The Longitudinal Coupling Impedance of a Slot on the SSC Collider Liner

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Abstract

The location of a liner inside the collider beam tube is being studied at the SSC Laboratory, in order to provide a synchrotron radiation intercept and to help enhance the vacuum. There will be wake fields propagating inside the liner following the beam fields incident on the pumping holes/slots on the liner. The effect of the wake fields may be expressed through coupling impedances. This paper describes a method to evaluate the longitudinal coupling impedance of slots/holes on the liner for a large range of frequencies (0–60 GHz).

I. INTRODUCTION

The Superconducting Super Collider (SSC) beam is designed to have an energy of 20 TeV. There will be synchrotron radiation to reckon with, even though this is a proton beam. The location of a liner inside the collider beam tube is being studied at the SSC Laboratory. The liner will serve as a synchrotron radiation intercept and also help enhance the vacuum. Suitable pumping holes or slots are required on the surface of the liner. These pumping holes will result in the propagation of wake fields inside the liner, following the incident beam fields. The effect of the wake fields on successive bunches may be evaluated through the coupling impedances, which will depend on the geometry and distribution of slots/holes on the liner. Coupling impedances valid for low frequencies have been presented by Gluckstern [1] and Kurennoy [2]. A semi-analytic expression for the longitudinal coupling impedance of slots/holes valid for a large range of frequencies (0–60 GHz) is derived here and the results are compared with those from Refs. 1 and 2 and with available measurements.

II. DESCRIPTION OF THE PROBLEM

A schematic of the beam pipe and the liner is shown in Figure 1. The liner of inner radius a and thickness Δ is located inside the beampipe of inner radius b. A slot of length w and width d is located on the liner. The center of the slot is at $z = 0$. Our analysis is valid for round holes also, and we will use d to denote the diameter of the hole. The coordinate system is also shown in Figure 1. A single charge q travels along the axis of the liner with the speed of light c, and the field at the slot is given by

$$E_r^b \;=\; \frac{q}{2\pi\epsilon_0 a}\delta(z - ct). \qquad (1)$$

If we Fourier transform Eq.(1) with $f(\omega) = \int f(t)\exp(j\omega t)dt$, we obtain

$$E_r^b(\omega) \;=\; \frac{Z_0 q}{2\pi a}\;\exp(jkz). \qquad (2)$$

In Eq.(2), $Z_0 = 120\pi$ is the impedance of free space. Our task is to calculate the diffracted field and its propagation inside the liner due to the incident field given by Eq.(2).

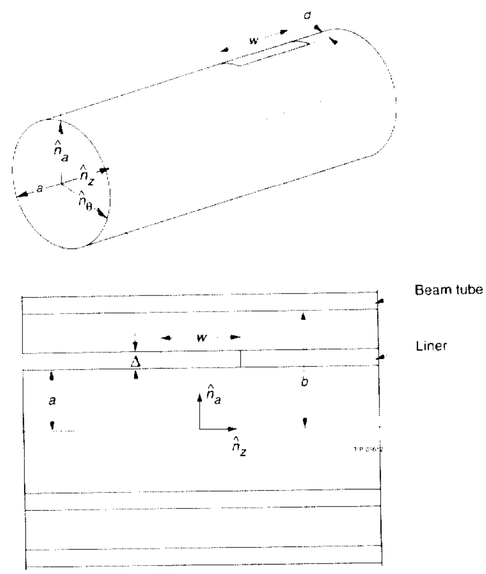

Figure 1. Slot details and coordinate system.

III. DERIVATION OF WAKE FIELDS

The charge relaxation time for our problem is of the order of 10^{-18} s, and the skin depth is of the order of a few microns. We will use the method outlined in Collin [3] for the solution of our problem using the wave guide normal modes and the Lorentz reciprocity theorem. We will follow the notation in Collin [3] and Plonsey and Collin [4]. We will assume a time variation of $e^{j\omega t}$ and a z variation of $e^{-j\beta z}$, with ω and β being the rotational frequency and the propagation factor, respectively. The fields are governed by the Helmholtz equation and can be found in [3] and [4]. The fields for the TM mode, which contributes to the longitudinal coupling impedance, can be expressed in terms of the longitudinal electric field e_z given by

$$e_z \;=\; J_n\big(k_c r\big)\Big(A\;\cos(n\theta)\;+\;B\;\sin(n\theta)\Big). \qquad (3)$$

