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The Effects of Coulomb Beam Interaction in Multiaperture Linac

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Abstract

In a multiaperture linac the Coulomb coherent beam oscillations are excited and thus stimulate particle losses. To analyse the coherent oscillations the model is proposed, in that the Coulomb fields of surrounded beams in relation to the considered bunch train are approximated by the fields of uniformly charged threads. The low order mode equations are derived by linearization of Coulomb and external fields. The beam interaction effects in the multiaperture alternating phase focused linac are studied. The dipole mode is shown to influence mainly the beam dynamics in this linac, and increasing of bunch sizes by the quadrupole mode is negligible.

1 INTRODUCTION

The basic method of total current increasing in ion linac is a use of multiaperture accelerating systems [1]. In a multiaperture linac the Coulomb coherent beam oscillations are excited and thus stimulate particle losses. The dipole and quadrupole modes are the low order modes of these oscillations. The dipole mode excites the bunch centre oscillations, and the quadrupole mode increases the bunch sizes. To analyse the coherent oscillations the model is proposed, in that the Coulomb fields of surrounded beams in relation to the considered bunch train are approximated by the fields of uniformly charged threads. The equations for dipole and quadrupole modes of coherent oscillations are derived by linearization of Coulomb and external fields. The beam interaction effects in the multiaperture alternating phase focused linac (APF) are studied.

2 THE EQUATIONS FOR DIPOLE AND QUADRUPOLE MODES OF COHERENT BEAM OSCILLATIONS

We derive the equations for dipole and quadrupole modes of coherent oscillations of a bunched beam in the given channel of a multiaperture drift tube linac. The considered beam is represented by a sequence of uniformly charged ellipsoids with semiaxes r_x,r_y,r_z following each other at distance L. The Coulomb fields of surrounded beams in relation to the considered bunch train are approximated by the fields of uniformly charged threads. The Coulomb field potential inside a beam propagating in the given channel is

$$U(x,y,z) = U_i(x,y,z) + U_e(x,y,z),$$

where x,y,z are coordinates originated from the given channel axis; U_i is the beam self-field potential; U_e is the field potential created by all other beams of the multiaperture linac.

The potential U_i is determined by the field superposition of all ellipsoidal charges. If we neglect the metal boundary influence the self-field potential inside the considered bunch in linear approximation to space-charge forces is described by the quadratic form

$$U_i(x,y,z) = -\frac{\rho}{2\varepsilon_0}[M_x^*(x-x_0)^2+M_y^*(y-y_0)^2+M_z^*(z-z_s)^2] \quad (1)$$

Here ρ is the space-charge density; ε_0 is the electric constant; x_0,y_0,z_s are coordinates of the given bunch centre; $M_{x,y,z}^*$ are ellipsoid form factors with mutual influence of the bunches [2].

Within the framework of the accepted model the potential defining the beam interaction in the accelerating gaps has the form

$$U_e(x,y,z) = -\frac{1}{4\pi\varepsilon_0\beta c}\sum_{\substack{j=1 \\ j\neq j_0}}^{N} I_j \ln[(x-x_j)^2+(y-y_j)^2]+const, \quad (2)$$

where β is the ratio of synchronous particle velosity to the speed of light; c is the speed of light; N is the number of channels of the multiaperture linac; j_0 is the number of the given channel; I_j and (x_j, y_j) are current and centre coordinates of the beam in the jth channel (fig. 1).

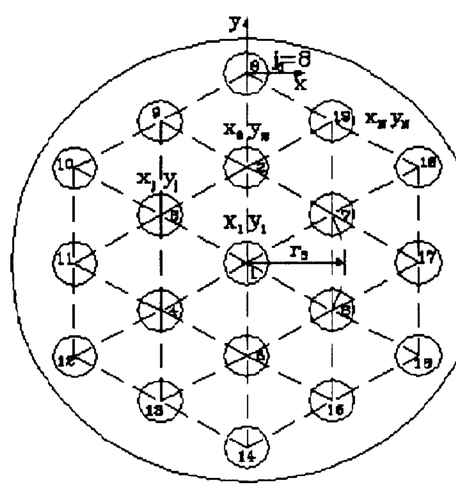

Figure 1: The arrangement of channels in the proton multiaperture APF linac.

We neglect the coupling between transverse degrees of freedom arising from the beam interaction. Then the linear equations of particles motion with the potential of the

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beam self-field (1) and the potential of the beam interaction (2) may be written as follows

$$\frac{d^2x}{d\tau^2}+Q_x^*(\tau)x-\frac{\alpha M_x^*}{r_xr_yr_z}(x-x_0)=f_x(\tau),$$
$$\frac{d^2y}{d\tau^2}+Q_y^*(\tau)y-\frac{\alpha M_y^*}{r_xr_yr_z}(y-y_0)=f_y(\tau), \quad (3)$$
$$\frac{d^2(z-z_s)}{d\tau^2}+Q_z^*(\tau)(z-z_s)-\frac{\alpha M_z^*}{r_xr_yr_z}(z-z_s)=0.$$

