The Ion Core Density in Electron Storage Rings with Clearing Electrodes

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Abstract

The low density of the ion core is required for efficient performance of electron storage rings. One of the effective methods of decreasing the density of ions in the electron beams circulating in storage rings is the application of electrostatic clearing electrodes. This report presents the results of analytic investigations of the ion core density distribution versus the magnetic field, the beam current, the density of the residual gas in the vacuum chamber and the distance between the sequential clearing sets. The numerical simulation of self-consistent evolution of the ion core has also been carried out. The results of simulation are in agreement with the theoretical predictions.

I. INTRODUCTION

The extraction efficiency of clearing electrodes depends on the longitudinal motion of ions because the electrodes do not cover all the ring circumference. They cannot be inserted inside the magnets of the ring. Hence, the parameters of the ion core trapping by the electron beam are dependent on ion motion between the clearing sets. As a rule, the clearing electrostatic electrodes are installed at dipole magnet ends. So, the present results for the ion core parameters are given just for this layout.

II. ION MOTION IN COMBINED MAGNETIC AND ELECTRIC FIELDS

The drift of charged particles in crossed electric and magnetic fields has been studied rather thoroughly [1]. However, implementation conditions for the drift theory significantly differ from those existing in storage rings, namely, the initial kinetic energy of ions is about the thermal energy of gas molecules ($\approx 0.03 \text{ eV}$), whereas the electrostatic potential of the beam reaches several hundred volts per ampere of the beam current. Therefore, the ion momentum changes significantly during a cyclotron period [2]. The cyclotron motion of ions can not be used for perturbation treatment. Here we shall derive ion trajectories in combined fields.

The ion trajectory in dipole magnets

Consider a singly charged ion born at rest in the uniform magnetic field $B_z = B_0$ and the normal to it electric field E_x . Let the electric field be expanded around the initial coordinates of the ion (x_0, y_0) :

$$E = E_0 + (x - x_0)E'$$
 (1)

This field is related to the normal component of the potential

$$\Phi = -(x - x_0)E_0 - \frac{(x - x_0)^2}{2}E'.$$
 (2)

The trajectory of the ion of mass M in these fields has the form

$$x = x_0 + \frac{P_0}{M\Omega} \left(1 - \cos\Omega t\right)$$

$$y = y_0 - \frac{\omega P_0}{\Omega M} t + \frac{\omega P_0}{\Omega^2 M} \sin \Omega t$$
(3)

$$P_0 \equiv -\frac{eE}{\Omega}; \ \Omega^2 \equiv \omega^2 + \frac{eE'}{M}; \ \omega \equiv \frac{eB_0}{Mc}$$

These expressions show the ion describing an ellipse that moves along the y axis with a direct velocity

$$v_y = -\frac{\omega P_0}{\Omega M}.$$
(4)

It should be noted that v_y is the same as the drift velocity is (see [1]), but the trajectory (3) differs from the drift one.

The amplitude of the *x*-oscillations is

$$a_{\chi} = \frac{|P_0|}{M\Omega} = \frac{v_{\chi}}{\omega}.$$
 (5)

It can be easily found that for a small nonuniformity of the electric field $(|a_x E'/E_0| \ll 1)$, the direct velocity v_y is determined by the electric field strength in the center of *x*-oscillations:

$$v_{y} = \frac{E_{0} + a_{x}E'}{B_{0}}c = \frac{E_{c}c}{B_{0}} = -\frac{c}{B_{0}}\frac{d\Phi}{dx}\Big|_{x=x_{o}-a_{x}}.$$
 (6)

Analysis of the ion motion

In the reference frame turned around the z-axis by an arbitrary angle, the direct ion velocity can be written in the canonical form:

$$v_x = \dot{x} = \frac{c}{B_0} \frac{\partial \Phi}{\partial y}; \quad v_y = \dot{y} = -\frac{c}{B_0} \frac{\partial \Phi}{\partial x}.$$
 (7)

