Head-On and Long-Range Beam-Beam Tune Shift Spread in the SSC

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Abstract

The head-on and long-range incoherent tune shifts for the Superconducting Super Collider (SSC) are estimated using the numerical integration of the analytical expression coming from the first order in the perturbation strength. The variation of the tune shift as a function of the displacements of the charged particle in the vertical and horizontal planes is studied with the nominal parameters for the SSC. A scaling expression is obtained for the parameters involved in the beam-beam tune shifts, which allows us to predict the effect in the incoherent tune shift spread under changes in these parameters.

I. INTRODUCTION

The tune shift spread (TSS) in a collider machine is one of the most important parameters in its design and operation. The spread must be limited to those values that avoid crossing dangerous resonances in the operation tune space of the machine. This crossing may produce instabilities in the beam, limiting the performance of the machine (beam lifetime and luminosity [1]), and creating radiation problems in the detectors themselves (beam-halo [2]). Therefore, it is desirable to keep the TSS as small as possible at the SSC. The most important contribution to the TSS comes from the beam-beam interaction (head-on and long-range). This TSS is due to the kicks given to the particles in one bunch by another bunch because of the nonlinear (in general) force felt by the particles of the two interacting bunches. One kick corresponds to the head-on collision (when actually the two interacting bunches are in the same line), which produces a negative TSS in both transverse directions. The other kick comes from the long range interaction in the intersection region (IR), where the beams have not yet been separated into different beam tubes. Assuming the crossing angle is in the vertical plane (y), this kick produces a negative tune shift in the horizontal plane (x) and a positive tune shift in the vertical plane, for y-amplitudes oscillate a little less than the separation of the close orbits of the two bunches, D_y. For a beam with Gaussian density distribution in both transversal planes, the exact expression for the tune shift experienced by a test particle due to head-on collision is well known [3]. This expression will be used in the study of the TSS.

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II. ANALYTICAL APPROXIMATION FOR HEAD ON AND LONG RANGE TSS

In general, the kick given to a particle (which has horizontal (x) and vertical (y) displacement from its own closed orbit) due to beam-beam interaction is given in the horizontal and vertical planes by

$$\Delta x' = -(\frac{4\pi\xi}{\beta})\frac{2\sigma^2}{r^2}\left(1-\exp(-r^2/2\sigma^2)\right)x \qquad (1a)$$

and

$$\Delta y' = -(\frac{4\pi\xi}{\beta})\frac{2\sigma^2}{r^2}\left(1-\exp(-r^2/2\sigma^2)\right)(y+D_y)\ , \qquad (1b)$$

where β is the Courant-Snyder beta function, σ is the standard deviation in the Gaussian distribution of the particles, r is defined by $r^2 = x^2+(y+D_y)^2$, and ξ (called the tune shift parameter) is defined as $\xi = -Nr_p\beta/4\pi\gamma\sigma^2$. N is the number of particles in the bunch, r_p is the classical radius of proton (e^2/mc^2), which has the value $1.5348\times10^{-18}m$, and γ is the relativistic factor, $\gamma^{-1} = \sqrt{1-v^2/c^2}$.

Out of any resonance, the tune shifts are given at first order in the perturbation strength by Reference [4]:

$$\frac{\Delta\nu_x}{\xi} = \int_0^1 du\ e^{-pu}\ \left[I_0(\alpha_1 u) - I_1(\alpha_1 u)\right] \left[I_0(\alpha_2 u) + \sum_{n=1}^{\infty}\frac{(a_y d_y u)^{2n} n!}{2^n(2n)!}\sum_{k=0}^{n}\frac{(-1)^k}{k!(n-k)!}\frac{d^k I_0(\alpha_2 u)}{d(\alpha_2 u)^k}\right] \qquad (2a)$$