Here $\tau=t/T_F$; T_F is the transition time of the focusing period; $Q_{x,y}^*(\tau)=Q_{x,y}(\tau)-g_{x,y}(\tau)$; the functions $Q_{x,y,z}(\tau)$ are proportional to the external force gradients with opposite sign; $\alpha=\frac{3\lambda^3}{\gamma^3}\frac{I}{I_0}k_F^2$; I is the beam pulsed current in the given channel; λ is the wavelength of the accelerating field; γ is Lorentz factor; I_0 is the characteristic current; $k_F=S_F/\beta\lambda$; S_F is the length of the focusing period. The functions $f_{x,y}(\tau)$ and $g_{x,y}(\tau)$ caused by mutual influence of beams are determined by the expressions in the accelerating gaps

$$f_t(\tau)=-\frac{1}{\beta(\tau)}\sum_{\substack{j=1\\ j\neq j_0}}^{N}\alpha_j^*\frac{t_j}{x_j^2+y_j^2},$$
$$g_t(\tau)=-\frac{1}{\beta(\tau)}\sum_{\substack{j=1\\ j\neq j_0}}^{N}\alpha_j^*\frac{2t_j^2-x_j^2-y_j^2}{(x_j^2+y_j^2)^2},\ (t=x,y)$$

and equal zero in drift tubes of a multiaperture linac. The parameters α_j^* are $\alpha_j^*=\frac{2\lambda^2}{\gamma^3}\frac{I_j}{I_0}k_F^2$. Note that factor $1/\gamma^2$ is added in the formulas for α, α_j^* to take into account influence of the magnetic fields created by beam currents.

We introduce coordinates originating from the beam centre in the given channel: $\tilde{x}=x-x_0$, $\tilde{y}=y-y_0$, $\tilde{z}=z-z_s$. Then from (3) we receive equations of motion of the bunched beam centre

$$\frac{d^2x_0}{d\tau^2}+Q_x^*(\tau)x_0=f_x(\tau),$$
$$\frac{d^2y_0}{d\tau^2}+Q_y^*(\tau)y_0=f_y(\tau), \quad (4)$$

and also equations for bunch semiaxes ($r_z<\frac{1}{2}L$)

$$\frac{d^2r_x}{d\tau^2}+Q_x^*(\tau)r_x-\frac{F_t^2}{r_x^3}-\frac{\alpha M_x^*}{r_yr_z}=0,$$
$$\frac{d^2r_y}{d\tau^2}+Q_y^*(\tau)r_y-\frac{F_t^2}{r_y^3}-\frac{\alpha M_y^*}{r_xr_z}=0, \quad (5)$$
$$\frac{d^2r_z}{d\tau^2}+Q_z^*(\tau)r_z-\frac{F_\ell^2}{r_x^3}-\frac{\alpha M_z^*}{r_xr_y}=0,$$

where F_t and F_ℓ are transverse and longitudinal beam emittances on the phase planes $(\tilde{x},\frac{d\tilde{x}}{d\tau})$ and $(\tilde{z},\frac{d\tilde{z}}{d\tau})$ respectively.

Transverse beam sizes in the channel are connected with bunch semiaxes by relations

$$R_{x_{min}^{max}}(\tau)=\pm r_x(\tau)+x_0(\tau),$$
$$R_{y_{min}^{max}}(\tau)=\pm r_y(\tau)+y_0(\tau). \quad (6)$$

Table 1: APF linac design parameters

Ion	H^+
Input energy	60 keV
Output energy	3 MeV
Frequency	148.5 MHz
Number of channels	19
Aperture radius of one channel	3.3 mm
Pulse current limit in one channel	13 mA
Phase acceptance	120°
Longitudinal phase advance	62°
Normalized transverse acceptance of one channel	0.13 $\pi cm\cdot mrad$
Transverse phase advance	67°
Number of focusing periods	7
Focusing period length	$3\beta\lambda$
Number of accelerating gaps	28
Peak field in gap	180 kV/cm
Accelerator length	1.45 m

Thus, in multiaperture accelerating systems the coherent beam oscillations caused by Coulomb beam interaction are excited. The proposed model of this interaction enables to study the dipole and quadrupole modes of coherent oscillations. The dipole mode excites the beam centre oscillations and obeys equations (4), and the quadrupole mode increases the bunch sizes according to (5).

3 THE COHERENT BEAM OSCILLATIONS IN THE ALTERNATING PHASE FOCUSED LINAC

We consider the coherent beam oscillations in the proton multiaperture alternating phase focused linac containing 19 channels (fig. 1). The main linac parameters are given in table 1.

In the central channel the Coulomb fields created by peripheral beams are compensated completely. The beam dynamics in the central channel of the multiaperture linac is studied by means of a set of equations (5). The periodic solutions of equations (5) in the first focusing period at $g_{x,y}(\tau)\equiv 0$ were accepted as injection conditions. The transverse beam emittance is chosen equal to 0.4 of the acceptance value. The transverse bunch sizes as functions of the longitudinal coordinate are shown in fig. 2 for zero and 7 mA injection currents.

