# Transverse Impedance of an Iris in a Beam Pipe*

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*Abstract*

In an earlier paper the longitudinal impedance of an iris was obtained in the form of an integral equation by matching the fields in the planes perpendicular to the axis at the ends of the iris. This equation was solved by expanding the field components into a complete set of azimuthally symmetric TM modes in the iris region and numerical results were obtained, particularly for large beam pipe radii. The same method is now applied to the calculation of the transverse impedance, where both asymmetric TM and TE modes are needed in the expansions. Once again the results are obtained for large beam pipe radius and a wide range of values of the relative values of the iris radius, the iris thickness, and the wavelength.

## I. INTRODUCTION

In previous papers we examined the longitudinal coupling impedance of an iris in a beam pipe. We first treated the case of an iris of zero thickness[1], obtaining a variational form for the impedance, and numerical values by expanding the trial function into a truncated orthonormal set. The results were presented as functions of $kb$ for various values of $(a-b)/b$, where the pipe radius is $a$, the iris radius is $b$ and the frequency is $kc/2\pi$.

In a subsequent paper[2] we considered the case of finite iris thickness $g$, again constructing a variational form for the impedance. In this case however, we used a set of matrix equations for the coefficients of the fields in the iris region. In this paper we were particularly interested in the limit $a/b \to \infty$ corresponding to the impedance of a circular hole of radius $b$ in a transverse conducting wall of thickness $g$. In all of these studies only TM modes are generated by the drive beam.

In the present paper we examine the transverse (dipole) coupling impedance for an iris of finite thickness. In this case we need both TM and TE modes to satisfy all the boundary conditions. Field matching leads to a set of equations for the field coefficients in the iris region and the equations resulting from a truncation of the field expansions are solved by matrix inversion. Results are obtained as a function of the three parameters $kb, a/b$, and $g/b$.

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## II. ANALYSIS

The appropriately normalized source fields for an ultrarelativitic beam are

$$\boldsymbol{E}_t{}^{(s)} = Z_0\boldsymbol{H}_t \times \boldsymbol{e}_z = (\cos\ kz - j\sin\ kz)\boldsymbol{\nabla}_t\phi_0 \qquad (1)$$

where we assume a time dependence $\exp(j\omega t)$ and where $\boldsymbol{e}_z$ is a unit vector in the $z$ direction. Here

$$\phi_0(r,\theta) = \left(\frac{r}{a^2} - \frac{1}{r}\right)\cos\theta \qquad (2)$$

in the pipe region ($|z| \geq g/2$), and

$$\phi_0(r,\theta) \to \tilde{\phi}_0(r,\theta) = \left(\frac{r}{b^2} - \frac{1}{r}\right)\cos\theta \qquad (3)$$

in the iris region ($|z| \leq g/2$). The separation of the drive current into an even and an odd part in $z$ allows the problem to be treated as the sum of two less complicated problems.

For the even part in Eq. (1), we write the transverse fields in the pipe region as

$$\begin{aligned}\boldsymbol{E}_t &= \sum_{\mu=1}^{\infty} A_\mu e^{-j\beta_\mu(|z|-g/2)}\boldsymbol{\nabla}_t\phi_\mu \\ &+ \sum_{\nu=1}^{\infty} B_\nu e^{-j\beta_\nu(|z|-g/2)}\boldsymbol{e}_z \times \boldsymbol{\nabla}_t\psi_\nu \\ &+ \cos kz\ \boldsymbol{\nabla}_t\phi_0 \qquad (4)\end{aligned}$$

and

$$\begin{aligned}Z_0\boldsymbol{H}_t \times \boldsymbol{e}_z = &\pm \sum_{\mu=1}^{\infty} \frac{A_\mu k}{\beta_\mu} e^{-j\beta_\mu(|z|-g/2)}\boldsymbol{\nabla}_t\phi_\mu \\ &\pm \sum_{\nu=1}^{\infty} \frac{B_\nu\beta_\nu}{k} e^{-j\beta_\nu(|z|-g/2)}\boldsymbol{e}_z \times \boldsymbol{\nabla}_t\psi_\nu \\ &- j\sin kz\ \boldsymbol{\nabla}_t\phi_0, \qquad (5)\end{aligned}$$

