

Since the particle velocity v=const, let us introduce the relative velocity

$$\frac{d}{dt}=v\cdot\frac{d}{vdt}$$

$$(1+kx)^2\cdot\dot{s}^2+\dot{x}^2+\dot{y}^2=1 \tag{2.2}$$

The corresponding canonical momentum can be represented by [3]

$$p_s=\vec{p}\cdot\hat{s}\cdot(1+kx) \tag{2.3}$$
$$=m\gamma v(1+kx)^2\cdot\dot{s}+eA_s$$
$$p_x=m\gamma v\dot{x}+eA_x$$
$$p_y=m\gamma v\dot{y}+eA_y$$

in the uniform field, let

$$\frac{e}{m\gamma v}B=k \tag{2.4}$$

the corresponding vector potentials are

$$A_s=\vec{A}\cdot\hat{S}\cdot(1+kx) \tag{2.5}$$
$$=-\frac{1}{2}\cdot\frac{p_o}{e}(1+kx)^2$$
$$A_x=A_y=0$$

the eq. (2. 3) becomes

$$H=c\left\{\frac{1}{(1+kx)^2}\left[p_s+\frac{1}{2}p_o(1+kx)^2\right]^2+p_x^2+p_y^2+m^2c^2\right]\}^{1/2} \tag{2.6}$$

Let us consider the four-dimensional emittance on the horizontal direction. In the canonical phase space, we have

$$du=dx.\,dp_x.\,ds.\,dp_s \tag{2.7}$$

in the emittance space, there is

$$dv=dx\cdot d\dot{x}\cdot ds\cdot d\dot{s} \tag{2.8}$$

they are related to each other by Jacobian determinant J:

$$du=Jdv \tag{2.9}$$
$$J=\frac{\partial(x,p_x,s,p_s)}{\partial(x,\dot{x},s,\dot{s})}$$

performing a derivitive to eq(2. 5), we know,

$$J=(m\gamma v)^2(1+kx)^2 \tag{2.10}$$

then

$$du_i=(m\gamma v)^2(1+kx_i)^2dv_i \tag{2.11}$$
$$du_k=(m\gamma v)^2(1+kx_k)^2dv_k$$

Liouville's theorem tell us,

$$du_k=du_i \tag{2.12}$$

therefore

$$dv_k=\frac{(1+kx_i)^2}{(1+kx_k)^2}dv_i \tag{2.13}$$

Which shows that the emittance element is not a con-

① Supported by the Netional Science Foundation of China stant. In other words, the nessesary and sufficient condition for the six-dimensional emittance conservation can not be obtained.

3. THE MOTION ON THE MEDIAN PLANE

In the case of the uniform field, the vertical motion is equivenlent to the motion in the free space. so that the vertical emittance is an invariant. In this way ,we need only to discuss the motion in the median plane of the magnet.

Under the hard-edge approximation, as shown in Fig1, when the initial conditions of the incident particles are $x_i, \dot{x}_i, s_i, \dot{s}_i$ respectively, where dot means the derivitive with respect to t over v. the straight line equation of the particle's orbit in free space is written as

$$x - x_i = x'_i \cdot (s - s_i) \tag{3.1}$$

where $x' = \frac{dx}{ds} = \dot{x}/\dot{s}$. the ξ-axis equation is

$$x = s \cdot tg(\frac{\pi}{2} - \theta_{ei}) = -s \cdot tg(\theta_{ei}) \tag{3.2}$$

performing the solution of the above two linear algebraic equation, we get the coordinate at the magnet entrance

$$\xi_i - \xi_{ei} = -[\frac{\dot{s}_i \cdot x_i - \dot{s}_i \cdot s_i}{\dot{s}_i \cdot \cos(\theta_{ei}) + \dot{x}_i \cdot \sin(\theta_{ei})}] \tag{3.3}$$

$$\Delta s_f = [(s_f - s_i)^2 + (x_f - x_i)^2]^{1/2} = \frac{x_i \cdot \sin(\theta_{ei}) + s_i \cdot \cos(\theta_{ei})}{\dot{x}_i \cdot \sin(\theta_{ei}) + \dot{s}_i \cdot \cos(\theta_{ei})}$$

where Δs_f stands for the travelling distance in the edge region.

Let us consider the transformation of the beam going through the uniform magnetic field region, as shown in Fig1, the particle's orbit in the uniform field is a circle. In the conditions given by the paper [1][2] the edge at the magnet exit is parallel to that at the entrance, and the center of the equilibrium orbit places on the exit edge. In that case, no one of the velocity is not parallel to each other at any place while they are parallel at the entrance.