and

$$\frac{\Delta\nu_y}{\xi} = \int_0^1 du\ e^{-pu}\ I_0(\alpha_1 u)\Bigg[I_0(\alpha_2 u) - I_1(\alpha_2 u) + \sum_{n=1}^{\infty}\frac{(a_y d_y u)^{2n}(n+1)!}{2^n(2n)!}\sum_{k=0}^{n+1}\frac{(-1)^k}{k!(n+1-k)!}\frac{d^k I_0(\alpha_2 u)}{d(\alpha_2 u)^k} - \sum_{m=1}^{\infty}\frac{a_y^{2m-2}d_y^{2m}u^{2m-1}m!}{2^{m-1}(2m-1)!}\sum_{k=0}^{m}\frac{(-1)^k}{k!(m-k)!}\frac{d^k I_0(\alpha_2 u)}{d(\alpha_2 u)^k}\Bigg]\ , \qquad (2b)$$

where $d_y = D_y/\sigma$ is the relative separation between the orbits of the colliding bunches, $s_i = \sin(\psi_i)$ and $\psi_i = \phi_i + 2\pi\nu_i t$ for $i = x, y$. $I_\nu(z)$ are the Bessel functions of imaginary argument; p and $\alpha's$ are defined as $p = \alpha_1+\alpha_2+d_y^2/2$ and $\alpha_i = a_i^2/4, i = x, y$.

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III. HEAD-ON AND LONG-RANGE TUNE SHIFT FOR THE LOW-β IR

Using the above analytical approximation and making a numerical integration of these expressions, the TSS can be calculated for the nominal parameters of the SSC collider. These calculations will be made mainly for the low-β IR, but the medium-β IR will be considered at the end of this section and in the next section.

A. *Head-On and Long-Range Tune Shift*

Making $d_y = 0$, the relative horizontal head-on tune shift predicted by Eq. (2) is shown in Figure 1 (for the vertical case, A_x and A_y are inverted). as a function of the amplitudes. Using the relation between the beam size and the Curant-Snyder parameters, the beta function (β) and the relative emittance (ϵ), $\sigma = \sqrt{\epsilon\beta/\gamma}$, the tune shift parameter is expressed as

$$\xi = -\frac{N r_p}{4\pi\epsilon} \; . \tag{3}$$

With the nominal values for the SSC, $N = 7.5\times10^9$ and $\epsilon = 10^{-6}m$, the value of the tune shift parameter obtained is $\xi = -0.000916$. Figure 2 shows the TSS due to the head-on case.

For the long-range tune shift, $d_y \neq 0$ and the tune shift parameter (3) has to be multiplied by the number of long-range interactions for the two low-β IRs, $2(2L/S_B)$, where S_B is the separation between the colliding bunches. This defines a new parameter, $\xi_L = \frac{4L}{S_B}\xi$, for the long-range tune shifts. Using the relation between the separation, D_y, the long-range interaction length, L, and the crossing angle, α ($D_y = \alpha L$), the relative closed orbits separation can be written in terms of the crossing angle, α ($D_y = \alpha L$), the long-range interaction length, L, the energy, γ, and the Courant-Snyder parameters, β and ϵ, as $d_y = D_y\sqrt{\gamma/\epsilon\beta} = \alpha L\sqrt{\gamma/\epsilon\beta}$. In addition, assuming a pure drift space during the long-range interaction, where the beta function is given by $\beta(L) = \beta^* + L^2/\beta^* \simeq L^2/\beta^*$, then, the relative closed orbit separation can be written in the form $d_y = \alpha\sqrt{\beta^*\gamma\epsilon}$. For the low-$\beta$ IR, β^*=0.5 m, and at the highest energy, $\gamma = 2\times10^4$, this parameter has the simple expression

$$d_y = 10^5\alpha \; . \tag{4}$$

Using the nominal crossing angle, α=75 μrad, Figure 3 and Figure 4 show the relative (with respect to ξ_L) tune shifts as a function of the amplitudes a_x for some a_y's. Finally, for the nominal values, L=75 m and S_b=5 m ($4L/SB = 60$ long-range collisions), the total TSS (head-on plus long-range) is shown in Figure 5 for the above nominal values.

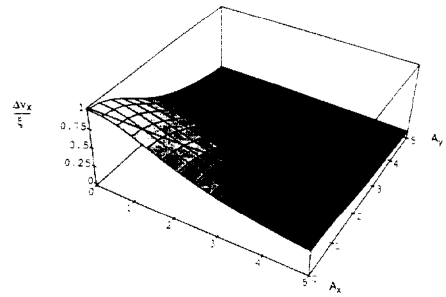

Figure 1. Relative horizontal head-on tune shift.

