THE PRINCIPLE OF ULTRA–FAST AUTOMATIC COOLING FOR BEAMS①

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Abstract

This paper is to illustrate that the transverse emittance of a beam with constant energy can be automatically shrinked in the particular static magnetic field,which can be used as an ultra–fast "cooler" to accelerators or storage rings.The analytical solution for the nonlinear equation of motion is obtained.The exact proof of the problem is given by means of the precisely analytical representation of Jacobean determinant corresponding to the nonlinear transformation of emittance.

1. INTRODUCTION

Concerning the motion for particles with constant energy under the influence of static magnetic fields, the beam emittance is an invariant[1],which was strictly proved in the linear approximation in 1958.It has not only become a fundamental principle in the modern beam dynamics and optics,but also been used to the classical and quantum optics[2].Therefore in 1991 V.V. Parkhomchuk and A.N.Skrinsky pointed out:"the phase density of the beam can not be increased by using any set of external electromagnetic fields, independent on the motion of specific particles of the beam."[3] In other words,in the modern beam dynamics the conservation of the beam emittance seems not to be changed forever.

In order to overcome the above severe restriction, people could not help looking for other ways reducing the beam emittance.The electron beam cooling was proposed by G.I.Budker in 1966[3].After then,the cooling time was contracted to 0.1s.Hence it is so–called "fast electron cooling"[4].In 1968,Simon van der Meer proposed stochastic cooling[5],which played an extremely important role in the discovery of W and Z bosons.So he was awarded 1984 Nobel Prize for physics[6].But both electron and stochastic cooling were so slow that people were forced to look for other even faster cooling method.In 1989 H.Ibegami proposed the cyclotron maser cooling[7], which is on the way to be investigated.Hence it is still in great need of looking for an ultra–fast cooling method.

D.E.Edwards and M.J.Syphers pointed out:"Since x′ is not the conjugate momentum to x,we can't make statements about phase space and energy,but is not a concern at present."[8]However,that is not the case in nonlinear motion.In order to distinguish it from the canonical phase space,we call the x–x′ space as Courant–Snyder(C–S) phase space or the emittane phase space in this paper.F.J.N.Wilson pointed out that Liouville's theorem in the emittance phase space is just to prove that the corresponding Jacobean determinant equals unity[9].It is worth pointing out that the emittance conservation in the nonlinear case has never been proved exactly,although none of exceptions have ever been found out in all accelerators or storage rings in the world at present. Hence,this problem has never been made any progress in several decades.

Fortunately,in the nonlinear case,we have found out a particular static magnetic field in which the emittance conservation is not true any longer,so the transverse emittance of a beam with constant energy will be shrinked rapidly.Of course,that is distinguished from both adiabatic and synchrotron radiation damping arising from the field or the energy change.Such a magnet with ultra–fast automatic cooling can be treated as a "cooler",which can be used to accelerators,storage rings and so on[10]. As compared with the stochastic cooling or "fast electron cooling",its cooling rate is much more rapid by six or seven orders of magnitude[10].It is technically simple and remarkably feasible.Its application fields are much more widespread than the others,especially to the fusion device of the continuous electron beam with intense current and so on.[10]

2. THE MOTION IN THE MEDEAN PLANE

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We are interested in the motion in the axis–symmitrical field, which in the median plane is only dependent on ξ.There is a Cartesian coordinate set of ξ,η,ζ ,The Hamilton equations in the median plane are represented[11]

$$\xi' = \frac{p_\xi}{\sqrt{(1-p_\xi^2)}} \tag{2.1}$$

$$p_\xi' = -(1-\delta)\cdot\frac{e}{p_0}\cdot B_\eta$$

$$p_\xi = \frac{\xi'}{\sqrt{[1+(\xi')^2]}}$$

$$H(\xi,p_\xi,s) = -\sqrt{(1-p_\xi^2)} - (1-\delta)\cdot\frac{e}{p_0}\cdot A$$

Because H does not contain the independent variable ζ explicitly,Hamilton H is a constant.As well known

$$\xi' = Tan(\frac{\pi}{2}-\theta) \tag{2.2}$$

where θ is an angle between the velocity vector and the $\xi-axis$. From the formulas (2.1) we obtain