where $\pm$ stand for $z \gtrless \pm g/2$. Here $\mu, \phi_\mu, \beta_\mu$ and $A_\mu$ are parameters associated with TM modes, with

$$\beta_\mu = (k^2 - p_\mu^2/a^2)^{1/2} = -j(p_\mu^2/a^2 - k^2)^{1/2} \qquad (6)$$

and $\nu, \psi_\nu, \beta_\nu$ and $B_\nu$ are parameters associated with TE modes, with

$$\beta_\nu = (k^2 - q_\nu^2/a^2)^{1/2} = -j(q_\nu^2/a^2 - k^2)^{1/2}. \qquad (7)$$

The functions $\phi_\mu$ and $\psi_\nu$ are products of $\sin\theta$ or $\cos\theta$ and $J_1(p_\mu r/a)$ with $J_1(p_\mu) = 0$, or $J_1(q_\nu r/a)$ with $J_1'(q_\nu) = 0$.

In the iris region we replace $a$ by $b$ and write

$$\begin{aligned} \boldsymbol{E}_t &= \sum_{\xi=1}^{\infty} F_\xi \cos\beta_\xi z \; \boldsymbol{\nabla}_t \phi_\xi \\ &+ \sum_{\eta=1}^{\infty} G_\eta \cos\beta_\eta z \; \boldsymbol{e}_z \times \boldsymbol{\nabla}_t \psi_\eta + \cos kz \; \boldsymbol{\nabla}_t \tilde{\phi}_0 \qquad (8) \end{aligned}$$

and

$$\begin{aligned} Z_0 \boldsymbol{H}_t \times \boldsymbol{e}_z = &- j\sum_{\xi=1}^{\infty} \frac{F_\xi k}{\beta_\xi} \sin\beta_\xi z \; \boldsymbol{\nabla}_t \phi_\xi \\ &- j\sum_{\eta} \frac{G_\eta \beta_\eta}{k} \sin\beta_\eta z \; \boldsymbol{e}_z \times \boldsymbol{\nabla}_t \psi_\eta \\ &- j\sin kz \; \boldsymbol{\nabla}_t \tilde{\phi}_0. \qquad (9) \end{aligned}$$

We now equate Eqs. (4) and (8) at $z = g/2$ for $\boldsymbol{E}_t$ over the range $0 \le r \le a$ and thereby obtain $A_\mu$ and $B_\nu$ as a sum over various terms involving $\phi_0, \tilde{\phi}_0, F_\xi$ and $G_\eta$. Then we equate Eqs. (5) and (9) at $z = g/2$ for $Z_0\boldsymbol{H}_t \times \boldsymbol{e}_z$ for $0 \le r \le b$ and obtain $F_\xi$ and $G_\eta$ as a sum over various terms involving $\phi_0, \tilde{\phi}_0, A_\mu$, and $B_\nu$. By eliminating $A_\mu$ and $B_\nu$ between the two resulting sets of equations, we obtain the matrix equations for $F_\xi$ and $G_\eta$

$$\sum_{\xi'} F_{\xi'} U_{\xi\xi'} + \sum_{\eta'} G_{\eta'} V_{\xi\eta'} = P_\xi \qquad (10)$$

$$\sum_{\xi'} F_{\xi'} V_{\xi'\eta} + \sum_{\eta'} G_{\eta'} U_{\eta\eta'} = Q_\eta \qquad (11)$$

where the parameters $U, V, W, P$, and $Q$ are explicit sums of integrals involving the Bessel functions. Equations (10) and (11) can then be solved for $F_\xi$ and $G_\eta$ once we truncate the sums over $\xi'$ and $\eta'$. A parallel analysis for the odd part of the source field, yields a similar set of equations.

Finally, we obtain an expression for the impedance as an integral over the three faces of the iris surface ($z = \pm g/2$ and $r = b$). This involves a term independent of $F_\xi$ and $G_\eta$ as well as ones proportional to $F_\xi$ and $G_\eta$ for both the even and odd source terms.

