Computer Simulation of the Maryland Transport Experiment.*

I. Haber
Plasma Physics Division, Naval Research Laboratory
Washington, DC 20375 USA

D. A. Callahan, A. B. Langdon
Lawrence Livermore National Laboratory, University of California
Livermore, CA 94550 USA

M. Reiser, D. X. Wang, J. G. Wang
Laboratory for Plasma Research, University of Maryland
College Park, MD 20742 USA

Abstract
The longitudinal dynamics of the high-perveance long-pulse electron beam in the Maryland Transport Experiment is examined for the special case of an initially parabolic bunch. Because the longitudinal dynamics can depend on details of time-dependent transverse beam parameters which are difficult to measure, sensitivity studies using r-z simulations have been used to demonstrate that the details of longitudinal beam evolution are insensitive to transverse mismatch, and the bunch length evolution can be accurately described by the one-dimensional envelope equation with the "geometry factor" appropriately chosen. Comparison of experimental data to r,z simulation and to the envelope solution is presented.

I. INTRODUCTION

The University of Maryland Transport Experiment is a flexible apparatus designed to explore the fundamental physics of space-charge-dominated beam transport. In the current configuration, a high perveance gridded gun injects an electron beam into a transport line with 38 interrupted solenoid focusing elements. Details of recent experiments, concentrated primarily on studying longitudinal and multi-dimensional beam physics, are described elsewhere[1-3]. One of the features of the apparatus which is important to the work described here is the gridded gun which is used to program the current waveform. This permits examination of longitudinal beam physics, which can be nonlinear and multi-dimensional, as the initial bunch shape is varied.

In view of past success in obtaining agreement between experiments, and simulation and theory, on the nonlinear transverse beam dynamics in the Maryland Experiment[4], comparisons are being extended to include the longitudinal and multi-dimensional physics in the recent experiments. However, it is difficult in the current apparatus to obtain detailed time-resolved data on the transverse beam characteristics. We therefore describe below the use of simulations to demonstrate, for the simple case of an expanding parabolic bunch, insensitivity of the longitudinal dynamics to the details of the transverse match. The r,z simulations, which have been performed using the WARP[5,6] PIC code, are compared to the experimental data as well as to the simple one-dimensional envelope model which can be used to describe the special case of a parabolic bunch.

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II. DYNAMICS OF A PARABOLIC BUNCH

For a beam bunch which is long compared to the radius of the beam pipe, the beam longitudinal self-electric field can be approximated by[7] $E_z \propto g\, \partial\lambda/\partial z$, where λ is the line density and g is a geometry factor which depends on the ratio of beam radius to pipe radius. If the bunch shape is parabolic, the longitudinal self-field is then a linear function of the distance from the bunch center. If the longitudinal velocity distribution of the bunch is appropriately chosen, an envelope equation[8] can then be derived to describe the bunch dynamics. In the experiment, the longitudinal beam temperature is sufficiently low that the thermal pressure, or emittance, contribution to the beam expansion is negligible compared with the space-charge contribution. Details of the longitudinal velocity distribution do not then significantly influence the bunch expansion.

If the one-dimensional description of the beam were adequate, an initially parabolic bunch would retain its parabolic bunch shape and its expansion would be well described by the longitudinal envelope equation. However, even in the one-dimensional description, the self-electric field depends on the beam radius through the geometry factor g, which multiplies the derivative of the line density. This "g-factor" can be written in the form $g = C + 2\ln(b/a)$ where b and a are the pipe and beam radius respectively, and C is a factor, generally between zero and unity, which will be further discussed below. It should be noted that, in general, g will vary along the bunch, as well as along the transport system, as the beam expands longitudinally and its radius decreases in order to remain in equilibrium with the transverse focusing forces.

Despite the possible influence of the beam radius on the longitudinal beam dynamics, no direct data are presently available on the time-resolved variation in the beam radius as the beam propagates down the transport line. However, the beam is approximately matched to the transport line by adjusting the matching lenses until some current loss is observed, presumably associated with the mismatched beam hitting the beam pipe, and then setting the matching lens values in the middle of the broad minimum for which little loss is observed.

If the beam is assumed matched to the transport line however, previously obtained experimental data on the magnet characteristics[9], as well as extensive data on transverse beam dynamics[4], allow confident prediction of the matched beam radius, if the average magnetic field is known. The average magnetic field can, in turn, be accurately related to the coil currents. For the 26.3 mA peak beam current, and the 1.91 A coil current used in the experiment described here, the calculated matched beam radius at the peak current is 6 mm. The beam pipe radius is 19 mm.