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The J_n is the Bessel function of order n with parameter k_c. The field at the metallic wall at $r = a$ is zero. This implies $J_n(k_c a) = 0$, and there are doubly infinite number of solutions p_{nm} given by

$$J_n(k_c a) = 0, \quad p_{nm} = k_c\, a, \quad m = 1, 2, 3 \ldots. \quad (4)$$

The corresponding propagation factor is given by

$$\beta_{nm} = \pm\sqrt{k^2 - \frac{{p_{nm}}^2}{a^2}}. \quad (5)$$

We will also need the propagation factor for rectangular slots with sides $2a_1, 2b_1$. It is given by [4]

$$\beta_{nm} = \pm\sqrt{k^2 - (\frac{n\pi}{a_1})^2 - (\frac{m\pi}{b_1})^2}, \quad (6)$$

where k is the wave number given by $\frac{\omega}{k} = c$. The propagation factor for the slot β^s will be high due to the small dimensions of the slot. This implies that the waves inside the slot/hole will he heavily damped. The tangential electric field is zero at the wall everywhere except at the slot/hole, where it is non-zero. We will assume a magnetic wall at the slot, and this implies that the tangential field at the slot/hole will be a maximum and the normal field will be zero. This will result in maximum possible power flow through the slot. Since the normal field is zero, the diffracted electric field at the slot/hole will be equal to the negative of the incident beam field, and the mode inside the slot/hole will be a TE mode. In order to calculate the other components of the diffracted field at the slot, we can use the field continuity criterion given by the following equation:

$$\frac{1}{e_{z,nm}} \frac{\partial e_{z,nm}}{\partial r}\bigg|_{liner} = \frac{J_n{}'(P_{nm} r/a)}{J_n(P_{nm} r/a)}\bigg|_{r=a} = \frac{1}{e_z}\frac{\partial e_z}{\partial r}\bigg|_{slot} = -\beta^s. \quad (7)$$

We have used the upper case P_{nm} in Eq.(7) to distinguish it from the lower case p_{nm} used in Eq.(4), which describes the boundary condition at the wall everywhere but at the slot/hole. If we assume the coefficients for the normal modes to be a constant, Eq.(7) will be satisfied for the sums of normal modes. The following equation results, and we can solve it to obtain the P_{nm}:

$$P_{nm} J_n'(P_{nm}) + a\beta^s\, J_n(P_{nm}) = 0. \quad (8)$$

The magnetic wall at the slot results in the following equation, where a subscript s denotes the diffracted (radiated) fields:

$$E_{sr} = \hat{n}_a . E_s = -E_r^b = \frac{-Z_0 q}{2\pi a} e^{jkz}. \quad (9)$$

Using the ratio of normal modes with P_{nm} in the Bessel argument, we get the following expression for the tangential diffracted electric field:

$$E_{sz} = \frac{Z_0 q}{2\pi a} e^{jkz} \frac{\sum_m J_0(P_{0m})}{\sum_n \sum_m \frac{j\beta_{nm}(P_{nm})}{P_{nm}} J_n'(P_{nm})}. \quad (10)$$

It should be noted that the numerator of Eq.(10) is summed over just $n = 0$ since other modes do not contribute to the longitudinal impedance. The diffracted field given by Eq.(9) will not satisfy the equation $\nabla \times E = 0$. There will be an imbalance, which we will call the magnetic current density J_m, and it will be given by [3]:

$$J_m = -\hat{n}_a \times E_s = \hat{n}_\theta E_{sz} - \hat{n}_z E_{s\theta}. \quad (11)$$

We can consider the J_m at the slot as an oscillating source and compute the radiated fields in the +ve and -ve directions using the Lorentz reciprocity theorem [3]:

$$\int\int (E_n^{\pm} \times H_s - E_s \times H_n^{\pm}) \hat{n} ds = \int\int\int (J.E_n^{\pm} - H_n^{\pm}.J_m) dv. \quad (12)$$

The radiated fields in the +ve and -ve directions can be expressed by the normal modes multiplied by appropriate coefficients [3], and we can derive the following coefficients for the wake fields. The coefficients have been generalized with two indices n, m corresponding to the Bessel modes and the roots corresponding to every mode. Further, the field E_{sz} has been multiplied by a factor G_{nm} to decompose it into components corresponding to the nth mode and normalized by a factor f_{nm}:

$$a_{nm} = \frac{1}{2}\int\int H_n^{-}.\hat{n}_\theta\; E_{sz} \frac{G_{nm}}{f_{nm}}\; a\; d\theta\; dz\bigg|_{slot}, \quad (13)$$

$$b_{nm} = \frac{1}{2}\int\int H_n^{+}.\hat{n}_\theta\; E_{sz} \frac{G_{nm}}{f_{nm}}\; a\; d\theta\; dz\bigg|_{slot}. \quad (14)$$

The factor G_{nm} is given by

$$G_{nm} = \frac{\frac{\beta_{nm}}{p_{nm}} J_n'(p_{nm})}{\sum_n \sum_m \frac{\beta_{nm}}{p_{nm}} J_n'(p_{nm})}. \quad (15)$$

