Synchronous Phase Changes Due to the Gap in the Bunch Train *

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Introduction

The B Factory bunch train in both beams is designed with a 250 *nsec* gap in order to minimize ion trapping. Possible differences in the synchronous phase between the bunches due to this gap in the bunch train are studied. Such an unequal phase shift between the bunches could lower the luminosity.

1 Synchronous Phase Calculation

The synchronous phase, which is the angle between the cavity voltage and the beam current, can be found from the beam loading vector diagram as seen in Figure 1. For a ring with evenly spaced bunches the synchronous phase ϕ_o is given by:

$$\phi_o = \arccos\left(\frac{V_a}{V_{co}}\right) = \begin{cases} 81.4^\circ \, H.E.R. \\ 86.37^\circ \, L.E.R. \end{cases}$$

where V_{co} is the cavity voltage and V_a is the loss per turn. The values of these parameters for both rings are given in Table 1. For strong coupling, like in super conducting cavities, the optimum tuning angle is $\psi_o \cong \phi_o$ [1] with the generator voltage V_g :

$$V_g = V_{co}\frac{\sin(\pi - 2\phi_o)}{\sin(\psi_o)} = \begin{cases} 10.46 \, MV \, H.E.R. \\ 1.52 \, MV \, L.E.R. \end{cases}$$

For a bunch train with a gap, the induced voltage is expected to change with time.

Ring and Cavity Parameters	H.E.R	L.E.R
N_b No. of bunches	230	230
I_o Beam current (Amp)	0.9	2.0
Number of RF cavities	12	4
V_{co} Total cavities voltage	35.	12.
V_a Energy Loss (MeV)	5.23	0.76
f_r Cavity Res. Freq. (MHz)	500.	499.9923
$\frac{\omega_{rf}-\omega_r}{\omega_r}$	$1.4 \cdot 10^{-5}$	$2.94 \cdot 10^{-5}$
$\frac{R}{Q}_{per\ cavity}$	89.	89.
Q_L Loaded quality factor	$2.4 \cdot 10^5$	$2.68 \cdot 10^5$
β_{opt} Coupling parameter	$4.1 \cdot 10^3$	$3.74 \cdot 10^3$
T_b time between bunches *nsec*	9.99986	9.99986

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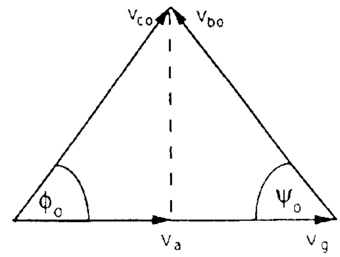

Figure 1: Vector Diagram Showing the cavity voltage as a superposition of the RF generator voltage V_g and the beam induced voltage V_{bo}

1.1 Beam Induced Voltage

The beam induced voltage for a continuous train of bunches (see P. Wilson [1]) can be found by taking the vectorial sum of the voltage induced by each bunch V_{bo}, including its decay by a time constant τ and rotation of an angle δ where

$$V_{bo} = \frac{\omega_r}{2}\frac{R}{Q}I_o T_b$$

$$\tau = \frac{T_b}{T_f}$$

$$\delta = T_b(\omega_r - \omega)$$

T_b is the time between bunches, T_f is the filling time, and ω_r is the cavity resonance frequency.

$$T_f = \frac{2 \cdot Q_o}{\omega(1+\beta)}$$

As time evolves, each bunch induced voltage can be represented by a vector $V_{bo}e^{n\xi}$ where $\xi = (-\tau + j\delta)$, n is the number of T_b passed since the bunch went through the cavity. The total induced voltage $V_{\bar{b}}$ is the vectorial sum $V_{\bar{b}} = \sum_{n=0}^{N_b-1} V_{bo}e^{n\xi}$. To include the effect of the gap we note that during the gap $n = [N_b, N_t - 1]$ (N_b number of bunches in the train, N_t number of evenly spaced bunches in one revolution-no gap) the train of bunches continues to propagate, but there are not any new bunches going through the cavity and the induced voltage given by

$$\sum_{n=N_b}^{N_t-1} V_{bo}\left(e^{n\xi} - e^{(n-N_b)\xi}\right)$$

starts to decay. In general for one revolution R_v of the bunch train

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$$\frac{V_{\bar{b}}}{V_{bo}} = \tag{1}$$

