On Limitations on Low-α Rings Performance Due To ∇Z-Instabilities

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Abstract

We analyze collective instabilities of a bunch due to the radial gradient of the longitudinal impedance. One of the specific features of these collective effects is that decrements of collective synchrotron modes do not depend on the value of momentum compaction factor of the ring. This effect can limit the performance of the so-called low-α storage rings. It can be used for the damping of unstable modes of the synchrotron coherent oscillations as well.

I. INTRODUCTION

To decrease the bunch length, it was suggested in [1] to use the rings with abnormally small value of the momentum compaction factor α. The performance of such low-α rings can be very sensitive to collective instabilities, when the dependence of the increments on α is weak. Usually, the analysis of these issues is focused on the cases, when the beam interacts with devices producing the nonzero impedance $Z(\omega)$ on the particle trajectory. In this case, the rates of unstable modes decrease with a decrease in α. The different type of collective effects may occur due to radial gradient of the coherent energy losses. If, for example, the bunch interacts with a matched plate, the increments of coherent modes do not depend on α but are determined by the local value of the dispersion function η [2,3]. With the suitable sign for the gradient of the energy losses, this fact can be used to enhance the damping of the synchrotron coherent oscillations of bunches. However, if such a gradient appears accidentally, the collective interaction redistributes the increments between the synchrotron, or horizontal coherent modes [4] and the instability gets the global character. This instability is one of the simplest results of the theorem of the sum of the decrements for collective modes [4]. If the bunch wake is described in terms of the coupling impedance, the radial gradient of the coherent energy losses is described by the radial gradient of $Z(\omega)$. Apart from the use of special devices [2,3], the gradient Z can occur due to nonsymmetric position of the closed orbit inside the vacuum chamber of the ring. The last case is specific, for instance, for the future B-factories. In this report we use a simple example to describe the main features of such ∇Z- collective instabilities.

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II. SYNCHROTRON OSCILLATIONS

Let us consider the single-turn interaction of the bunched beam with a low-Q system of electrodes, which has a radial gradient of the coupling impedance. Therefore, we take the relationship between the harmonics of the longitudinal wake field $E_\vartheta(n,\omega)$ and the azimuthal harmonics of the bunch distribution function $f_{n,\omega}$ in the form

$$E_\vartheta(n,\omega) = -\frac{Ne\omega_0}{\Pi}\int d\Gamma Z_n(x,\omega) f_{n,\omega}. \qquad (1)$$

Here, $\Pi = 2\pi R_0$ is the perimeter of the orbit. If the unperturbed oscillations of the particle are described by the equations ($\mathcal{E} \simeq pc$)

$$\begin{gathered} x = x_b + \eta\frac{\Delta p}{p}, \quad x_b = a_x\cos\psi_x, \\ \theta = \omega_0 t + \varphi, \quad \varphi = \varphi_s\cos\psi_s, \quad \dot\varphi = -\omega_s\varphi_s\sin\psi_s, \\ \dot\psi_x = \omega_x = \omega_0\nu_x, \quad \dot\psi_s = \omega_0\nu_s, \\ I_x = \frac{p}{2R_0}\nu_x a_x^2, \quad I_s = \frac{\mathcal{E}\nu_s}{2\omega_0\alpha}\varphi_s^2, \end{gathered}$$

the integral equation for synchrotron coherent oscillations ($f(I_s,\psi_s,t) = f_0(I_s) + f_{m_s}\exp(im_s\psi_s - i\omega t)$, $\omega \simeq m_s\omega_s + \Delta\omega_m$) reads

$$\begin{gathered} \Delta\omega_m f_m = \frac{Ne^2\omega_0 m_s}{2\pi}\frac{\partial f_0}{\partial I_s}\int\limits_{-\infty}^{\infty} dn J_{m_s}(n\varphi_s)\int\limits_0^\infty d\varphi_s'\varphi_s'\times \\ \left\{\frac{iZ_n(x,n\omega_0+m_s\omega_s)}{n+m_s\nu_s}\exp(in\varphi_s\cos\psi_s)\right\}_{m_s} f_m. \end{gathered} \qquad (2)$$

For the sake of simplicity, we assume that $Z_n(\omega)$ is a pure resistive impedance, which linearly depends on x

$$Z_n(x,\omega) = (\partial Z_n/\partial x)_0 x = (\nabla Z_n)_0\eta\frac{\Delta p}{p},$$

and that for typical harmonics $|\partial Z_n/\partial\omega| \ll |Z_n/\omega|$. Then, the calculation in Eq.(2) of the harmonics over ψ_s results in

$$[\Delta p/p\exp(in\varphi_s\cos\psi_s]_{m_s} = \frac{m_s\nu_s}{\alpha n}J_{m_s}(n\varphi_s),$$

