# Strong-Weak Beam-Beam Simulation with a Six Dimension Symplectic Code

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*Abstract*

By using a symplectic code in the six dimensional phase space, we have tried to study beam blow-up due to the combined effect of the beam-beam force and the other non-linear forces from the lattice. Tune dependence of beam blow-up was studied. With the nominal beam-beam parameters of the KEK B factory (0.05), some beam blow-up was observed in the simulation. The simulation showed that the beam-beam interaction does not affect dynamic aperture.

## I. INTRODUCTION

The motivation of this study arises mainly from designs of future B factories. The designers of B factories want to know the answers to the following questions; (1)How fast damping rate is required to keep the effects of the beam-beam interaction within a tolerable region? (2)Which tunes should we choose to minimize the harmful effects of the beam-beam interaction? (3)How long bunch length should we use ? (4)How large (horizontal) crossing angle for the purpose of beam separation is allowable from the viewpoint of the beam-beam interaction? and so on. In studying the above issues, we should study not only the effects of the beam-beam interaction alone but also non-linearity of a lattice in conjunction with the beam-beam interaction. In this study we aimed at simulating beam behavior taking both effects of the beam-beam interaction and non-linearity of the lattice into consideration simultaneously. For this purpose we added the beam-beam force to the tracking code "SAD"[1] which has been developed at KEK. As for the beam-beam interaction we followed the method proposed by K. Hirata et al.[2] where both of the bunch length effect on the collision points and the energy change caused by the electric field of the counter-rotating beam are considered. The energy change is necessary to keep the map symplectic in the six dimensional phase space.

Our study is still at an early stage and in this paper some preliminary results on the head-on collision case are described. We used the lattice of the KEK B factory which is being designed for its low energy ring[3]. Although this study has been done on the KEK B factory ring, our method is, of course, in principle applicable to any ring collider.

## II. FORMULA FOR THE BEAM-BEAM MAP

In this simulation a strong bunch is divided into some slices so that each slice contains the same number of particles. Each slice is represented by a infinitesimally thin disc which is located at the barycentre of the particles. We assume that the strong bunch has gaussian distribution in the six dimension. In the following, we consider a map for a weak particle affected by the strong (thin) slice. As is mentioned above, we used a beam-beam map proposed by K. Hirata et al.[2]. Here, we briefly describe this beam-beam map.

The motion of a particle belonging to the weak beam is described by the coordinate

$$\boldsymbol{x} = (x, p_x, y, p_y, z, \epsilon)$$

as a function of the distance,$s$ from the IP. (The values for the strong slice are specified by asterisks in subscript.) Here, $x$ and $y$ are the transverse position deviations from the nominal orbit. And $p_x$ and $p_y$ are associated momenta normalized by $p_0$ (the absolute value of three-momentum $\boldsymbol{p}$ for a reference particle;*i.e.*

$$(p_x, p_y) = \frac{m\gamma}{p_0}(\frac{dx}{dt}, \frac{dy}{dt})$$

where $\gamma$ is the Lorentz factor for the considered particle. In the longitudinal direction, we use

$$z = c(t_0 - t)$$

where $c$ is the light velocity and $t_0 - t$ is the difference in the arrival times at $s$ between the relevant particle and the reference particle. And we also use

$$\epsilon = \frac{p - p_0}{p_0} = \frac{E - E_0}{E_0}$$

where we assumed the ultra-relativistic beam for briefness. The map has the following form;

$$\begin{aligned}
x^{new} &= x + S(z, z_*) f_X(X, Y, Z), \\
p_x^{new} &= p_x - f_X(X, Y, Z), \\
y^{new} &= y + S(z, z_*) f_Y(X, Y, Z), \\
p_y^{new} &= p_y - f_Y(X, Y, Z), \\
z^{new} &= z \\
\epsilon^{new} &= \epsilon - \frac{1}{2} f_X(X, Y, Z)[p_x - \frac{1}{2} f_X(X, Y, Z)]
\end{aligned}$$

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$$-\frac{1}{2}f_Y(X,Y,Z)[p_y - \frac{1}{2}f_Y(X,Y,Z)]$$
$$-g(X,Y,Z). \qquad (1)$$

In these expressions, we use the usual Bassetti-Erskine formula for a gaussian bunch which gives the transverse (2 dimension) beam-beam kick[4];

$$f_y(x,y,\sigma_x(s),\sigma_y(s)) + if_x(x,y,\sigma_x(s),\sigma_y(s))$$
$$= -\frac{N_* r_e}{\gamma_0}\sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}}\{w(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}})$$
$$-exp(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2})w(\frac{\frac{\sigma_y}{\sigma_x}x+i\frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}})\}. \qquad (2)$$