## III. NUMERICAL RESULTS

We have obtained the transverse impedance for several sets of parameters. In Fig. 1 we show the real and imaginary parts of the impedance as a function of $kb$ for $a/b = 2, g/b = 1$. A matrix size of $40 \times 40$ appears to be sufficient for convergence in this case. The real part of the impedance appears to start at $kb \sim .9$ corresponding to the onset of a propagating $TE_{11}$ pipe mode at $ka = 1.84$. The onset of the propagating $TM_{11}$ mode at $kb \sim 1.9$ corresponding to $ka = 3.83$ also shows clearly.

In Fig. 2 we show the results for $a/b = 10, g/b = 1$. It now appears that the contribution from the $TE_{11}$ mode for $1.84 < ka < 3.83$ is of the order $10^{-3}$ compared with that from the $TM_{11}$ mode for $ka > 3.83$. For reasons not yet well understood, it appears that the TE modes are suppressed for large $a/b$. It should be noted that, for large $a/b$, larger matrices are needed for accurate numerical computation.

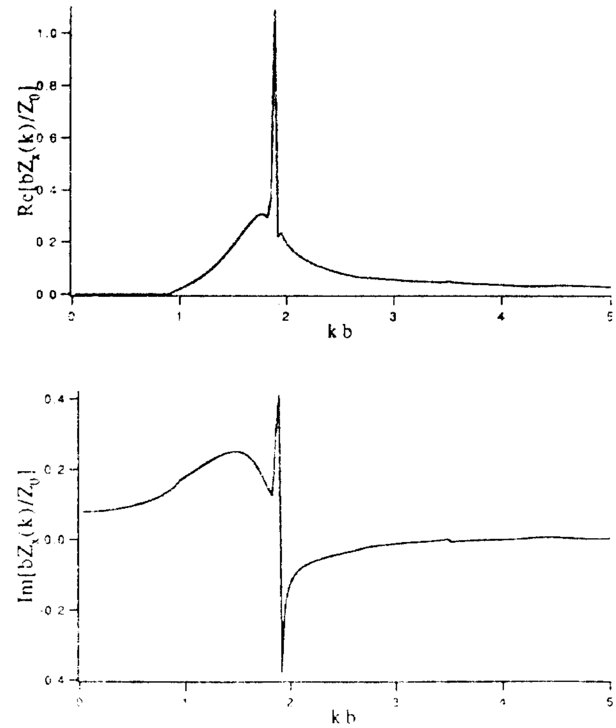


Figure 1: $bZ_x(k)/Z_0$ vs. $kb$ for $a/b = 2$, $g/b = 1$.

In Fig. 3 we show the results for $a/b = 100, g/b = 1$. The $TE_{11}$ mode is now of order $10^{-5}$ compared with the $TM_{11}$ mode. And in this case we need a matrix of $250 \times 250$ to obtain suitable numerical accuracy.

Finally, we explore the case of a zero thickness wall by doing numerical calculations for $g/b = 0$ as well as for very small $g/b$. In Fig. 4 we show the result for $a/b = 10, g/b = 0$. Once again the contributions from the TE modes are much less than those for the TM modes.

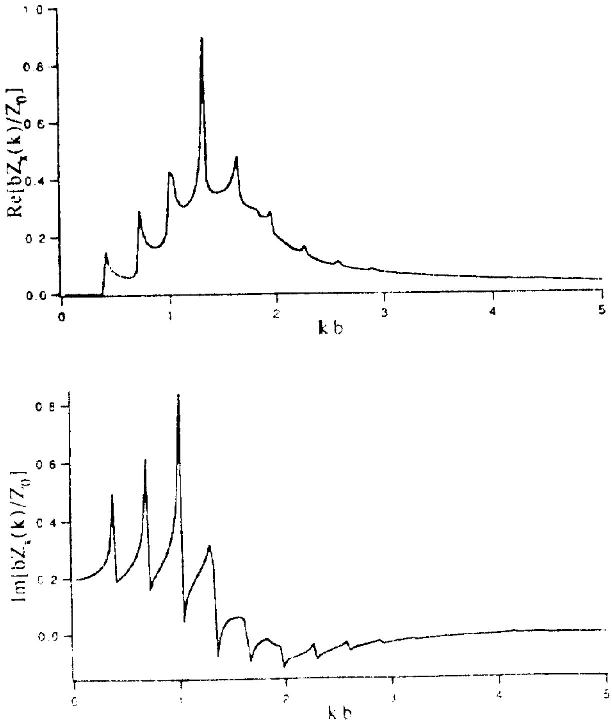


Figure 2: $bZ_x(k)/Z_0$ vs. $kb$ for $a/b = 10$, $g/b = 1$.

