Longitudinal Beam-Beam Effects in Circular Colliders

M. Hogan and J. Rosenzweig
Department of Physics
University of California, Los Angeles, CA 90024

Abstract

The longitudinal beam-beam interaction, which can lead to incoherent heating, synchrobetatron coupling, and coherent longitudinal instabilities in circular colliders, is examined. This analysis discusses two types of energy kicks, those due to the transverse particle motion coupling to the electric portion of the transverse kick, and those derived from the inductive electric field induced near the interaction, which is obtained from the transverse kick through use of a generalization of the Panofsky-Wenzel Theorem. Implications for low energy e^+e^- colliders (φ & B factories) with beams crossing head-on, and at finite angles, with and without crab crossing, are discussed.

I. INTRODUCTION

Table 1: Notation used in this paper.

e	electron charge
r	radial position
x, y	transverse coordinates
z	longitudinal coordinate
c	speed of light
$s = z - ct$	beam coordinate
p	particle momentum
$'$	d/ds
σ_i	rms. beam size in i dim.
β	Beta-function
a	Courant-Snyder amplitude
N	# electrons in bunch
*	denotes evaluation at IP

An ultra-relativistic particle with longitudinal coordinate s_0 has E-M fields which are nearly normal to the direction of motion, and may be approximated as[1]

$$E_\perp = \frac{2e}{r}\delta(s - s_0) \tag{1}$$

where the notation is given in Table 1. Since particles do not all collide head on there is some longitudinal kick given during the beam-beam interaction. The net longitudinal kick can be obtained for a single particle by taking the projection of the transverse fields from the opposing beam onto the design orbit of the particle and integrating over the betatron phase space of the opposing beam. The resulting energy kick can be thought of as arising from two sources: a) longitudinal fields, and b) the work done by the transverse motion of the particle against the transverse fields.

For beams that do not collide at the nominal interaction point (IP), there is a time dependent beta function, $\beta(s)$, and the beam size variation gives rise to an inductive longitudinal field. The longitudinal momentum has been previously derived from a straight forward retarded relativistic calculation[2]. In the limit that the beams are shorter than β^* (this limit is assumed Ed throughout this paper), these kicks can also be derived from a form of the Panofsky-Wenzel Theorem which is generalized to include fields arising from free charges [see Appendix],

$$\bar{\nabla}_\perp(\Delta p_z) = \frac{\partial(\Delta \bar{p}_\perp)}{\partial s} \quad . \tag{2}$$

We will employ the Panofsky-Wenzel theorem method in this paper, as it is simpler and more powerful than doing the straight forward calculation, especially for off-axis particles.

II. THE LONGITUDINAL BEAM-BEAM INTERACTION

The longitudinal kick due to transverse motion is the sum of the individual kicks a particle receives traversing the opposing bunch, calculated by integrating over the phase space of the opposing bunch. The resulting differential equation for the transverse acceleration involves complex error functions for elliptical beams.

A focused gaussian beam is described by a time dependent charge density and has associated transverse currents given by the continuity equation. By integrating the charge and current densities we obtain the corresponding scalar and vector potentials, which intern describe associated electric and magnetic fields. The Lorentz force law may then be used to compute the instantaneous acceleration felt by a test particle traversing these fields. The resulting equations contain very tedious integrals that may be solved numerically [2].

Both cases yield results that are not intuitive and do not allow for an estimate of the size of these kicks which could determine the importance of this analysis for circular colliders. Therefore we will now consider certain limiting cases.

