

# Simulation of Emittance Dilution in Electron Storage Ring from Compton Backscattering\*

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## Abstract

A Monte-Carlo simulation of Compton backscattered  $\lambda_L = 3.2\text{-}\mu\text{m}$  ( $E_L = .38494\text{ eV}$ ) photons from an IR-FEL on 75-MeV electrons in a storage ring yields an RMS electron energy spread of  $\Delta_E = 11.9\text{-keV}$  for a sample of  $10^7$  single scattering events. Electrons are sampled from a beam of natural energy spread  $\sigma_E = 5.6\text{-keV}$  and damped transverse angle spreads  $\sigma_{x'} = .041\text{-mrad}$  and  $\sigma_{y'} = .052\text{-mrad}$  (100% coupling), scaled from the 200-MeV BNL XLS compact storage ring. The Compton-scattered X-Rays are generated from an integral of the CM Klein-Nishina cross-section transformed to the lab. A tracking calculation has also been performed in 6-dimensional phase space. Initial electron coordinates are selected randomly from a Gaussian distribution of RMS spreads  $\sigma_{x_0} = .102\text{-mm}$ ,  $\sigma_{x'_0} = .041\text{-mrad}$ ,  $\sigma_{y_0} = .018\text{-mm}$ ,  $\sigma_{y'_0} = .052\text{-mrad}$ ,  $\sigma_{\phi_0} = 22\text{-mrad}$  and  $\sigma_{E_0} = 6\text{-keV}$ . A sample of 10000 electrons were each following for 40000 turns around the ring through an RF cavity of  $f_r = 211.54\text{-MHz}$  and peak voltage  $V_m = 300\text{-keV}$ . Preliminary results indicate that the resulting energy distribution is quite broad with an RMS width of  $\Delta_E = 124\text{-keV}$ . The transverse widths are only slightly increased from their original values, i.e.  $\Delta_x = .106\text{-mm}$  and  $\Delta_x = .043\text{ mrad}$ . The scaled energy spread of  $\Delta_E \sim 360\text{-keV}$  for  $\sim 350,000$  turns desired in a 10-msec X-Ray angiography exposure is well within the RF bucket used here; even  $V_m < 50\text{-kV}$  is adequate. Further, the electron energy spread adds a negligible RMS X-Ray energy spread of  $\Delta_{EX} = .32\text{-keV}$ . The electron energy damping time of  $\tau_E = 379\text{-msec}$  at 75-MeV in an XLS-type ring allows for damping this induced spread and top-off of the ring between heart cycles.

## 1. INTRODUCTION

The production of X-Rays by Compton backscattering of laser photons by electron beams has been considered recently by several laboratories. At BNL we have numerically studied scattering of an infrared photon beam produced by a Free Electron Laser (IRFEL) from an electron beam in a low energy (75-570 MeV) Storage Ring [1-3]. Sprangle, et al. at NRL have proposed a configuration based on a  $1\text{-}\mu\text{m}$  terawatt Nd:YAG laser beam scattering from low energy ( $\sim 40\text{ MeV}$ ) electrons from a rapid-cycling betatron [4]. An experiment using another approach, an electron-linac-driven IRFEL photon beam scattered from a linac electron beam is underway at Vanderbilt [5]. The Compton backscattering studies at