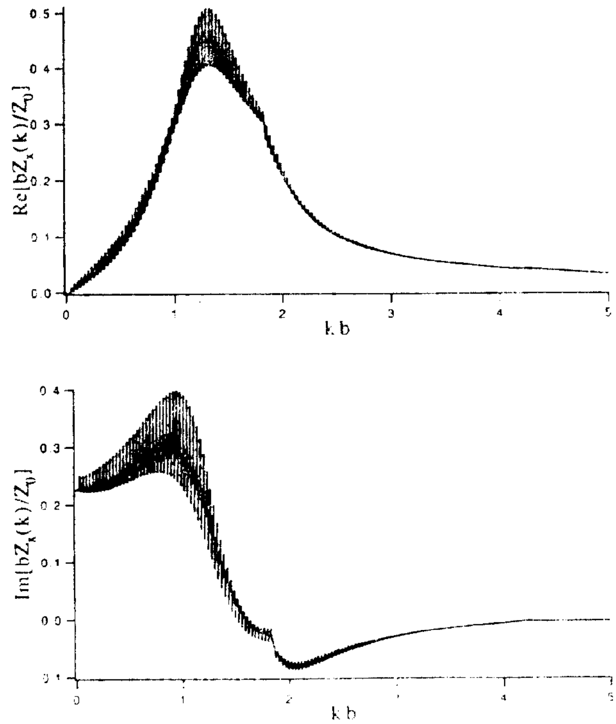


Figure 3: $bZ_x(k)/Z_0$ vs. $kb$ for $a/b = 100$, $g/b = 1$.

## IV. SUMMARY

We have briefly outlined the analysis for the transverse coupling impedance of an iris in a beam pipe and have implemented a numerical procedure to obtain values for different parameters. It appears that both TE and TM modes in the pipe region are needed to satisfy the boundary conditions. However the TE contributions fall rapidly to zero as $a/b$ becomes large.

In future work we hope to obtain a variational formulation for the impedance, particularly in the case where $g/b \to 0$. In addition we would like to obtain the limiting forms for $a/b \to \infty$ for arbitrary $g/b$.

## References


[1] R.L. Gluckstern and W.F. Detlefs, Proceedings of the Particle Accelerator Conference, San Francisco, CA, May 1991, p. 1600.

[2] R.L. Gluckstern and S. Jiang, Proceedings of the Linac Conference, Ottawa, Canada, Aug. 1992, p. 477.


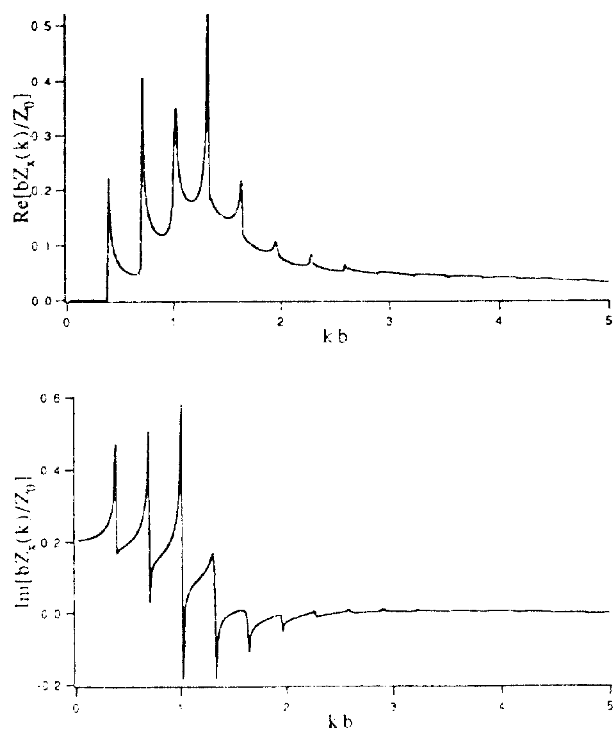


Figure 4: $bZ_x(k)/Z_0$ vs. $kb$ for $a/b = 10$, $g/b = 0$.