One should note that the arguments of $f_x, f_y$ in the map are different from the above. We have to replace x,y and s in eq. (2) by X,Y, and S that are defined as follows:

$$S(z,z_*) = (z-z_*)/2,$$
$$X = x + p_x S(z,z_*),$$
$$Y = y + p_y S(z,z_*),$$
$$Z = z.$$

The last term in eq.(1) comes from the longitudinal electric field due to the strong slice and are expressed for a gaussian distribution as

$$g(X,Y,\sigma_x(S),\sigma_y(S)) = (-\alpha_x+\gamma_x S)A_x + (-\alpha_y+\gamma_y S)A_y$$

where $\alpha$ and $\gamma$ are twiss parameters at the IP and $A_{x,y}$ are defined by

$$A_{x,y}(X,Y,\sigma_x(S(Z)),\sigma_y(S(Z)))$$
$$= -\frac{1}{2(\sigma_x^2-\sigma_y^2)}\{Xf_X + Yf_Y$$
$$+ \frac{2N_* r_e}{\gamma_0}\left[\frac{\sigma_{y,x}}{\sigma_{x,y}}exp(-\frac{X^2}{2\sigma_x^2}-\frac{Y^2}{2\sigma_y^2})-1\right]\}.$$

## III. CHOICE OF BASIC TRACKING PARAMETERS

By using the tracking program, we have mainly studied beam blow-up and its (betatron) tune dependence. We also studied the effect of the beam-beam interaction on dynamic aperture. Prior to the tracking studies, we examined some basic parameters used in the trackings. To see beam blow-up multiple particles are tracked simultaneously. In this study we tracked 100 super particles which represent the weak bunch for each case. (Although we compared a case that 100 particles are tracked to a case of 1000 particles for one damping time (8000 turns), we found no essential difference between them.) In our simulation code, the strong bunch is composed of some thin slices. With some different number of slices we tracked 100 weak particles for five damping times (40000 turns). In the tracking we included the effects of radiation damping and quantum excitation. Fig. 1 shows the vertical beam size as a function of the number of turns. (In this condition beam blow-up in the horizontal or longitudinal direction was not remarkable.) As is seen in the figure beam blow-up does not depend significantly on the number of slices except in the case of one slice. However, in the following simulation we used 20 slices, since almost all computing time is devoted to the tracking in the lattice and increasing the number of the slices does not contribute to the total computing time. As is also seen in Fig. 1, 10000 or 20000 turns is maybe enough to estimate roughly the equilibrium beam size. In the following simulation we tracked 100 particles for 10000 turns for economy of time. The initial distribution for weak bunch is generated by a random number method with 6 dimension gaussian distribution. We use nominal machine parameters for the standard deviations of the distributions.

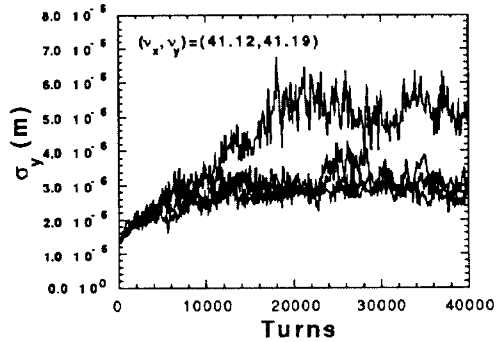


Figure 1: The vertical beam size as a function of the number of turns with different number of slices. The cases of 1, 3, 5, 10 and 20 slices were examined.

## IV. SOME RESULTS OF THE SIMULATION

### *A. Dependence of beam size on betatron tunes*

The nominal tune of the KEK B factory of the present design is $(\nu_x, \nu_y) = (41.12, 41.19)$. With some different vertical tunes we made trackings and observed equilibrium beam sizes under the condition that the horizontal tune was fixed at 41.12. The result of the tune survey is shown in Fig. 2. The solid line denotes the case that the beam-beam interaction exists. The dotted line designates the case that the beam-beam interaction is removed. Even without the beam-beam interaction, the vertical beam size increases around the coupling resonance. With the beam-beam interaction there are three peak. They might be assigned to the resonances of $\nu_x = \nu_y, 4\nu_y = 165$ and $2\nu_x + 2\nu_y = 165$. The nominal tune was chosen considering dynamic aperture and the coupling correction. Fortunately this tune is also good from the viewpoint of the beam-beam interaction. However, even with this good tune some beam blow-up is seen.