The coordinates (x, y) of the direct motion are canonically conjugated, Hamilton function is independent of time

$$H = \frac{c\Phi}{B_0}.$$
 (8)

Using the hamiltonian treatment of direct ion motion we come to the conclusion that in a uniform magnetic field the ions move along the curves of a constant electrostatic potential Φ . Expression (8) is very useful as a basic solution for the perturbation treatment. By this way one can easily obtain the gradient drift at the magnet ends:

$$v_x = \frac{\partial H}{\partial y} = -\frac{c\Phi}{B^2} \frac{\partial B}{\partial y} \tag{9}$$

It explains the acceleration of ions moving out and reflection of ions moving into a magnet. Similar effects occur if the longitudinal electric field is applied (asymmetric electrodes, etc. [3]). Examples of the direct trajectories of ions are presented in Fig.1. The magnet edge and the clearing electrode are at the right hand side. The isolines in this figure have been drawn at regular intervals of the Hamiltonian, so the velocity is inversely proportional to the spaces between the lines. The repulsion of ions by a positive electrode (bottom figure) has been observed and obtained in the simulations [3].

Kinetic description

The above-written Hamilton function (8) generating ion trajectories allows us to describe the evolution of the ion core by the Vlasov-like kinetic equation:

$$\frac{\partial n_i}{\partial t} + \{H, n_i\} = \sigma n_b n_0 c; \quad \Delta \Phi = 4 \pi |e| (n_b - n_i). \quad (10)$$

Here σ is the ionization cross section; n_b is the beam density averaged over the period of the bunch sequence; n_0 is the density of residual gas; {} represent the Poisson brackets.



For the case of a low core density $n_i/n_b \ll 1$ and the azimuthally uniform electron beam, one obtains the core density at a distance y downstream the clearing electrodes:

$$n_i = \sigma n_0 B_0 y \frac{n_b}{E_x} \,. \tag{11}$$

In this case, the density of the ion core is independent of the beam current, because the electric field E_x is proportional to the beam density n_b . Taking into account both the upstream and the downstream ion flows, we can easily obtain that the number of ions between two clearing sets is proportional to the distance between these sets, to the residual gas pressure and the magnetic field strength for any beam current. For an arbitrary neutralization, when all ions reach the clearing electrodes, the neutralization factor cannot exceed one half

$$\eta \equiv \frac{N_i}{N_b} \le 0.5$$

as can be derived from (11). This is twice as small as that for the absence of clearing. It should be noted that the near-axis ions, that move slowly, cannot reach the electrodes. So the ion core density at $x \approx 0$ is limited by the conditions of ion confinement by the beam in the magnet-free space [3].

(12)

The core-to-beam density ratio is presented in Fig.2 for the Gaussian distribution of the x beam density.



Fig.2. The relative core density. y and z axes are in arbitrary units.

The shape of the ion core density differs from that of the beam both in transverse and longitudinal directions.

III. SIMULATIONS

Simulation of the ion core evolution has been carried out. The code is based on equation (10) without the RHS in the kinetic equation. The initial density shape of the ion core was the same as the beam -- Gaussian transverse density distribution and the uniform longitudinal one. The neutralization factor for singly charged nitrogen ions was 0.1. The clearing electrodes are placed at both sides. Fig.3 depicts the density distribution after time interval of 6.6 μ sec. As it can be seen from this figure, the longitudinal density shape becomes 'three-angular,' the transverse shape differs from the Gaussian one.

IV. CONCLUSIONS

Here we have studied some aspects of ion core formation between the clearing sets in the magnetic field. The results presented can be summarized as follows.

i -- The longitudinal velocity of ions in the magnets is independent of the initial momentum and can be sufficiently higher than the thermal velocity of the residual gas atoms.



Fig.3. Simulated ion core density distribution, x-axis is directed vertically.

- ii -- The neutralization factor is limited from above by the value of one half comparing to unity in the free space.
- iii- At a low neutralization, the density of the ion core is independent of the beam current.

V. REFERENCES

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