This estimate of the matched radius along with the measured initial peak current and bunch length can be used to run an r,z WARP simulation which can be used to compare with the experiment. Lacking time-resolved data about the transverse beam characteristics, however, some assumption must still be made regarding the axial variation of transverse beam conditions away from the bunch center. The beam in the simulation is assigned a local emittance proportional to current and is therefore assumed to have a constant tune depression along the beam. The beam is matched along its length to the focusing force in the simulation, which is applied uniformly along the transport line, by adjusting the local radius so that the charge density is a constant. Once these conditions are met there are no free parameters in comparing the simulation to the experiment.

The top curve in Fig. 1 shows the evolution of the rms bunch length, from the simulation, as a function of distance propagated. Also plotted on the same axes are the experimental bunch widths measured at each of the five current monitors. Both rms bunch length and the bunch length of a best fit parabola are shown. In both cases a small tail in the current distribution at the edge of the bunch shape has been neglected.

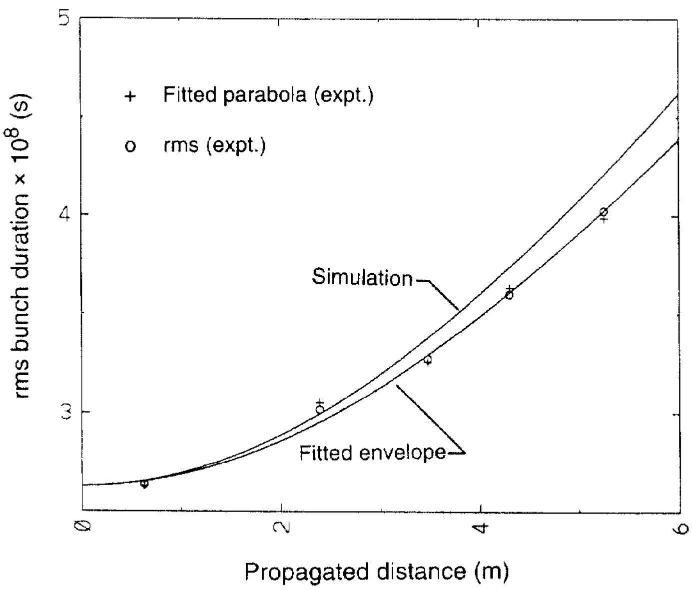

Fig. 1. Bunch length vs. distance propagated for a bunch with initially parabolic shape. Data from an r,z WARP simulation is plotted, as the top curve, on the same axes as experimental measured points, as well as, an envelope solution fitted to the expansion data.

In view of the lack of data on the transverse beam dynamics, it is difficult to say whether any of the difference between simulation and experiment is a consequence of an inappropriate choice of the initial transverse beam distribution. Simulations were therefore performed to examine the sensitivity of the longitudinal beam expansion rate to a transverse mismatch. When the beam is initially mismatched, transverse betatron oscillations are observed which, as expected, vary in frequency along the bunch as the expansion results in a differential in the beam velocity between the head and tail of the bunch. Even when the beam is initialized with a 50% mismatch that varies along the bunch, and which is sufficient to cause halo formation along the beam, only a 0.6% change is observed in the rms bunch expansion after the bunch has propagated six meters. Furthermore, the insensitivity to transverse mismatch observed for the rms average, also extends to local details of the longitudinal beam evolution. Examination of the longitudinal phase space and various projections of that phase space, such as the line density variation along the bunch, show almost no evidence of the transverse mismatch.

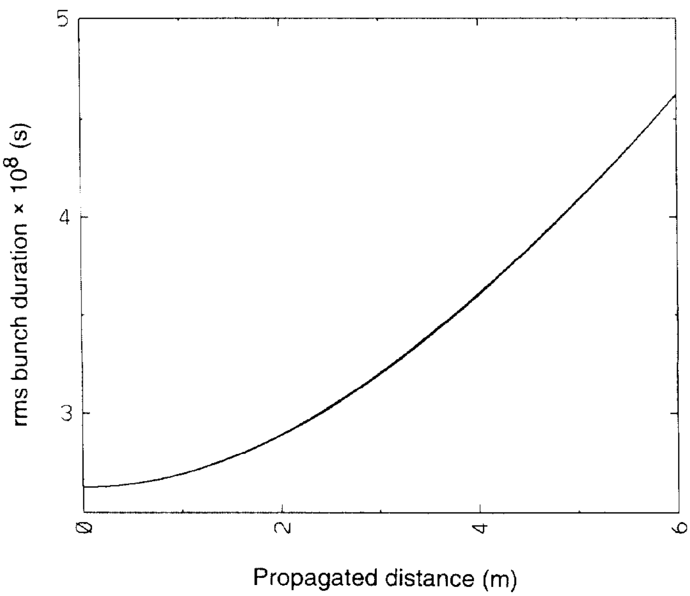

Fig. 2. Solution of the one-dimensional envelope equation overlayed onto the curve of rms bunch length obtained from the r,z simulation.