A. Round Beams

The energy change (ΔE) an off-axis test particle which travels at a nonzero angle with respect to the axis receives passing through an opposing beam is given by projecting the orbit of the particle onto the field of the opposing beam particles, as given by Eq. (1). Assuming a round beam with a gaussian distribution, Eq. (1) can be integrated to give the total energy change per passage:

$$\Delta E = -\frac{Ne^2}{r^2}\left[1 - \exp\left(-\frac{r^2}{2\sigma_r^2}\right)\right](xx' + yy'). \tag{4}$$

Averaging over a betatron oscillation (which is assumed to be much shorter than a synchrotron oscillation), and expanding equation (4) for $r < \sigma_r$ reduces this to

$$\Delta E = \frac{Ne^2\beta'(\frac{s}{2})}{2\beta(\frac{s}{2})}\left[\frac{a_x + a_y}{2\varepsilon}\right], \tag{5}$$

where we have introduced the Courant-Snyder amplitudes of the particle. Note that if the beams do not collide at the nominal IP, implying that $\beta' \neq 0$, the energy kick averaged over a betatron oscillation is non-vanishing.

The energy kick due to the inductive field is computed using the Panofsky-Wenzel Theorem:

$$\frac{\partial(\Delta p_z)}{\partial r} = \frac{\partial(\Delta p_r)}{\partial s} \quad . \tag{6}$$

For round beams the transverse kick is, from equation (4),

$$\Delta p_r = \frac{2Ne^2}{cr}\left[1-\exp\left(-\frac{r^2}{2\sigma_r^2}\right)\right]. \tag{7}$$

Since

$$\sigma_r = \sigma_0\left[1+\left(\frac{s}{2\beta^*}\right)^2\right] \tag{8}$$

has an s dependence, the longitudinal kick is obtained using Eqs. (7) and (8), and integrating the Panofsky-Wenzel expression (taking the boundary condition $\Delta p|_{r=\infty}=0$) to yield

$$\Delta E = \Delta p_z c = \frac{Ne^2\beta'(\frac{s}{2})}{2\beta(\frac{s}{2})}\exp(-\frac{r^2}{2\sigma_r^2}) \tag{9}$$

Averaging over a betatron oscillation for $r < \sigma_r$ we have

$$\Delta E \cong \frac{Ne^2\beta'(\frac{s}{2})}{2\beta(\frac{s}{2})}\left[1-\frac{a_x+a_y}{2\varepsilon}\right],$$

and the amplitude dependence of this kick is canceled by the term found in Eq. (5). Previous analysis[1] has not included this cancellation, which makes the energy kick more uniform as a function of position. The resulting total energy kick for round beams is

$$\Delta E = \Delta p_z c = \frac{Ne^2\beta'(\frac{s}{2})}{2\beta(\frac{s}{2})}. \tag{10}$$

This energy kick, which is now correlated only to the relative longitudinal position of the oncoming bunch, can contribute to a coherent instability, as discussed in section III.

B. Flat Beams

Other relevant aspects of the beam-beam interaction can be analyzed in the limit that $\sigma_x >> \sigma_y$. In this quasi-one dimensional case the Panofsky-Wenzel theorem reads

$$\frac{\partial(\Delta p_z)}{\partial y} = \frac{\partial(\Delta p_y)}{\partial s}. \tag{11}$$

The transverse kick in a gaussian beam is given by

$$\Delta p_y = \frac{\sqrt{2\pi}Ne^2}{c\sigma_x}erf\left(-\frac{y}{\sqrt{2}\sigma_y}\right), \tag{12}$$

where

$$\sigma_y = \sigma_0\left[1+\left(\frac{s}{2\beta_y{}^*}\right)^2\right]^{\frac{1}{2}}. \tag{13}$$

The contribution to the energy kick is now due to transverse motion can be found in analogy to Eq. 4. Also, the inductive energy kick is, using Eq. (11),

$$\Delta E = \Delta p_z c = \frac{Ne^2\sigma_y\beta_y'(\frac{s}{2})}{2\sigma_x\beta_y(\frac{s}{2})}\exp\left(-\frac{y^2}{2\sigma_y^2}\right). \tag{14}$$

Again, the transverse dependence is proportional to the current density, a variation which is canceled exactly by the energy kick due to transverse motion derivable from Eq. 12 for small amplitude ($y < \sigma_y$) particles. Note that this expression is smaller than the equivalent round beam formula (Eq. (10)) by a factor of $R = \sigma_y/\sigma_x$. This factor is due to larger average distances between particles (R/2) and weaker focusing in the x-dimension (2).