BNL have been directed specifically at the medical problem posed by trans venous digital subtraction coronary angiography at the Iodine K-edge ( $E_x = 33.17\text{-keV}$ ). This problem has heretofore been pursued experimentally using high field wiggler radiation from high energy electron storage rings by the Stanford-BNL group [6,7] and the DESY group [8] and there have been proposals for a somewhat lower ( $\sim 1.3\text{ GeV}$ ) storage ring using a superconducting 8-T wiggler [9] or a pulsed  $\sim 15\text{T}$  wiggler [10]. Previous BNL studies of Compton backscattering sources [2,3] have considered an in-ring FEL; however, it is clear for the intensities needed in angiography that the required FEL power exceeds the Renieri limit [11] and gives an electron energy spread too large for the FEL to operate. Renieri's estimate has been confirmed in numerical simulations by Luccio and Pellegrini [12]. In the present study we consider Blum's proposal [1] of an external, Linac-driven IRFEL in an optical resonator configuration colliding with a storage ring beam at the low-beta interaction region of the ring straight section. The model for the ring is the compact, 8.5-m circumference, 200-MeV XLS ring operated at BNL with peak currents in excess of 1A [13]. The ring parameters used here are scaled down to 75-MeV. The calculations were performed in two parts, (1) calculations of the angle and energy distributions resulting from a sample of  $10^7$  randomly selected single scattering events, and (2) a full 6-dimensional tracking calculation of 10000 randomly selected initial electron coordinates, with each electron followed for 40000 turns around the ring. The collision probability on each encounter of the electron with the photon bunch is computed from the total (Thomson) cross section  $\sigma_0$  and the number of photons per bunch NL. For the present calculation there are an average of 87 collisions per electron.

## 2. FORMULATION

Initial electron phase space coordinates are computed using a random Gaussian generator [14] and input values of the RMS widths of the damped distributions,  $\sigma_{x_0}$ ,  $\sigma_{x'_0}$ ,  $\sigma_{y_0}$ ,  $\sigma_{y'_0}$ ,  $\sigma_{\phi_0}$ , and  $\sigma_{E_0}$  of the XLS storage ring [13], scaled down to 75-MeV for the present application. The scattering angle  $\theta$  of the X-Ray relative to the electron direction is then obtained by random selection from a table of the normalized integrated Compton differential cross section vs. polar angle  $\theta$  [15],

$$\sigma_N(\theta) = 2\pi \int_0^\pi (d\sigma/d\Omega)_{cm} J(\theta) \sin\theta d\theta / \sigma_0 \quad (1)$$

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where  $\sigma_0$  is the total cross section from the integral of Eq. 1

from 0 to  $\pi$ . For the energies considered here  $\sigma_o \approx 665$  mb. The expressions for  $(d\sigma/d\Omega)_{cm}$  and the Jacobian transformation  $J(\theta)$  from the electron rest frame to the lab were taken from Sandorfi, et al. [16]. The energy of the X-Ray is then evaluated as

$$E_x(\theta) = 4\gamma^2 E_L / (1 + 4\gamma E_L / mc^2 + \gamma^2 \theta^2) \quad (2)$$

and the recoil electron energy is then  $E = E_o - E_x$ . A random azimuthal angle  $\phi$  is then generated in the interval  $0 \leq \phi \leq 2\pi$ . New momentum coordinates of the recoil electron are then obtained in the transverse (X-Y) and longitudinal (z) direction and new angle projections of the electron are computed as  $x' = P_x/P_z$ ,  $y' = P_y/P_z$ . It is not necessary to modify the position coordinates  $x, y$  since we assume (unlike the XLS) that the interaction point is at the center of a dispersionless straight section with  $\eta = \eta' = 0$ . The electron is then transported around the ring by the one-turn transport matrix  $M_o$  to obtain new transverse coordinates  $x, x', y, y'$

$$M_o = \begin{pmatrix} \cos 2\pi\nu & \beta_o \sin 2\pi\nu \\ -(\sin 2\pi\nu)/\beta_o & \cos 2\pi\nu \end{pmatrix} \quad (3)$$

and then passes through an RF cavity prior to re-entering the interaction region. The electron phase angle relative to the RF phase is also modified by the one-turn phase advance

$$\Delta\phi = 2\pi\alpha h(E - E_o)/E_o \quad (4)$$

where  $\alpha = (\Delta C/C_o)/(\Delta p/p_o)$  is the momentum compaction factor and  $h$  is the harmonic number. We take  $\alpha = .32$  and  $h = 6$  from the XLS [13]. The RF cavity energy increment  $E_c = eV_m \sin \phi$  also changes the longitudinal momentum component  $p_z$ ; we therefore modify the angles  $x'$  and  $y'$  after passage through the cavity. Finally, when the electron re-enters the interaction region, the probability of another collision with a photon of the laser bunch is evaluated as

