Effects of Magnetic Focusing on Longitudinal Emittance and Energy Dispersion of an Intense Short Accelerating Electron Pulse

J-M. DOLIQUE and J.C. COACOLO
Université Joseph Fourier-Grenoble I and CEA, Centre d'Etudes de Bruyères-le Châtel. France.

Abstract

Magnetic focusing by coils appropriately disposed and fed has been widely proposed and used as a transverse emittance recovering technique. In particular for the accelerating electron beam of FEL RF-photoinjectors. After a new investigation of the effect on radial emittance, we address the question of the consequences of magnetic focusing on longitudinal emittance and energy dispersion. With the theoretical method described in a previous article, the radial, transverse, longitudinal, and 3D- rms emittances of the whole beam pulse are computed from the cathode onwards, for sample beam parameters, and various magnetic field intensities. Obviously beneficial in terms of beam radius, magnetic focusing seems to induce only a moderate and transitory radial emittance reduction and, on the other hand, a strong degradation of longitudinal emittance and energy dispersion.

I. INTRODUCTION

The situation under consideration is the one which occurs for instance in the photoinjector of a high-intensity RF-FEL (e.g. [1]). A short (10-100 ps) intense (some hundred of A) electron beam, extracted from the cathode, is accelerated in a first RF cell, about 0.1 m long, from thermal energy to a few MeV. During extraction and at acceleration beginning, the beam undergoes radial expansion and radial emittance growth under the influence of strong self-field effects (often called space charge effects though they are not electrostatic but in fact intricate electromagnetic effects).

To focus the beam and remedy the emittance growth, a magnetic lens is often placed at photoinjector exit, before the beam enters further accelerating cavities. The beneficial effect on radial emittance of such magnetic focusing has been studied in various publications (e.g. [2]). We propose on the one hand to come back to this effect, and on the other hand to investigate the consequences of magnetic focusing for longitudinal emittance and energy dispersion.

II. THEORETICAL TOOLS

A. Self-field calculation

The theory has been described in [3],[4]. With view of an analytical study, the electromagnetic field introduced in the electron motion equation is calculated as a direct relativistic electron-electron interaction field obeying the Liénard-Wiechert formulas. Retardation and acceleration field effects, which play an important role, are thus taken into account more simply and transparently than in the way generally followed by computer simulations, which consists in solving the set of Maxwell's equation. Boundary conditions on the cathode are satisfied using images.

B. Definitions used for the emittances

The emittances are normalized rms emittances, linked to the first symplectic linear invariant [5]

$$I = \sum_{i,j} < \Delta x_i\, \Delta x_j >< \Delta p_{x_i} \Delta p_{x_j} > - < \Delta x_i\, \Delta p_{x_j} >< \Delta x_j\, \Delta p_{x_i} >$$

where < > means an average taken over the whole beam

$$< G >= \int G(\mathbf{x},\mathbf{p}|t)\, f(\mathbf{x},\mathbf{p}|t)\, d^3x\, d^3p$$

where $\mathbf{x}$ is the position, $\mathbf{p}$ the impulsion. The distribution function f is normalized to 1. $\Delta\mathbf{x}=\mathbf{x}-<\mathbf{x}>$, $\Delta\mathbf{p}=\mathbf{p}-<\mathbf{p}>$, where $<\mathbf{x}>$ and $<\mathbf{p}>$ are the centroid position and impulsion respectively.

In the absence of magnetic field :

$$I = (\frac{mc}{4})^2\, [\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2]$$

where ε_x and ε_y are the two usual transverse emittances :

$$\varepsilon_x = 4\,[< x^2 >< (\frac{p_x}{mc})^2 > - < x\,\frac{p_x}{mc} >^2]^{1/2}$$

(similar expression for ε_y), and ε_z the longitudinal emittance :

$$\varepsilon_z = 4\,[< \Delta z^2 >< (\frac{\Delta p_z}{mc})^2 > - < \Delta z\,\frac{\Delta p_z}{mc} >^2]^{1/2}$$

And for an axisymmetric beam :

$$\varepsilon_x = \varepsilon_y = \varepsilon_r = 2\,[< r^2 >< (\frac{p_r}{mc})^2 > - < r\,\frac{p_r}{mc} >^2]^{1/2}$$