$$= \sum_{R_v N_t}^{R_v N_t + N_b - 1} e^{n\xi} \left[1 - e^{-N_b \xi} + e^{-N_t \xi} - e^{-(N_t + N_b)\xi} + e^{-2N_t \xi} - \ldots \right]$$

$$= \sum_{R_v N_t}^{R_v N_t + N_b - 1} e^{n\xi} \left[\frac{e^{-N_t \xi (R_v + 1)} - 1}{e^{-N_t \xi} - 1} - e^{-N_b \xi} \frac{e^{-N_t \xi R_v} - 1}{e^e - N_t \xi - 1} \right]$$

and in the gap interval:

$$\frac{V_{\bar{b}}}{V_{bo}} = \tag{2}$$

$$= \sum_{R_v N_t + N_b}^{(R_v+1)N_t - 1} e^{n\xi} \left[1 - e^{-N_b \xi} + e^{-N_t \xi} - e^{-(N_t + N_b)\xi} + e^{-2N_t \xi} - e^{-(2N_t + N_b)\xi} + \ldots \right]$$

$$= \sum_{R_v N_t + N_b}^{(R_v+1)N_t - 1} e^{n\xi} \left[1 + e^{-N_t \xi} + e^{-2N_t \xi} + \ldots \right] - e^{-N_b \xi} \left[1 + e^{-N_t \xi} + e^{-2N_t \xi} \ldots \right]$$

$$= \sum_{R_v N_t + N_b}^{(R_v+1)N_t - 1} e^{n\xi} \frac{e^{-N_t \xi (R_v + 1)} - 1}{e^{-N_t \xi} - 1} \left[1 - e^{-N_b \xi} \right]$$

The vectorial sum of the induced voltage of a train of 230 bunches with a gap of $25T_b$ for the high energy ring is seen in figure 2a. The saw tooth shape of the line is due to voltage decay during the gap. The voltage seen by the bunches is:

$$\overline{V}_b = V_{\bar{b}} + \frac{1}{2} V_{bo} \tag{3}$$

and its absolute value is shown in figure 2b. This voltage is not constant even at steady state as for the case of continuous train, but changes with time. Figure 3 shows the high energy ring and Figure 4 shows the low energy ring. The oscillations during the build up time are due to the difference between the RF driving frequency ω_{rf} and the resonance frequency of the cavity ω_r. The larger the difference $\omega_{rf} - \omega_r$ the amplitude of oscillation of the transient response is larger, but the steady state value is smaller.

1.2 Cavity Voltage Calculation

Since the generator voltage V_g and the tuning angle ψ can not be regulated on a time scale of $10\,nsec$ (time between bunches) the voltage on the cavity V_c is temporarily changed when V_b is changed .

$$\overline{V}_c = \overline{V}_g - \overline{V}_b$$

The change in $|V_c|$ corresponding to the changes in V_b is also shown in Figures 3 and Figure 4.