By means of the similar relation, the increased quantity of the distance of travels in the magnet region can be written as[1,2]

$$\Delta S_m = \int_{\xi_i}^{\xi_k} [\frac{1}{\sqrt{1-f^2}} - \frac{1}{\sqrt{1-f_e^2}}] d\xi \tag{3.4}$$

we know

$$\dot{x}_k = -\sin(\delta\theta_k) \tag{3.5}$$

$$\sin(\delta\theta_k) = [1 - \dot{s}_i \cdot \sin(\theta_{ei}) + \dot{x}_i \cdot \cos(\theta_{ei}) + \sin(\theta_{ei})]$$

since the coordinates and the velocity are continuous, by means of the similer ways to the paper[1], the final results can be written as

$$x_k = -\left[\frac{\dot{s}_i \cdot x_i - \dot{x}_i \cdot s_i}{\dot{s}_i \cdot \cos(\theta_{ei}) + \dot{x}_i \cdot \sin(\theta_{ei})}\right] + \delta x \tag{3.6}$$

$$\dot{x}_k = -[1 - (\sqrt{1 - \dot{x}_i^2}) \cdot \sin(\theta_{ei}) + \dot{x}_i \cdot \cos(\theta_{ei}) + \sin(\theta_{ei})]$$

$$s_k = \Delta s_f + \Delta s_m = \frac{x_i \cdot \sin(\theta_{ei}) + s_i \cdot \cos(\theta_{ei})}{\dot{x}_i \cdot \sin(\theta_{ei}) + \dot{s}_i \cdot \cos(\theta_{ei})} + \Delta Sm$$

$$\dot{s}_k = \sqrt{1 - \sin^2(\delta\theta_k)}$$

$$\sin(\delta\theta_k) = -[1 - \dot{s}_i \cdot \sin(\theta_{ei}) + \sqrt{1 - \dot{s}_i^2} \cdot \cos(\theta_{ei}) + \sin(\theta_{ei})]$$

$$\delta x = -\int_{\xi_i}^{\xi_k} [\frac{f}{\sqrt{1-f^2}} - \frac{f_e}{\sqrt{1-f_e^2}}] d\xi$$

$$f = \sin(\theta_{ei} + \delta\theta_i) + k \cdot \xi$$

$$f_e = \sin(\theta_{ei}) + k \cdot \xi$$

$$k = \frac{e}{p_0} \cdot B$$

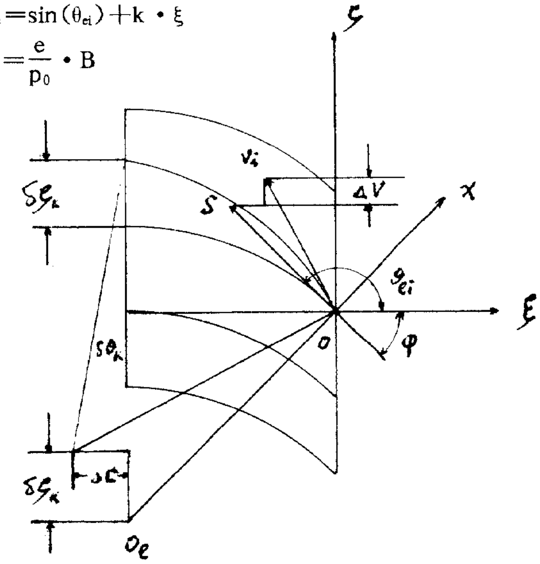

Fig. 1, Diagram of Particle's Motion in the Influence of Uniform Field

they are the precise solutions in the four dimensions. In the case of the two dimensions, those become the same as eq. (3.9) in the paper [1]

4. JACOBIAN TRANSFORMATION OF THE EMITTANCE SPACE IN THE FOUR DIMENSIONS

By means of differentiating the above precise formulas, the corresponding Jacobian determinant in the four dimensions can be obtained. Since $\dot{x}_k$ is only dependent on $\dot{x}_i$, and $\dot{s}_k$ is only dependent on $\dot{s}_i$, we know

$$dx_k \cdot d\dot{x}_k \cdot ds_k \cdot d\dot{s}_k = J \cdot dx_i \cdot d\dot{x}_i \cdot ds_i \cdot d\dot{s}_i$$

$$J = \begin{vmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ 0 & J_{22} & 0 & 0 \\ J_{31} & J_{32} & J_{33} & J_{34} \\ 0 & 0 & 0 & J_{44} \end{vmatrix} = J_{22} \cdot J_{44} \cdot (J_{11} \cdot J_{33} - J_{13} \cdot J_{31}) \tag{4.1}$$

$$J_{11} = \frac{(\partial x_k}{\partial x_i} = -\dot{s}_i / [\dot{s}_i \cdot \cos(\theta_{ei}) + \dot{x}_i \cdot \sin(\theta_{ei})]$$