When a magnetic field is present, there is an azimuthal motion. Again for an axisymmetric beam, besides the radial emittance, one has to consider a transverse emittance $\varepsilon_\perp$ given by :

$$\varepsilon_\perp^2 = \varepsilon_x^2 + \varepsilon_y^2 = 2\varepsilon_r^2 + 8 < r^2 >< (\frac{p_\theta}{mc})^2 >.$$

In any case, it is interesting to consider I itself, conveniently reduced to the dimension of a length, as a 3D emittance :

$$4\sqrt{I}/mc = (\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2)^{1/2} = \varepsilon_{3D}$$

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III. EFFECT OF MAGNETIC FOCUSING ON RADIAL EMITTANCE

A. Evolution of the emittances inside the magnetic field zone

Figures 1 and 2 show the evolution of the emittances along the beam (t is the time passed from the beginning of the photoemission : at time 0 the beam front is at z=0), without magnetic focalisation (1), or with (2). The parameters are : RF electric field on the cathode E_0=15 MV/m, beam current, section, and length I =100 A, S=1 cm^2, τ = 50 or 100 ps.

The photoinjector exit is at z = 10 cm, which corresponds to t = 430 ps.

The magnetic lens center is at z=12,5 cm, which corresponds to t = 516 ps.

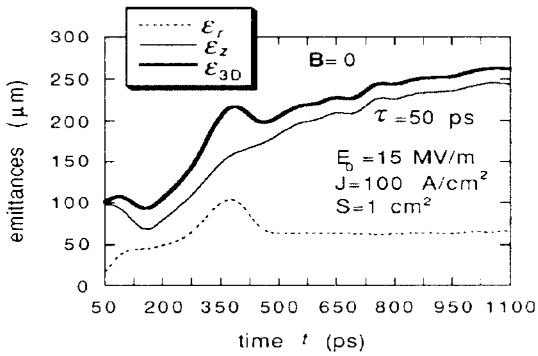

Figure 1a (τ =50 ps)

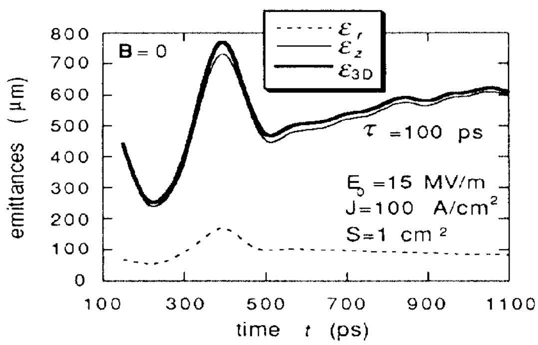

Figure 1b (τ =100 ps)

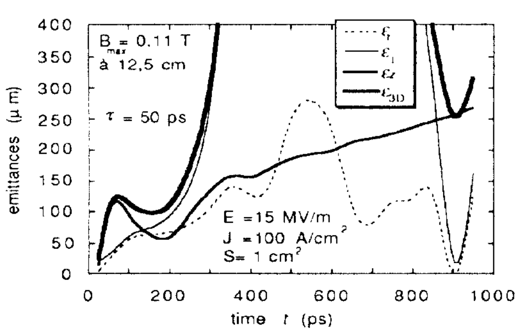

Figure 2a (τ =50 ps)

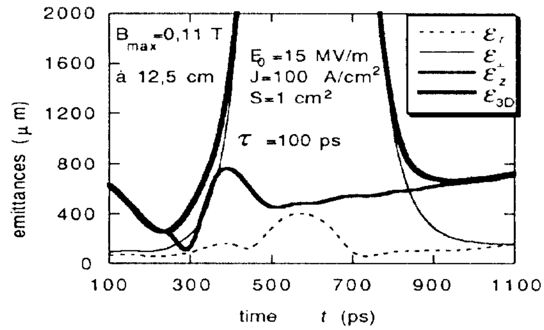

Figure 2b (τ =100 ps)

Figure 1 (no magnetic field), shows that the emittances grow and oscillate inside the photoinjector, where electrons are accelerated (0<t<430 ps), whereas in the following ballistic phase ε_r remains practically constant, while ε_z (and therefore ε_{3D}) keeps on growing but slower and slower. This corresponds : a) in the photoinjector accelerating phase, to an electromagnetic self-field strongly non-linear in both radial and axial directions ; b) in the ballistic phase, to a radial dynamics decoupled from the longitudinal one, and with negligible non-linear effects, while for the longitudinal dynamics the latter remain and slowly decrease.