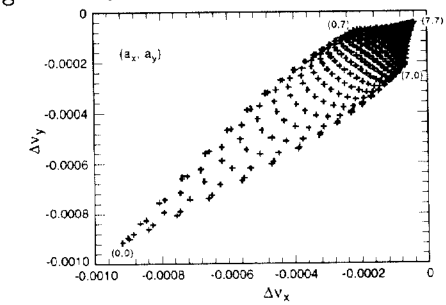

Figure 2. Head-on tune shift with the nominal SSC low-β IR parameters.

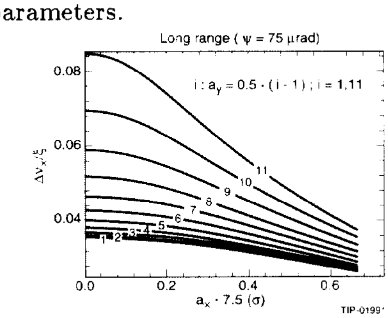

Figure 3. Relative horizontal long-range tune shift.

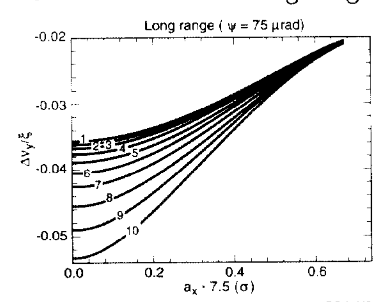

Figure 4. Relative vertical long-range tune shift.

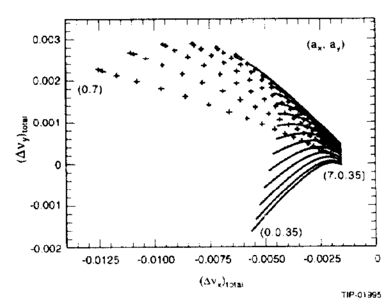

Figure 5. Total TSS with one low-β IR, α=75 μm.

B. Maximum Long Range Tune Shift

The relative horizontal long-range tune shift reaches its maximum value when the horizontal amplitude is zero and the vertical amplitude (almost true for the vertical plane) has its maximum allowed value ($a_x = 0$ and $a_y = a_y^{max}$). This observation helps to simplify the analysis of the maximum relative tune shift as a function of the crossing angle, since the maximum tune shifts can be calculated from the following expressions. (See Reference [4].)

$$\left(\frac{\Delta\nu_x}{\xi}\right)_L^{max} = \frac{1}{\pi}\int_0^{2\pi} \frac{\left(1 - e^{-s_{xy}/2}\right) d\psi_y}{s_{xy}} \qquad (5a)$$

and

$$\left(\frac{\Delta\nu_y}{\xi}\right)_L^{max} \simeq \frac{2}{\pi}\int_0^{2\pi} \frac{\left(1 - e^{s_{xy}/2}\right)\left(s_y^2 + \frac{d_y}{a_y^{max}} s_y\right) d\psi_y}{s_{xy}}, \qquad (5b)$$

where s_{xy} is defined as $s_{xy} = -(a_y^{max} s_y + d_y)^2$.

The maximum long-range tune shifts expected for the SSC as a function of d_y and for several maximum amplitudes a_y^{max} are shown in Figure 6. In the same way, the maximum tune shifts expected as a function of the maximum vertical amplitude are shown in Figure 7.

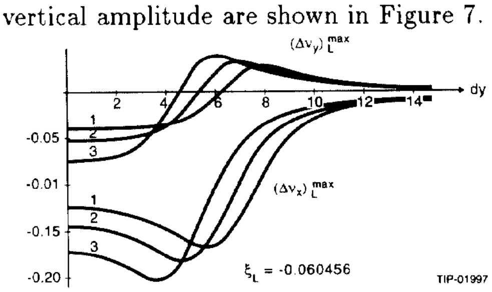

Figure 6. Maximum LRTS in one low-β IR, $a_x = 0$; and 1: a_y=7; 2: a_y=6; 3: a_y=5.