$$P_c = N_L \sigma_o / A_{eff} \quad (5)$$

where  $N_L$  is the number of laser photons per bunch and  $A_{eff}$  is the effective overlap area of the electron and photon bunches. Here we assume equal transverse size of the bunches and  $A_{eff} = 4\pi\sigma_x\sigma_y$ .

### 3. RESULTS

For the single scattering case we show the resulting  $x'$  electron angle distribution in Fig. 1 for  $10^7$  scattering events and the corresponding electron energy distribution in Fig. 2. The resulting RMS width of the  $x'$  distribution,  $\Delta x' \approx .041$ -mrad, is unchanged from the input width  $\sigma_x$ , within the accuracy of this calculation. The resulting electron energy distribution is decreased by  $\delta E = 16.7$ -keV, as expected since this calculation has no mechanism for restoring the electron energy lost in the electron-photon collisions. The resulting

RMS width of the electrons is  $\Delta E = 11.85$ -keV, a modest increase from the initial RMS spread of  $\sigma_E = 5.6$ -keV. The double peak exhibited in Fig. 2 is not understood.

The realistic estimate of beam emittance dilution requires a multiturn calculation in which the six-dimensional phase volume ( $x, x', y, y', \phi, E$ ) is tracked for the desired  $\sim 10$  msec duration of the X-Ray pulse. For the  $f_R = 35.257$ -MHZ rotation frequency of the ring model used here this is 352,570 turns. The collision probability per turn from Eq. 5 using  $N_L = 3.24 \cdot 10^{18}$  photons/bunch (0.2-J/bunch) previously estimated [3] to obtain the required X-Ray flux for coronary angiography, is  $P_c = 2.0757 \cdot 10^{-3}$ . Thus, each electron will suffer about  $N_c = 732$  collisions on average. Since the energy of this ring is relatively low there is insignificant radiation damping during the 10 msec spill, and we have indeed not included synchrotron radiation loss in the calculation. The preliminary results quoted here are only for 40000 turns due to computer time limitations but we feel that the RMS spreads can be scaled by  $\sqrt{N_c}$ . The results for the  $\phi$  and  $E$  distributions from a 10000 electron sample are shown in Fig. 3 and 4 and yield RMS spreads  $\Delta\phi = .1155$  rad and  $\Delta E = .124$ -MeV. The  $E$  distribution is surprisingly uniform perhaps due to the limited statistical sample.

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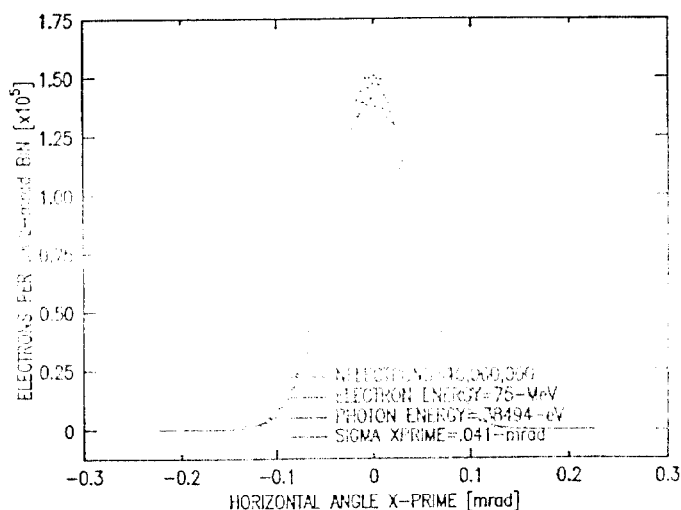