In Figure 2, $\varepsilon_\perp$ and ε_{3D} exhibit, as expected in consideration of the azimuthal motion, a strong maximum around the lens center. Radial and azimuthal dynamics coupling drives ε_r oscillations. As expected again $\varepsilon_z(t)$ remains unchanged.

B. Influence of the lens magnetic field intensity on radial emittance downstream from the lens

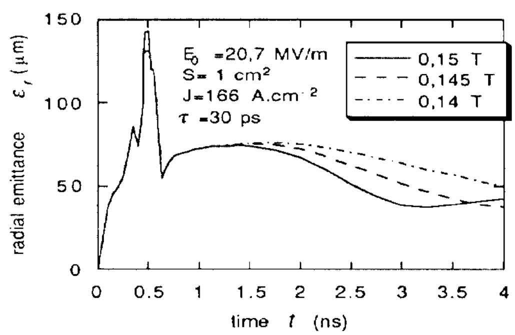

Figure 3

Figure 3 shows the evolution of ε_r downstream from the lens, in a region of ballistic propagation, as a function of the magnetic field intensity. The beneficial effect observed from t=1.5 to 3.2 ns (for the above particular values of the parameters), i.e. from z =0.4 to 0.9 m , disappears later : in the more strongly focused beam, non-linear effects drive a new emittance growth. In actual RF-FEL further accelerating cavities are not so distant from the lens center. But taking a new acceleration into account does not modify the above result : the beneficial effect on ε_r of a stronger B_0 is only transitory.

IV. EFFECTS OF MAGNETIC FOCUSING ON LONGITUDINAL EMITTANCE AND ENERGY DISPERSION

Figures 4, 5 and 6 show the influence of the magnetic field intensity on the longitudinal and 3D emittances ε_z and ε_{3D}, as also on the energy dispersion $\sigma_\gamma/<\gamma>$, where σ_γ is the γ standard deviation.

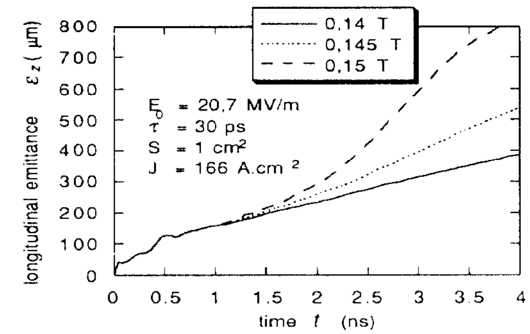

Figure 4

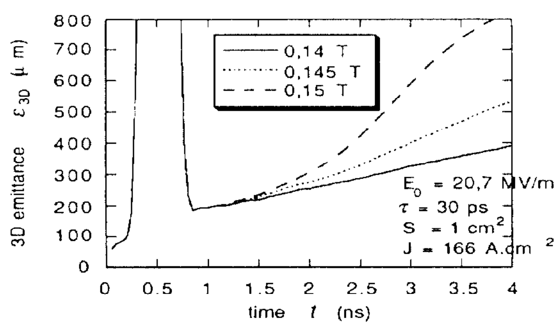

Figure 5

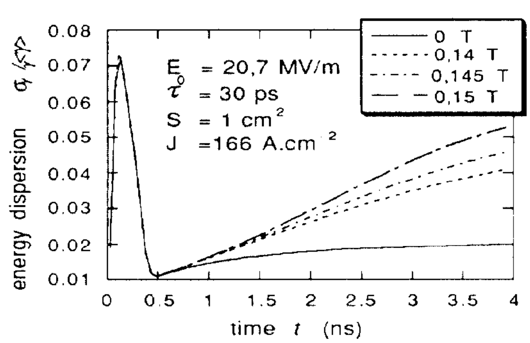

Figure 6

The meaning of these curves is clear : magnetic focusing drives strong growths for both longitudinal emittance and energy dispersion. The larger the magnetic field intensity, the stronger the growth.

V. CONCLUSION

If magnetic focusing has an evident practical interest to avoid an unacceptable radial expansion, its beneficial effect on emittance, sometime emphasized, seems to be questionable. For radial emittance it is moderate and transitory. For longitudinal emittance (and for energy dispersion), magnetic focusing induces a strong growth.

VI. REFERENCES

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