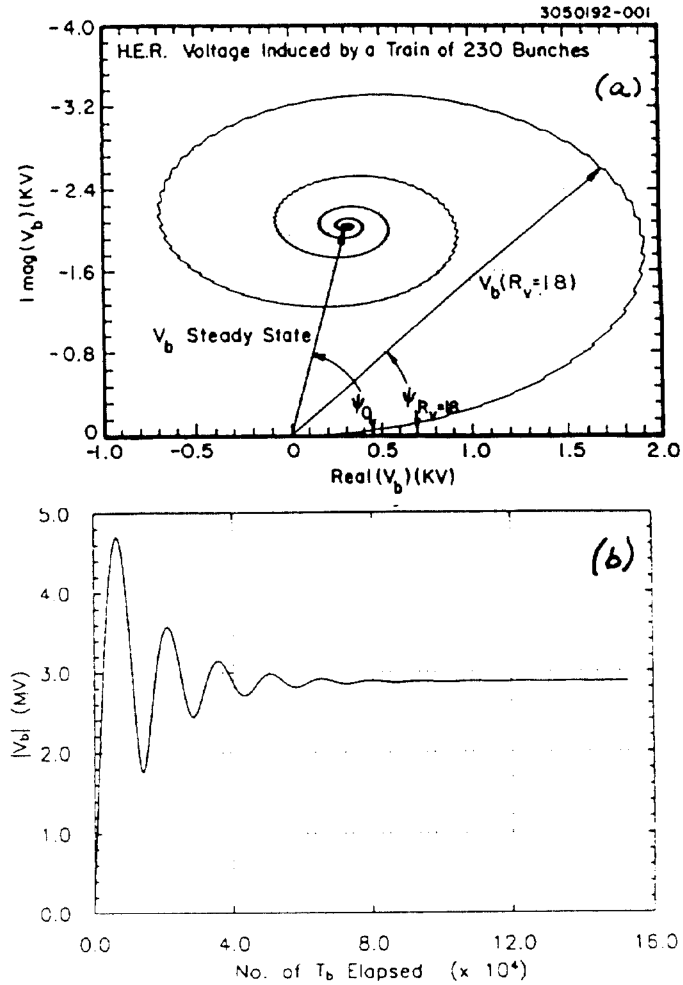

Figure 2: Evolution of the Beam Induced Voltage. a) As vectorial propagation. b) As function of time passed.

By looking at figure 3 and figure 4 one can follow the changes of the vector diagram of a bunch train with a gap. The beam induced voltage $|V_b|$ increases during $n = [R_v N_t, R_v N_t + N_b - 1]$ and reaches its maximum at the end of the train, while the cavity voltage decreases to its minimum during this period. Figure 5a and Figure 5b shows the position of $|V_c|$ and $|V_b|$ at the beginning of the train $|V_{ci}|$, $|V_{bi}|$ and at the end of the train (beginning of the gap) $|V_{ce}|$, $|V_{be}|$ correspondingly, both imposed on a diagram of a continuous train with equal current.

In reality the bunch spacing will be slightly different to keep the loss V_a constant in the steady state for all the bunches. Allowing this change on the vector diagram, while keeping $V_a = constant$, creates a new direction for the current. The associated synchronous phase measured relative to this new direction is:

$$\phi_b = \arccos\left(\frac{V_a}{V_c}\right)$$

Assuming that the change in V_c due to the change in the spacing is very small and using the values obtained in Fig-

Assuming that the change in V_c due to the change in the spacing is very small and using the values obtained in Figures 3 and Figure 4, the change in the synchronous phase $\Delta\phi_b = \phi_b - \phi_o$ was calculated. The result is shown in Figure 6.

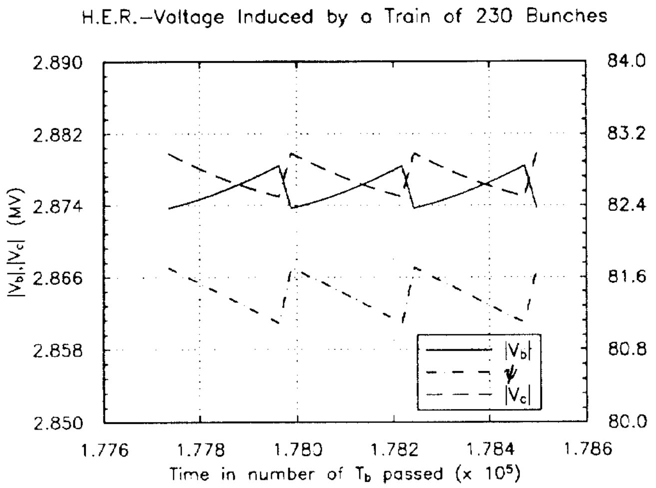

Figure 3: Beam Induced Voltage seen by the bunch train at steady state for H.E.R, its phase ψ and the corresponding cavity voltage.