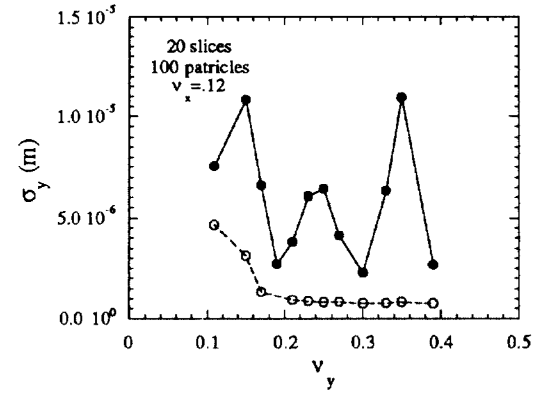


Figure 2: Tune dependence of the equilibrium beam size.

*B. Dependence of beam size on the beam-beam parameters*

At the nominal tune of $(\nu_x, \nu_y) = (41.12, 41.19)$, dependence of beam size on the beam-beam parameters were examined. The result is shown in Fig. 3. If the beam-beam parameter is small, the vertical beam size decreases as time goes by. This is because the initial beam size for which we used the nominal value of x-y coupling (1%) is different from the value which is determined by the lattice. In this study we did not consider any machine error and the calculated emittance ratio $(\varepsilon_y/\varepsilon_x)$ was 0.1%. With the nominal beam-beam parameters of 0.05, some beam blow-up was observed in the simulation.

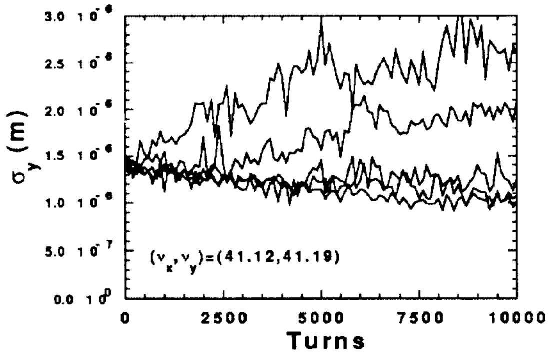


Figure 3: The vertical beam size as a function of the number of turns with different number of particles in the strong bunch. The cases with $\xi_x = \xi_y$ of 0, 0.01, 0.03, 0.04 and 0.05 are shown.

*C. Dynamic aperture*

At the nominal tune, we investigated dynamic aperture in two cases; with and without the beam-beam interaction. For each case, both the horizontal and the vertical apertures were studies. We found that dynamic aperture in both directions is not affected by the beam-beam interaction almost at all. In Fig.4 the horizontal aperture is depicted as a function of momentum deviation. As is shown in the figure, there is no signature that dynamic aperture is influenced by the beam-beam interaction as far as the present case is concerned. However, this seems to be not so surprising, since unlike usual non-linear forces the beam-beam force decreases rapidly as the amplitude of the betatron oscillation becomes large.

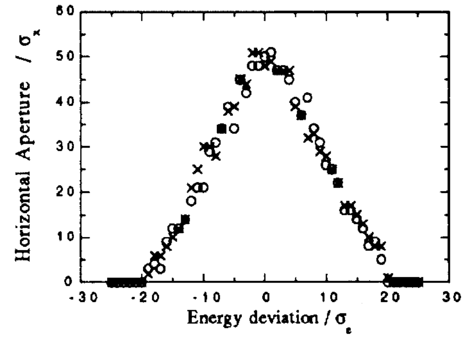


Figure 4: Horizontal dynamic aperture as a function of momentum deviation.

## V. SUMMARY AND REMAINING PROBLEMS

By using a symplectic code in the six dimensional phase space, we have tried to study beam blow-up due to the combined effect of the beam-beam force and the other non-linear forces from the lattice. Beam blow-up depends on betatron tunes and the resonance lines which seem to induce the beam blow-up were found. However, to what extent the lattice non-linearity is responsible to the beam blow-up is not clear from the present study. To see this, it is important to compare the present results with the case that the non-linearity of the lattice is removed from the simulation and only the beam-beam force is a non-linear element in the ring. With the nominal beam-beam parameters of the KEK B factory (0.05), some beam blow-up was observed in the simulation. Aiming at suppression of this beam blow-up, shortening the radiation damping time may be helpful. The effect of the damping can be and should be simulated using the present code. The simulation showed that the beam-beam interaction does not affect dynamic aperture so far as the present case is concerned. Other interesting issues such as relationship between synchro-betatron resonance and crossing angle or bunch length can also be dealt with the present method and will be investigated shortly.

## VI. REFERENCES

[1] K. Oide, Nucl. Instr. Meth. A276 427(1989).
[2] K. Hirata, H. Moshammer and F. Ruggiero, KEK Preprint 92-117(1992).
[3] B-Factory Accelerator Task Force, KEK Preprint 90-24(1990).
[4] M. Bassetti and G. Erskine, CERN-ISR-TH/80-06(1980).