$$J_{13} = \frac{\partial x_k}{\partial s_i} = \dot{x}_i / [\dot{s}_i \cdot \cos(\theta_{ei}) + \dot{x}_i \cdot \sin(\theta_{ei})]$$

$$J_{22}=\frac{\partial \dot{x}_k}{\partial \dot{x}_i}=-\left[\cos(\theta_{ei})+\frac{\dot{x}_i}{\sqrt{1-\dot{x}_i^2}}\cdot\sin(\theta_{ei})\right]$$

$$J_{31}=\frac{\partial s_k}{\partial x_i}=\sin(\theta_{ei})/[\dot{s}_i\cdot\cos(\theta_{ei})+\dot{x}_i\cdot\sin(\theta_{ei})]$$

$$J_{33}=\frac{\partial s_k}{\partial s_i}=\cos(\theta_{ei})/[\dot{s}_i\cdot\cos(\theta_{ei})+\dot{x}_i\cdot\sin(\theta_{ei})]$$

$$J_{44}=\frac{\partial \dot{s}_k}{\partial \dot{s}_i}$$

$$=-\frac{tg(\delta\theta_k)}{tg(\delta\theta_i)}\left[\cos(\theta_{ei})+\frac{\dot{x}_i}{\sqrt{1-\dot{x}_i^2}}\cdot\sin(\theta_{ei})\right]$$

we have

$$J=-\frac{tg(\delta\theta_k)}{tg(\delta\theta_i)}\cdot\frac{1}{(1-\dot{x}_i^2)}[\dot{s}_i\cdot\cos(\theta_{ei})+\dot{x}_i\cdot\sin(\theta_{ei})]$$

$$tg(\delta\theta_k)=\frac{\sin(\delta\theta_k)}{\sqrt{1-\sin^2(\delta\theta_k)}} \tag{4.2}$$

$$\sin(\delta\theta_k)=1-\dot{s}_i\cdot\sin(\theta_{ei})+\dot{x}_i\cdot\cos(\theta_{ei})+\sin(\theta_{ei})$$

this result gives us such inspirations as follows.

1. Jacobian J is less than unity 1. while $\dot{x}_i\neq 0$, the emittance in the four dimensions will be always shrinked. In other words, it is cooling indeed.

2. From the formula (4.1) we know, $J_{11}\cdot J_{22}=1$, which signifies that the extended rate of the transverse coordinate x is just equal to the shrinked rate of the transverse velocity $\dot{x}$. so the transverse emittance in the six dimensions is an invariant, too. Therefore, the beam cooling is only taken place in the longitudinal direction.

3. Since the vertical emittance is an invariant, but one of the four degree daughter space is shrinked, it is sure that the emittance in the six dimensions is decreased.

5. THE PHYSICAL MECHANISM

The physical mechanism of the cooling can easily be seen. It is the basic characteristics of the motion that all of the transverse velocity not to equal zero are always decreased and transferred to the longitudinal. Firstly, $J_{11}>1$, the dimension of the transverse amplitude for the parallel incident particles is increased; secondly, $J_{22}<1$, the transverse velocity is decreased; and thirdly, $J_{11}J_{22}=1$, the transverse emittance for the six dimensions is an invariant. In that case, the transverse velocity is continuously transferred to the longitudinal direction to make the longitudinal velocity continuously approximate to the equilibrium one. However. J_{33} is approximately equal to unity 1, but $J_{44}<1$, the longitudinal dimension is approxi-mately invariant, while the longitudinal emittance is obviously decreased. Since the longitudinal cooling comes from that the transverse velocity is transferred to the longitudinal direction, we can call it the transfer cooling.

In the paper[1][2], in the case of simplifying the problem to the four-dimesional one, transverse emittance is decreased, which is led by the longitudinal velocity transferred to the transverse. we know, $\frac{dx_k}{ds_k}=\frac{dx_k}{vdt}/\frac{ds_k}{vdt}=\dot{x}_k/\dot{s}_k=-tg\delta\theta_k$, and we have seen that the decreased rate of the transverse velocity $\dot{x}_k$ is equal to the increased rate of x_k, also we think about that the longitudinal velocity $\dot{s}_k$ is increased because $\dot{x}$ is transferred to the longitudinal direction, so the decreased rate of x' must be less than the decreased rate of $\dot{x}$ because of the increase of the denominator $\dot{s}$. Therefore, in this case, the transverse cooling comes from the longitudinal transfer.

the mechanism gives us an inspiration to find a way of improving the cooling rate. If we make the edge focusing angle φ at the exit become minus, which plays a role in focusing for the transverse motion, the $\dot{x}_k$ at the exit is rather approximating to zero, so that the cooling rate must be improved, which will be tested latter.

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