Neglecting in Eq.(2) the higher powers of $m_s\nu_s$, we trans-

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form Eq.(2) into

$$\Delta\omega_m f_m = i\frac{Ne^2\omega_0}{2\pi}\frac{m_s^2\nu_s}{\alpha}\frac{\partial f_0}{\partial I_s}\eta\int\limits_{-\infty}^{\infty}\frac{dn}{n^2}(\nabla Z_n)_0\times$$
$$\times J_{m_s}(n\varphi_s)\int\limits_0^{\infty}d\varphi_s'\varphi_s' J_{m_s}(n\varphi_s')f_m, \quad |m_s\nu_s|\ll 1. \tag{3}$$

Substituting here $u=\sqrt{\varphi_s^2/2}$ and

$$\Delta\omega_m = -i\Lambda m_s^2\frac{Ne^2\omega_0}{\Pi p}, \quad f_m=\sqrt{-\frac{\partial f_0}{\partial u}}\chi_m, \tag{4}$$

we transform Eq.(3) into an integral equation with a symmetric kernel

$$\Lambda\chi_m = \eta\int\limits_{-\infty}^{\infty}\frac{dn}{n^2}(\nabla Z_n)_0\int\limits_0^{\infty}du' K_m(u,u')\chi_m(u'),$$
$$K_m(u,u')=\sqrt{\frac{\partial f_0}{\partial u}\frac{\partial f_0}{\partial u'}}J_{m_s}(n\sqrt{u})J_{m_s}(n\sqrt{u'}). \tag{5}$$

The kernel $K_m(u,u')$ will be positively defined, and hence, all its eigennumbers Λ will be real positive numbers if

$$\eta\int\limits_{-\infty}^{\infty}dn(\nabla Z_n)_0 > 0. \tag{6}$$

Due to Eq.(4) the last condition yields the criterion, when synchrotron coherent oscillations decay. This has the obvious physical sense. The interaction with electrodes damps the coherent synchrotron oscillations if the coherent energy losses increase with the amplitude of oscillations. In our case, this takes place if condition (5) is satisfied.

Eq.(5) is too complicated for direct solution. In fact, it is not necessary, since the general conclusion concerning the collective stability of a bunch can be obtained by the calculation of the relevant sums of the decrements [4]. The sum of decrements of the synchrotron modes near the given m_s is

$$\delta_{m_s} = -\eta\frac{Ne^2\omega_0^2}{2\pi pv}\int\limits_0^{\infty}du\frac{\partial f_0}{\partial u}\int\limits_{-\infty}^{\infty}\frac{dn}{n^2}(\nabla Z_n)_0 m_s^2 J_{m_s}^2(n\sqrt{u}). \tag{7}$$

Using the identity

$$\sum_{m=-\infty}^{\infty} m^2 J_m^2(x) = \frac{x^2}{2},$$

we obtain

$$\delta_\Sigma^{(s)} = \sum_{m_s=-\infty}^{\infty}\delta_{m_s} = \eta\frac{Ne^2\omega_0^2}{2\pi pv}\int\limits_{-\infty}^{\infty}dn(\nabla Z_n)_0. \tag{8}$$

III. HORIZONTAL OSCILLATIONS

Apart from the effect on synchrotron coherent oscillations the interaction with such an element of the vacuum chamber will modify the increments of radial coherent modes. The variation of the sum of the decrements for the dipole horizontal coherent oscillations ($m_x=\pm 1$) can be calculated using the relationship

$$\dot{I}_x = \int\limits_0^{2\pi}\frac{d\tau}{2\pi}\frac{\partial I_x}{\partial\Delta p}eE_\vartheta(\tau+\varphi,t), \quad \tau=\omega_0 t.$$

Substituting this expression and

$$f=f_0(I_x,I_s)+f_m\exp(im_x\psi_x+im_s\psi_s-i\omega t),$$
$$f_0(I_x,I_s)=F_0(I_x)\rho(\varphi_s)$$

in the linearized Vlasov equation, we obtain the integral equation for horizontal collective modes

$$(\omega-m_x\omega_x-m_s\omega_s)f_m = -i(\dot{I}_x)_{m_x,m_s}\rho(\varphi_s)\frac{\partial F_0}{\partial I_x}. \tag{9}$$