$$p_\xi = Cos(\theta) \tag{2.3}$$

$$Sin(\theta) = f(\xi,\xi_i,\theta_i)$$

$$f(\xi,\xi_i,\theta_i) = Sin(\theta_i) + (1-\delta)\cdot\frac{e}{p_0}\cdot[A(\xi_i)-A(\xi)]$$

$$\frac{d\zeta}{d\xi} = -\frac{f}{\sqrt{(1-f^2)}}$$

$$\zeta-\zeta_i = -\int_{\xi_i}^{\xi}\frac{f}{\sqrt{(1-f^2)}}\cdot d\xi$$

Let us consider the motion of two particles with differently initial conditions,there are two invaraints

$$Sin(\theta+\delta\theta) - Sin(\theta) = Sin(\theta_i+\delta\theta_i) - Sin(\theta_i) \tag{2.4}$$

$$\delta\zeta = \delta\zeta_i$$

When the particles from free space pass through the magnetic field region, by means of the transformation from Cartesian coordinate system to the natural coordinate set of x,y,s,as shown in Fig.1,the solution is represented

$$x_k = \{\frac{x_i}{Cos(\theta_{ei}) + x_i'\cdot Sin(\theta_{ei})} + \int_{\xi_i}^{\xi_k}[\frac{f}{\sqrt{1-f^2}} - \frac{f_e}{\sqrt{1-f_e^2}}]\cdot d\xi\} \cdot [Cos(\theta_{ek}) + x_k'\cdot Sin(\theta_{ek})] \tag{2.5}$$

$$x_k' = -\frac{Sin(\delta\theta_k)}{\sqrt{1-Sin^2(\delta\theta_k)}}$$

$$Sin(\delta\theta_k) = [C_i + Sin(\theta_{ek})]\cdot Cos(\theta_{ek}) - Sin(\theta_{ek})\cdot\sqrt{1-(C_i+Sin(\theta_{ek}))^2}$$

$$C_i = \frac{Sin(\theta_{ei})}{\sqrt{1+(x_i')^2}} - Cos(\theta_{ei})\cdot\frac{x_i'}{\sqrt{1+(x_i')^2}} - Sin(\theta_{ei})$$

$$f = Sin(\theta_{ei}+\delta\theta_i) + (1-\delta)\cdot\frac{e}{p_0}\cdot[A(\xi_i)-A(\xi)]$$

$$f_e = Sin(\theta_{ei}) + (1-\delta)\cdot\frac{e}{p_0}\cdot[A(\xi_i)-A(\xi)]$$

$$Sin(\theta_{ek}+\delta\theta_k) = Sin(\theta_{ei}+\delta\theta_i) + Sin(\theta_{ek}) - Sin(\theta_{ei})$$

$$x_i' = -Tan(\delta\theta_i)$$

That is the exactly analytical solution of the nonlinear motion in the natural coordinate system.

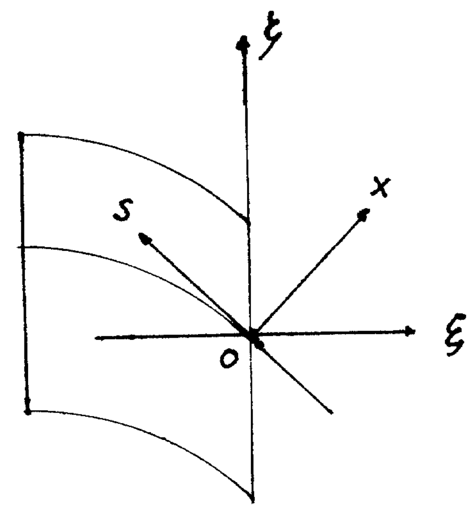

Figure 1. Cartesian and natural coordinate

3.THE PROOF FOR THE COOLING

According to the fundamental method given by F.J.N.Wilson[9],it is the necessary condition for the cooling that the corresponding Jacobean determinant must be smaller than unity.By means of differentiating the above formula,the Jacobean determinant can be obtained.From the above formula we know

$$J_{21} = \frac{\partial x_k'}{\partial x_i} = 0 \tag{3.1}$$

Then we have

$$J = J_{11}\cdot J_{22} \tag{3.2}$$

$$J_{11} = [Cos(\theta_{ei}) + x_i'\cdot Sin(\theta_{ei})]^{-1} \cdot [Cos(\theta_{ek}) + x_k'\cdot Sin(\theta_{ek})]$$