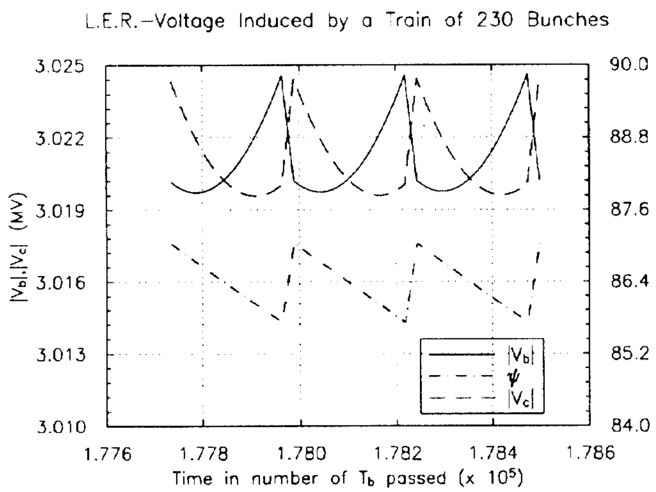

Figure 4: Beam Induced Voltage seen by the bunch train at steady state for L.E.R, its phase ψ and the corresponding cavity voltage.

2 Conclusions

A maximum change of $\Delta\phi_b = \pm 0.007^\circ$ and $\pm 0.0025^\circ$ was obtained for the H.E.R. and L.E.R. respectively which is equivalent to deviation of $1.2 \cdot 10^{-3} cm$ and $4. \cdot 10^{-4} cm$. Since the change in phase for both rings goes in the same direction, the maximum relative deviation between the bunches in the two rings is $8. \cdot 10^{-4} cm$ compared to the $.1 cm$ bunch length. Since there are only 4 cavities in the L.E.R. as opposed to 12 in the H.E.R. this leads to a smaller induced voltage even though the current is more than twice as large. The reason that a gap of 250 $nsec$ does not cause

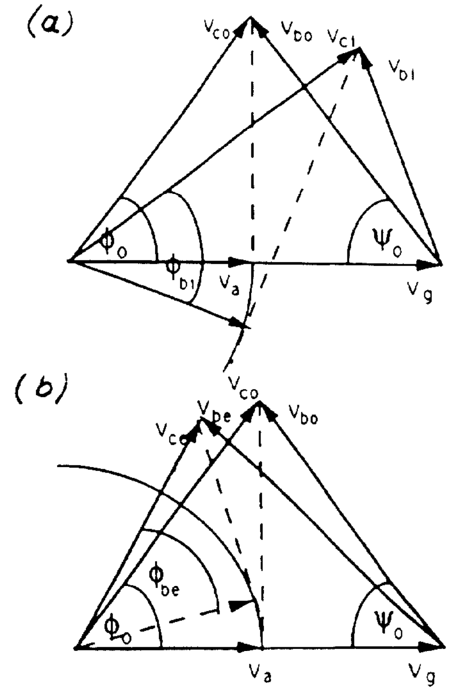

Figure 5: Vector diagram of a bunch train with a gap. a) at the end of the train. b) At the beginning of the train.

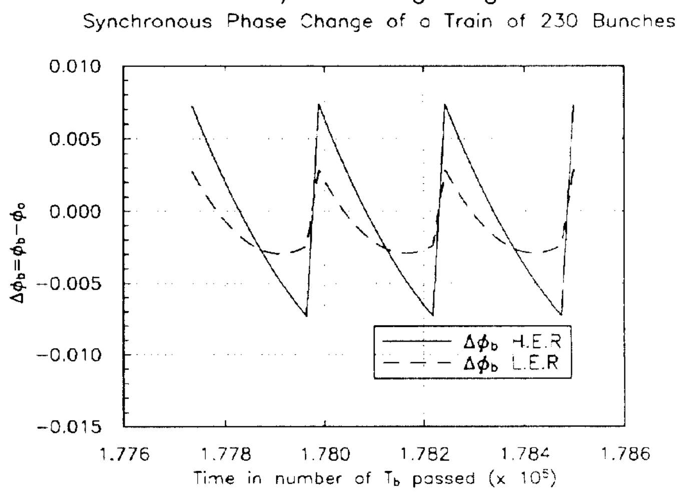

Figure 6: Change in the synchronous phase $\Delta\phi_b = \phi_b - \phi_o$.

a measurable change in the synchronous phase is that the time constant of each cavity $\frac{2\omega_r}{Q_L} = 38.2\mu\ sec$ is much larger than the $0.25\mu\ sec$ gap length which means that the cavity voltage does not discharge significantly during the gap causing only small change in ΔV_b.

Acknowledgment

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References

[1] P. Wilson, SLAC-PUB-2884, February 1982.