IV. THE LONGITUDINAL COUPLING IMPEDANCE

The longitudinal coupling impedance $Z(\omega)$ is defined as follows [1,2]:

$$Z(\omega) = -\frac{1}{q}\int_{-\infty}^{\infty} E_z(r = 0, \theta = 0) e^{-jkz}\; dz. \quad (16)$$

Using the expressions for the wake fields derived before, we get

$$Z(\omega) = \sum_m \frac{Z_0 d}{2\pi^2 a} \frac{e^{j\beta_{0m}(p_{0m})w} \sin(kw)}{p_{0m}^2}$$
$$\times \frac{\sum_m J_0(P_{0m})}{\sum_n \sum_m \frac{-j\beta_{nm}(P_{nm})\, a}{P_{nm}} J_n'(P_{nm})}$$
$$\times \frac{1}{\sum_n \sum_m \frac{\beta_{nm}(p_{nm})\, a}{p_{nm}} J_n'(p_{nm})}. \qquad (17)$$

In the low frequency limit, $\beta_{nm} = \frac{jp_{nm}}{a}$ and $\sin(kw) \approx kw$, and the impedance is found to be

$$Z(\omega) \approx -j[\frac{c_1 Z_0 w dk}{2\pi^2 a} + \frac{c_2 Z_0 w^2 dk}{2\pi^2 a^2} + \ldots], \qquad (18)$$

where c_1, c_2 are constants. The second term reduces to the expression in [1,2] with $w = d = 2R$ for a round hole with radius R, but for the constant of proportionality.

V. RESULTS AND DISCUSSION

Calculation of the impedance for given slot/hole dimensions involves solution of Eqs.(4) and (8) for the roots p_{nm} and P_{nm} and summing up the series in Eq.(17) for a given k. Computations were carried out for various slots and holes for a liner radius of 0.0165 m. The impedances for 2-, 3-, and 4-mm holes in the frequency range 0–5 GHz are shown in Figure 2; they are compared with the impedance Z_g from [1] and also with measured data from E.Ruiz et al. [5]. The measured values are less than Z_g and the impedances from the present computations are less than even the measured values. The differences are much less for holes of 2, 3-mm diameter. The impedances of slots of different depths d and same width w are shown in Figure 3. The slot areas have been maintained the same for comparison. The impedance is found to decrease with the depth d. The behavior of the real and imaginary parts of the impedances Z_r and Z_i for 2- and 3-mm square slots for a range of 0–60 GHz is shown in Figure 4. The impedance is inductive for lower frequencies, capacitive for larger frequencies, and reduces to zero for very large frequencies. The impedances decrease as k^{-1} for large frequencies, as shown by Eq.(17). The Z_r is negative for certain frequencies ; this is due to the truncation error in the series (with $n = 7$ and $m = 39$) amplified by the term $\sin(kw)$. The advantage of the present formulation is that it is valid for a large range of frequencies and is valid for slots or holes.

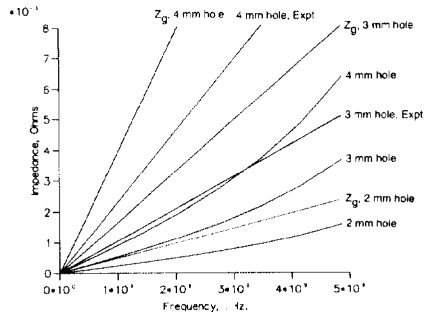

Figure 2. Impedances for 2-, 3-, and 4-mm holes.

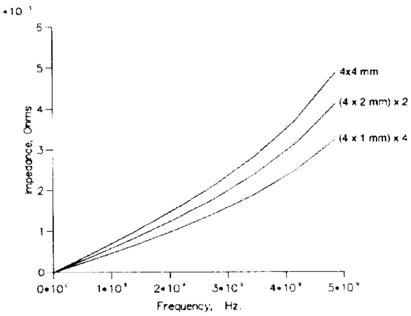

Figure 3. Impedances of slots with different depths.

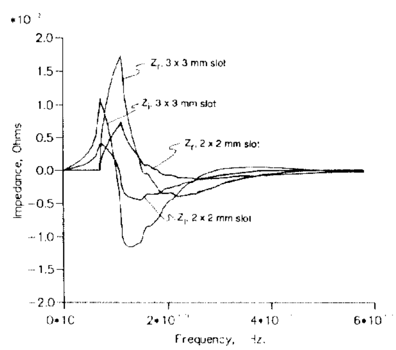

Figure 4. Behavior of the real and imaginary parts of the impedances.

VI. ACKNOWLEDGMENTS

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VII. REFERENCES

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