$$J_{22}=\{Cos(\theta_{ek})+Sin(\theta_{ek})\cdot\frac{C_i+Sin(\theta_{ek})}{\sqrt{1-[C_i+Sin(\theta_{ek})]^2}}\}$$

$$\cdot[Cos(\theta_{ei})+x_i'\cdot Sin(\theta_{ei})]$$

$$\cdot[Cos(\delta\theta_k)]^{-3}\cdot[1+(x_i')^2]^{-\frac{3}{2}}$$

When $\theta_{ek}=\pi$, The above formula becomes

$$J=[Cos(\delta\theta_k)]^{-3}\cdot[1+(x_i')^2]^{-\frac{3}{2}} \qquad (3.3)$$

$$Cos(\delta\theta_k)=1-Sin(\theta_{ei})+\frac{x'_i\cdot Cos(\theta_{ei})+Sin(\theta_{ei})}{\sqrt{1-x'^2_i}}$$

When we select the parameters for the equilibrium orbit

$$\theta_{ei}-\frac{\pi}{2}=\frac{\pi}{6};\ \frac{\pi}{4};\ \frac{\pi}{3};\ \frac{5\pi}{12}\ \ and\ \ \theta_{ek}=\pi$$

and $x_i'=-0.01\div0.01$ Radian,the results for calculus of J are illustrated in Firgure 2.That suggest us:

1.The Jacobean determinant J is not dependent on the initial coordinate x_i,but on x_i'.

2.It is only when $x_i'=0$ that J is equal to 1.By other words, it is only when the beam emittance equals zero that such emittance can really becomes an invariant of motion.Therefore,in a general way,the emittance conservation is not true any longer.

3.If $x_i'\neq0$,the Jacobean determinent J is always smaller than 1. In that case,after a beam passes through such magnetic field region, its transverse emittance must be shrinked automatically.That results in the ultra–fast automatic cooling for the beam.

4.The bigger the angular width of the bending magnet is,the smaller the Jacobean determinent J is.

5. The decisive factor for the ultra–fast automatic cooling is only dependent on the proper edge–focusing angle of the bending magnet rather than the field distribution.The necessary conditions are $\theta_{ek}=\pi$rather than the field distribution.The necessary conditions are $\theta_{ek}=\pi$ and the two edges of the bending magnet are parallel.Therefore it is technically simple and remarkably feasible.

6. That "cooler" is fitted for ions,electron,especially for the ultra–low energy fields.It can be used to accelerators,storage rings and so on.

7. An example of its application has been given in the forthcoming paper[10]. As compared with the stochastic cooling or the fast electron cooling, its cooling rate is much more rapid by six or seven orders of magnitude.

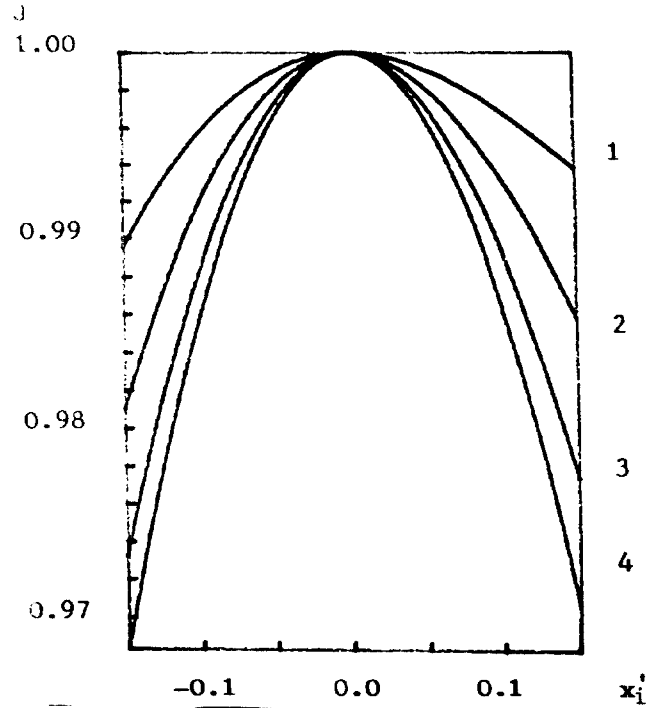

Figure 2. Jacobean determinant corresponding to the nonlinear transformation of emittance

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