III. COHERENT BEAM-BEAM OSCILLATIONS

Longitudinal beam-beam effects can drive a coherent longitudinal oscillation. While this subject has been analyzed before[1], it has never been understood that the longitudinal beam-beam kick is nearly independent of x and y. The coupled equations of motion for the beam centroids ($s_{1,2}$) are

$$\begin{aligned} s_1'' + \omega_s^2 s_1 &= (\pm)k_{bb}(s_1 - s_2) \\ s_2'' + \omega_s^2 s_2 &= (\pm)k_{bb}(s_2 - s_1) \end{aligned} \quad . \tag{15}$$

Here, $k_{bb} \equiv V'_{bb}/V'_{rf}$, the rf gradient $V'_{rf} = k_{rf}V_{rf}$, and V'_{bb} (the effective beam-beam gradient) includes components due to both parallel and transverse motion of the beam particles:

$$eV'_{bb} = \frac{Ne^2}{2R\beta^{*2}}. \tag{16}$$

The + (-) sign refers to operation above (below) transition. Above transition, we obtain the dispersion relation

$$\omega = \omega_s(1-2k_{bb})^{\frac{1}{2}}. \tag{17}$$

Thus the instability threshold, which occurs when ω=0, is given by $2V'_{bb} = V'_{rf}$. If the vertical beta function is lowered by a factor η, then V'_{bb} will increase by a factor of $\eta^{\frac{3}{2}}$. This is a strong dependence and may indicate trouble with higher luminosity designs.

If the machine is run below transition, the frequency of the coupled mode becomes

$$\omega = \omega_s(1+2k_{bb})^{\frac{1}{2}} \tag{18}$$

and there is no possibility of this coherent longitudinal instability.

IV. IMPLICATIONS FOR LOW ENERGY COLLIDERS

A. φ Factories

For the UCLA φ-factory design parameters, the expected energy kick for a one σ_z particle is ~ 1 keV. The rf voltage gradient V'_{rf} = 1 MeV/m is quite small due to the quasi-isochronous condition being employed. With N=1.6 x 10^{11}, R=7, β_y^*= 4 mm, and emittances of 1.1 x 10^{-6} m-rad in both x and y, the effective beam-beam gradient is

$$eV'_{bb} = \frac{Ne^2}{2R\beta^{*2}} = 1 \text{ MeV/m} . \tag{19}$$

If the machine is run above transition, this gradient is longitudinally defocusing, and one would expect serious bunch lengthening, since to first order there is complete longitudinal defocusing,

$$\frac{\sigma_z}{\sigma_{z0}} = \left(1 - \frac{V'_{bb}}{V'_{rf}}\right)^{-\frac{1}{2}} = \infty \tag{20}$$

This system is also above threshold for the longitudinal instability by a factor of two in beam charge.

If the machine is designed to operate below transition, the beams have stable coherent longitudinal motion. The beam-beam effects reinforce the rf focusing because

$$\frac{\sigma_z}{\sigma_{z0}} = \left(1 - \frac{V'_{bb}}{V'_{rf}}\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}. \tag{21}$$

This implies that the bunches could be shortened allowing shorter bunches than present designs indicate.

B. B Factories with Crab Crossing

Crab crossing schemes (Figure 1) may be necessary to provide the high luminosity's (~3 x 10^{33}) required for B-factories.