Because of the degree of insensitivity of the bunch expansion to substantial transverse mismatch, a comparison was undertaken to determine how well the bunch length in the simulation would conform to the envelope equation prediction. Since it is difficult to calculate what g-factor is appropriate for a nonuniform bunch whose radius varies with time as the beam expands, the procedure which was employed was to consider g to be a free parameter, and to find the value of g which would result in a beam expansion which matches the r,z simulation at the end points of 6 m in the simulation. The curves of rms bunch length from the simulation, and the bunch length calculated using the g value which matches the end points, were then plotted on the same set of axes as shown in Fig. 2. As can be seen from the curves, the intermediate points coincide to approximately the width of the line on the plot. If the value of g is written in the form $g = C + 2 \ln(b/a)$, then the value of C is found to be 0.791. This comparison between envelope solution and simulation was also performed for a bunch with the same current, but with the externally applied transverse focusing lowered, so that the matched radius at the beam center is doubled to 12 mm. In this case, the envelope solution and simulation curves also agreed closely and a value of 0.775 was obtained for C. This weak dependence means that the rms bunch dynamics for an initially parabolic bunch may be accurately predictable using a simple envelope model, although whether this procedure remains valid over a larger range of parameters or whether it breaks down if the beam is given an initial inward (bunching) head-to-tail differential velocity, and is then allowed to compress longitudinally, remains to be tested.

Since the expansion calculated by the envelope solution closely matches the simulation, it becomes convenient to use the envelope equation to compare against the experimental data. The bottom curve in Fig. 1 is from a solution of the envelope equation with g = 2.7, chosen to match the data points. This compares to the value of 3.11 used in Fig. 2 to match to the simulation curve. This is the value of g which would be calculated if the beam matched radius were approximately 23% larger than the 6 mm matched radius calculated from the strength of the externally applied magnetic field. Because only the product of g and the initial current appears in the envelope equation, this curve would also be generated by a 15% reduction in the intimate peak current from what was measured.

The difference in expansion rate between simulation and experiment appears to be larger than the uncertainty in either the equilibrium matched radius or the initial beam current. However, under other experimental conditions much closer agreement has been obtained. Further experimental uncertainty can also arise from the degree to which the bunch tail, whose amplitude approaches the ripple in the current waveform, is included in the definition of the bunch length. As more of the tail is included, the bunch expansion comes somewhat closer to the simulation.

Another possible source of measurement uncertainty is possible loss in beam as the bunch propagates. There is in fact some evidence that part of the beam is lost, and particularly if this loss occurs primarily at the bunch ends, can account for a decrease in the expansion rate compared to what is predicted in the simulation. The fact that the current measured at the monitor 2.39 m from the gun consistently is above the expansion curve which fits the other data points, may in fact be evidence that beam particles are being lost.

In addition to the difference in expansion rate between the simulation and the experiment there are several details of the observed bunch behavior which are not the same as the simulation. For example, the initial current waveform is not precisely a parabola and this effects the evolution of the bunch pulse shape which shows deviations from parabolic shape, including the formation of tails not seen in the simulation, as the beam propagates. There is also a noticable difference between the phase space in the simulation and the observed variation in beam velocity along the bunch. The simulations, shortly downstream from the gun show an "s" shape in the phase space not observed experimentally.

III. CONCLUSIONS

The description above is concentrated on the use of simulations, together with experiment, to examine evolution of the bunch length during free expansion of an initially parabolic bunch. Many details of the comparisons between simulation and experiment must await a more comprehensive description of the work conducted. The use of simulation, as described here, in the conduct of the relatively simple investigation of a relatively simple experiment has nevertheless yielded interesting insight into the bunch dynamics. It was found that the longitudinal dynamics is insensitive to the details of transverse beam match, and the bunch length evolution is well described by a simple envelope equation, notwithstaniding the expectation that the "g-factor," which multiplies the current in that description, would vary along the beam and along the transport line as the beam expands, so that the envelope description would not be accurate. As the comparisons are expanded to include beams which fill a greater fraction of the beam pipe, whose shapes deviate substantially from the parabolic shape employed here, and which are subject to an initial velocity "tilt" which causes the bunch to compress, a very rich set of phenomena can be explored.

IV. REFERENCES

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