Fig. 1. ELECTRON HORIZONTAL ANGLE DISTRIBUTION

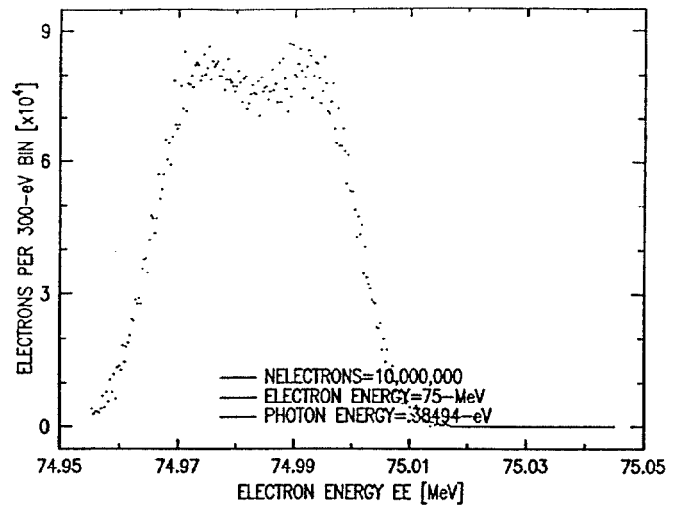
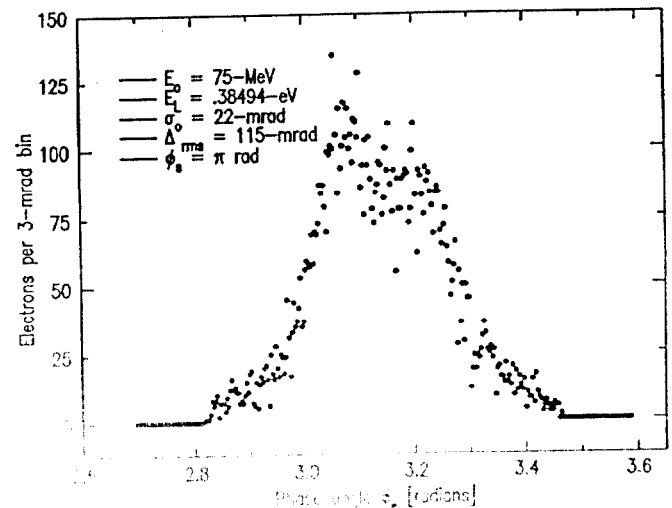


Fig. 2. ELECTRON ENERGY DISTRIBUTION IN COMPTON BACKSCATTERING



ELECTRON PHASE DISTRIBUTION AFTER 40000 TURNS IN XLS

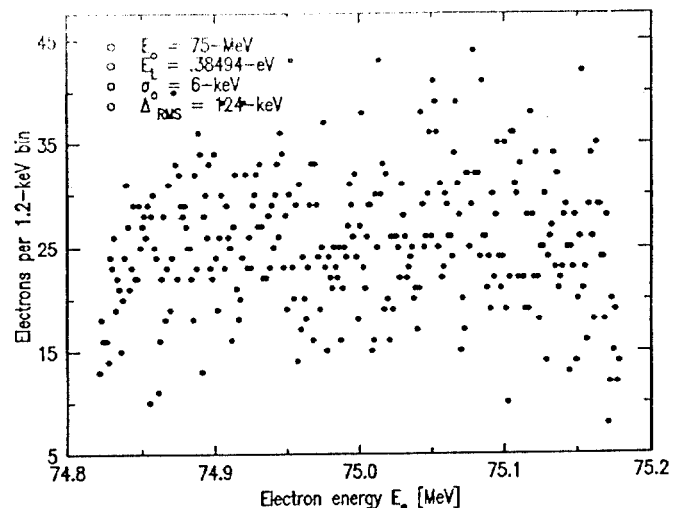


Fig. 4. ELECTRON ENERGY DISTRIBUTION AFTER 40000 TURNS IN XLS