As is seen in fig. 2, the envelope maximums are oscillated along the accelerator. Modulation of beam envelope maximums is caused by discontinuities of instantaneous values of transverse tunes at the joint points of focusing periods. In spite of existing beam mismatching the accelerated particle losses are absent in the central channel for injection currents not exceeding 7 mA.

The peripheral beams located in the vicinity of the drift tube boundary suffer the largest influence of Coulomb in-

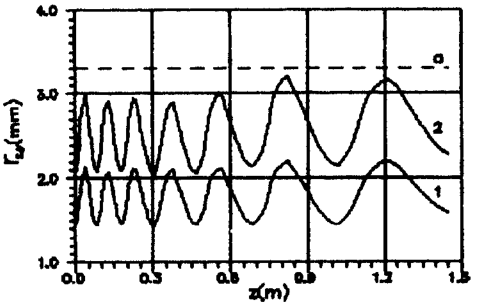
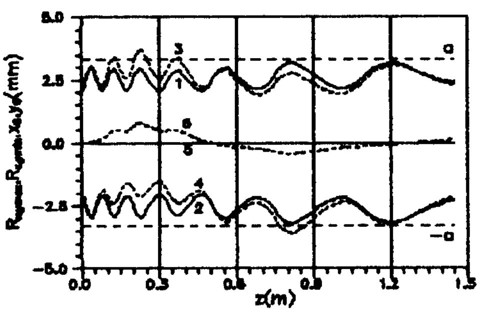
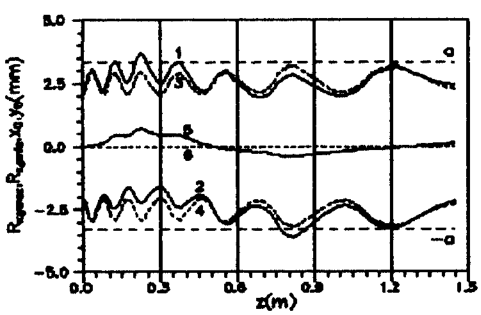


Figure 2: The transverse bunch semiaxes $r_{x,y}$ in the central channel 1 vs the longitudinal coordinate z for zero (1) and 7 mA (2) injection currents.

teraction forces. Due to the symmetry of channels arrangement in the multiaperture linac it is enough to consider the dynamics of interacting beams in channels 8 and 17 (fig. 1). We neglect displacement of bunch centres in the electrostatic injector of a multiaperture linac and accept beam emittances and initial bunch sizes equal to the corresponding values for the 7 mA central beam. The results of the solution of equations (4),(5) are plotted in fig. 3 for peripheral 7 mA beams propagating in channels 8 and 17.



(a)



(b)

Figure 3: The transverse sizes $R_{x\,max}$ (1), $R_{x\,min}$ (2), $R_{y\,max}$ (3), $R_{y\,min}$ (4) and bunch centre position x_0 (5), y_0 (6) in peripheral channel 8 (a), 17 (b) vs the longitudinal coordinate z for 7 mA current in each beam.

The distance between the centres of channels is chosen equal to $r_c = 7.6\,mm = 2.3a$, where a is an aperture radius of the channel. The given packing of channels provides high density of their arrangement and sufficient construction rigidity of multiaperture drift tubes. As is seen in fig. 3, the bunch centre oscillations (dipole oscillations) are stable and are accomplished in the radial plane passing over the centre of the considered channel. This polarization of dipole oscillations is accounted for by the fact that in the direction perpendicular to the radial plane the forces of beam interaction are compensated completely because of the channels symmetrical arrangement. For the given packing of channels the amplitudes of peripheral beam centre oscillations in channels 8 and 17 are $0.79\,mm\,(0.24a)$ and $0.72\,mm\,(0.22a)$ respectively.

The comparison of the curves in figs. 2,3 shows that increasing of semiaxes of peripheral bunches relative to the central ones is less than 0.3%. Therefore influence of the quadrupole mode on the beam dynamics in the linac is negligible.

The dipole mode of coherent oscillations mainly contributes in the effect of the beam size growth $|R_{x,y\,min}|$, $R_{x,y\,max}$ (6) in the channels of the multiaperture linac. For the given packing of beams the dipole mode excitation results in particle losses in peripheral channels (fig. 3).

One of the possible ways of particle loss reduction in linac is decreasing of the channel arrangement density. In particular, to transport beam in the considered multiaperture linac without particle losses the distance between the channels must be increased approximately 2.6 times [2].

4 CONCLUSION

To analyse coherent oscillations in a multiaperture linac the model is proposed, in that the Coulomb fields of surrounded beams in relation to the considered bunch train are approximated by the fields of uniformly charged threads. The equations for the low order modes of coherent oscillations are derived by linearization of Coulomb and external fields. The beam interaction effects in the multiaperture alternating phase focused linac are studied. The Coulomb beam interaction is shown to be substantial for packing of channels with high density ($r_c = 2.3a$). The dipole mode of coherent oscillations mainly influences the beam sizes in the channel. In the considered accelerating structure the bunch centre oscillations result in particle losses in peripheral channels. To decrease particle losses in a multiaperture linac it is necessary to depress the dipole mode.

5 REFERENCES

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