Using here $(\partial I_x/\partial\Delta p) = -(\nu_x\eta a_x\cos\psi_x/R_0)$, $\Delta\omega_m = \omega\pm\omega_x-m_s\omega_s$ and $f_m=\chi_m(\varphi_s)\sqrt{I_x}\partial F_0/\partial I_x$, we find that χ_m obeys the following equation

$$\Delta\omega_m\chi_m = i\rho(\varphi_s)\frac{Ne^2\omega_0\eta}{2\Pi p}\int\limits_{-\infty}^{\infty}dn(\nabla Z_n)_0\times$$
$$J_{m_s}(n\varphi_s)\int\limits_0^{\infty}d\varphi_s'\varphi_s' J_{m_s}(n\varphi_s')\chi_m(\varphi_s'). \tag{10}$$

The direct calculation of the trace of the kernel in Eq.(10) yields the total sum of decrements ($\delta=-\mathrm{Im}\omega$) of the horizontal betatron and synchrobetatron modes

$$\sum_{m_s=-\infty}^{\infty}(\delta_{1,m_s}+\delta_{-1,m_s}) = -\eta\frac{Ne^2\omega_0^2}{2\pi pv}\int\limits_{-\infty}^{\infty}dn(\nabla Z_n)_0. \tag{11}$$

The comparison of Eqs.(8) and (11) gives the sum rule

$$\delta_\Sigma = \delta_\Sigma^{(s)} + \sum_{m_s=-\infty}^{\infty}(\delta_{1,m_s}+\delta_{-1,m_s}) = 0. \tag{12}$$

This equation describes a particular case in a more general statement concerning the sum of the decrements of coherent oscillations, which previously was found in [4]. Eq.(12) shows that without special efforts the interaction of the beam with a device producing the horizontal gradient of the coupling impedance results in a global coherent instability of the bunch, when the synchrotron, or the horizontal synchrobetatron collective modes, are unstable.

Let us also mention an example, where Eq.(10) can be solved directly. The model, which will be described below, can be useful for analysis of many others single-bunch instabilities [5]. We take as $\rho(\varphi_s)$ a Lorentz distribution: $\rho = \sigma_s/(\sigma_s^2 + \varphi_s^2)$ and assume that ∇Z does not depend on n. Then, we can find from Eq.(10) that a function

$$w(z) = \sqrt{z}\int_0^\infty dt t J_{m_s}(2zt)\chi_m(2t), \quad t = \frac{\varphi_s}{\sigma_s} \tag{13}$$

satisfies the equation [5]

$$\frac{d^2w}{dz^2} + \left\{-\frac{1}{4} + \frac{\Lambda}{z} - \frac{m_s^2 - 1/4}{z^2}\right\} w = 0. \tag{14}$$

Here, $z = n\sigma_s/2$ and

$$\Lambda = 2i\eta\frac{Ne^2\omega_0}{2\Delta\omega_m \Pi p}(\nabla Z)_0. \tag{15}$$

The solution of Eq.(14) reads

$$w = z^{|m_s|+1/2}e^{-z/2}F(|m_s| + \Lambda + \frac{1}{2}, 2|m_s| + 1, z), \tag{16}$$

where $F(\alpha, \gamma, z)$ is the confluent hypergeometric function. The eigenfunctions w do not grow at $z \to \infty$ if

$$|m_s| + \Lambda + 1/2 = -l, \quad l = 0, 1, \ldots$$

The substitution of Λ from this equation in Eq.(15) yields the increments of modes

$$\delta_{m_s,l} = -\frac{Ne^2\omega_0^2}{2\pi pv}\left(\eta\frac{\partial Z}{\partial x}\right)_0 \frac{1}{2(|m_s| + l) + 1}. \tag{17}$$

In contrast with the "ordinary" instabilities, the increments in Eq.(17) and relevant decrements of the synchrotron modes do not depend on the momentum compaction factor of the ring. In particular, this fact will limit the performance of rings with very low α.

Described instability occurs due to a redistribution of decrements between the synchrotron and horizontal coherent modes. If, for some reason, the decrements of the horizontal coherent oscillations exceed the increments due to such a redistribution, the wideband systems of the electrodes with a proper sign for the horizontal gradient of the longitudinal impedance can be used to damp synchrotron coherent oscillations of a single bunch. For instance, this can be done using systems of the matched plates [2]÷[5].

We thank prof. A. Skrinsky, who originally drew our attention to these problems. One of the authors (D.V.P.) is indebted to KEK and its B-factory group for their hospitality.

IV. REFERENCES

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