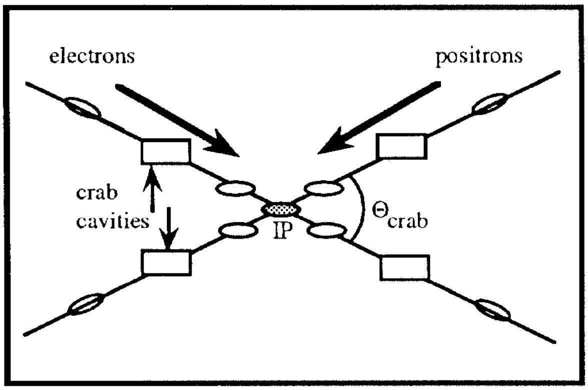

Figure 1: Schematic representation of crab crossing. Crab cavities apply time dependent rf kicks which tilt the bunches. After the collision, another set of crab cavities kick the beams back to there original orientations.

The longitudinal beam-beam interaction could become important in this type of scheme. Recall that the energy kick due to transverse motion for round beams contains terms proportional to xx', yy'. For crab crossing the angle x' is now essentially to the crossing angle. The resulting energy kick may again lead to large longitudinal effects. This issue needs to be investigated further.

V. CONCLUSION

This paper has analyzed two types of longitudinal kicks arising from the longitudinal beam-beam interaction: those due to the transverse particle motion coupling to the transverse portion of the electric kick, and those derived from the inductive electric field induced near the interaction point. These effects may become important in low energy or high luminosity colliders (φ & B factories) since they may lead to coherent longitudinal instabilities. These effects can be minimized by the use of flat beams. In addition, if coherent instabilities become a problem, it may be necessary to operate below transition. The effect on low energy colliders such as B-factories which may utilize crab crossing to improve luminosity needs to be addressed further.

Appendix: Generalized Panofsky-Wenzel Theorem

The Panofsky-Wenzel theorem gives a relationship between the integrated longitudinal and transverse momentum kicks a particle receives as it traverses a medium or device excited in the wake of another particle [4]. This appendix will generalize this theorem to include fields arising from free charges, assuming that fields and potentials vanish at infinity, and that the particle receiving the kick travels parallel to the z axis. In general, an electric field $\vec{E}$ may be described in terms of a scalar potential (Φ) and vector potential ($\vec{A}$):

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\Phi \ . \tag{22}$$

Inserting (22) into the Lorentz force equation

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}_b}{c} \times \vec{B}\right) \tag{23}$$

and noting that for a particle traveling parallel to the z-axis

$$\frac{1}{c}\vec{v}_b \times \vec{B} = \beta_b \hat{z} \times \left(\vec{\nabla} \times \vec{A}\right) = \vec{\nabla}\left(\beta_b A_z\right) - \beta_b \frac{\partial A_z}{\partial z} \tag{24}$$

we obtain the following expression for W, the force per unit charge q:

$$\vec{W} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\Phi - \left[\vec{\nabla}\left(\beta_b A_z\right) + \hat{z}\beta_b \frac{\partial A_z}{\partial z}\right]. \tag{25}$$

By noting that

$$\frac{1}{c}\left(\frac{\partial \vec{A}}{\partial t} + v_b \frac{\partial \vec{A}}{\partial z}\right) \Leftrightarrow \frac{1}{c}\frac{\partial \vec{A}}{\partial t} = \beta_b \frac{\partial \vec{A}}{\partial z}, \tag{26}$$

equation (25) can be rewritten as

$$\Delta\vec{p} = \frac{q}{c}\int_R \left[\beta_b \frac{\partial \vec{A}}{\partial z} + \vec{\nabla}\left(\beta_b A_z - \Phi\right)\right] dz \ . \tag{27}$$

Outside of region R, $\vec{A} = 0$, leaving

$$\Delta\vec{p} = \vec{\nabla}\left[\frac{q}{c}\int_R \left(\beta_b A_z - \Phi\right) dz\right]. \tag{28}$$

Since it can be derived from a potential, Δp satisfies the relation

$$\vec{\nabla} \times \left(\Delta\vec{p}\right) = 0 \ . \tag{29}$$

In shorthand notation, this can be written as

$$\vec{\nabla}_\perp \left(\Delta p_z\right) = \frac{\partial\left(\Delta\vec{p}_\perp\right)}{\partial z}. \tag{30}$$

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