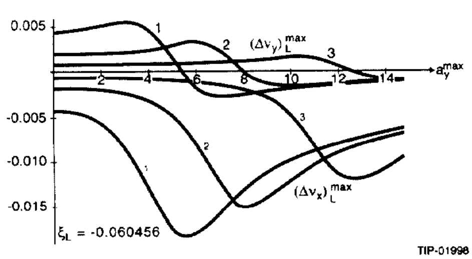

Figure 7. Maximum LRTS in one low-β IR, $a_x = 0$; and 1: α=50 μrad; 2: α=75 μrad; 3: α=120 μrad.

IV. MAXIMUM TUNE SHIFT SCALING

The maximum head-on tune shift has the following dependence: $(\Delta\nu_{x,y})_{ON}^{max} \sim -\frac{N}{\epsilon}$. The maximum long range tune shifts vary, for not very high vertical amplitudes ($a_y \leq 5d_y/6$), as $(\Delta\nu_x)_L^{max} \sim -N\beta(L)/LS_B\gamma\alpha^2$ and $(\Delta\nu_y)_L^{max} \sim +N\beta(L)/LS_B\gamma\alpha^2$. Therefore, the total maximum tune shifts in the horizontal and vertical planes are

$$(\Delta\nu_x)_T^{max} = -N\left(\frac{f_1}{\epsilon} + \frac{f_2\beta(L)}{LS_B\gamma\alpha^2}\right) \qquad (6a)$$

and

$$(\Delta\nu_y)_T^{max} = -N\left(\frac{f_1}{\epsilon} - \frac{f_2\beta(L)}{LS_B\gamma\alpha^2}\right), \qquad (6b)$$

where f_1 and f_2 are integration factors. Using the approximation $\beta(L) \simeq L^2/\beta^*$ and the nominal values for the SSC collider, and considering two low-β and two medium-β IR ($n_{low\beta} = n_{med\beta} = 2$), the following scaling relation is obtained for the maximum horizontal tune shift (similar relation for the vertical):

$$(\Delta\nu_x)_T^{max} = -0.000916 n_o^* N_o/\epsilon_o - \frac{0.0125\ N_o}{S_{Bo}\gamma_o}\left[\left(\frac{n_o L_o}{\beta_o^*\alpha_o^2}\right)_{low\beta} + \frac{1}{10}\left(\frac{n_o L_o}{\beta_o^*\alpha_o^2}\right)_{med\beta}\right], \qquad (7)$$

where the subindex (o) in $N_o, \epsilon_o, S_{Bo}, \gamma_o, L_o, \beta_o^*, \alpha_o$, and $n_o^* = n_{olow\beta} + n_{omed\beta}$, means its relative values with respect to the nominal ($N = 7.5 \times 10^9$, $\epsilon = 10^{-6}$ m, $S_B = 5$ m, $\gamma = 2 \times 10^4$, $L_{low\beta} = 75$ m ($L_{med\beta} = 150$ m), $\beta^*_{low\beta} = 0.5$ m($\beta^*_{med\beta} = 10$ m), $\alpha = 75$ μrad, and $n_o^* = 4$).

Making $\alpha_o \geq 1.6$ ($\alpha \geq 120$ μrad) and keeping all other parameters equal to one, the maximum TSS can be reduced to a tolerable value for the SSC (Figure 8). Notice from these expressions that collision at lower energies worsens the long-range tune shift.

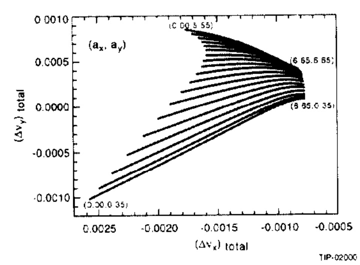

Figure 8. Total TSS with one low-β IR, α=120 μm.

V. CONCLUSIONS

Using the first approximation in the incoherent perturbative beam-beam strength, the tune shift spread was estimated for the SSC collider. The results suggests that this TSS may be large for the SSC collider with the nominal SSC parameters, and the beam-beam tune shift moves toward or outward the line $\nu_x = \nu_y$ in the tune shift space (depending of the location of the selected operational point). They establish that by increasing the crossing angle up to at least 120μrad, it is possible to bring down the TSS to tolerable values for the SSC.

VI. REFERENCES

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