Suppression of Longitudinal Coupled-Bunch Instabilities by a Passive Higher Harmonic Cavity*

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Abstract

A double RF-cavity system with a passive higher-harmonic cavity is considered for the purpose of preventing coupled-bunch instabilities and/or increasing the bunch lifetime. Expressions are presented for the onset of the equilibrium phase instability, the frequency and damping rate of the Robinson instability, the synchrotron frequency, synchrotron frequency spread, and bunch length. An algorithm is presented for evaluating the performance of a passive higher-harmonic cavity, and applied to the SRRC electron storage ring, which is being installed.

1 Introduction

The performance of an electron storage ring may be limited by coupled-bunch instabilities [1] and the Touschek lifetime. A passive RF cavity with resonant frequency near a harmonic of the fundamental RF cavity may be used to increase Landau damping of synchrotron oscillations and/or increase the bunchlength [2, 3], thereby suppressing coupled-bunch instabilities and increasing the Touschek lifetime. However, unwanted side-effects such as the equilibrium phase and Robinson instabilities should be avoided. In Section 2, we present an algorithm which evaluates a higher harmonic cavity [4]. The algorithm is applied to the SRRC storage ring in Section 3. We use the notation of Sands [5].

2 Analysis algorithm

We consider a ring with a passive higher-harmonic cavity in which the fundamental RF-cavity is operated in the "compensated condition" [5] with tuning angle adjusted so that the generator current is in phase with the voltage. The following values must be input to the algorithm: V_{T1} : peak effective RF voltage in Cavity 1; Q_1^o : unloaded quality factor of Cavity 1; R_1^o: unloaded impedance of Cavity 1 at resonance; β : RF-coupling coefficient for Cavity 1; α : momentum compaction; T_o : revolution period; ω_g : generator angular frequency; E : electron energy; σ_E : electron energy spread from synchrotron radiation emission; I : average beam current magnitude; V_s: synchronous voltage; ν : harmonic number of Cavity 2; Q_2 : quality factor of Cavity 2; R_2 : resonant impedance of Cavity 2; ϕ_2 : tuning angle of Cavity 2; τ_L : longitudinal radiation damping time; $Z(\omega_{C.B.})$: parasitic impedance driving coupled-bunch oscillations; and $\omega_{C.B.}$: frequency of the parasitic mode.

Let ω_1 be the resonant frequency of Cavity 1, $Q_1 = Q_1^o/(1+\beta)$ the loaded quality factor, $R_1 = R_1^o/(1+\beta)$ the impedance at resonance, and ϕ_1 the tuning angle, defined by $\tan\phi_1 = 2Q_1(\omega_g - \omega_1)/\omega_1$. This tuning angle is the negative of that used by some authors. Robinson oscillations are dependent upon the additional angles $\phi_{1\pm}$ which obey $\tan\phi_{1\pm} = 2Q_1(\omega_g \pm \Omega - \omega_1)/\omega_1$. Cavity 2 is a passive higher harmonic cavity with resonant frequency ω_2 near $\nu\omega_g$ where ν is its harmonic number. ϕ_2 is its tuning angle, given by $\tan\phi_2 = 2Q_2(\nu\omega_g - \omega_2)/\omega_2$. As with Cavity 1, Robinson oscillations involve additional angles $\phi_{2\pm}$ which obey $\tan\phi_{2\pm} = 2Q_2(\nu\omega_g \pm \Omega - \omega_2)/\omega_2$.

Let Ω denote the Robinson instability angular frequency, α_R the instability damping rate (negative for growth), $e > 0$ the electron charge magnitude, while F_1 and F_2 are bunch form factors for the fundamental and harmonic cavities. We initially set $F_1 = 1$ and $F_2 = 0.1$, and iterate until the form factors are consistent with the bunchlength. Our algorithm proceeds as follows:

1. Calculate ψ_1, the phase angle of the bunch center, which equals zero for a bunch at the voltage peak:

$$V_s = F_1 V_{T1}\cos\psi_1 - 2IR_2F_2^2\cos^2\phi_2 \tag{1}$$

If this equation can only be solved with $|\cos\psi_1| > 1$, then there is no possible equilibrium phase of the bunch in Cavity 1, and the calculation is discontinued.

2. Calculate the tuning angle of Cavity 1 for operation in the "compensated condition" [5]:

$$\tan\phi_1 = \frac{2F_1IR_1}{V_{T1}}\sin\psi_1 \tag{2}$$

3. Calculate the coefficients of the Taylor expansion of the effective synchrotron potential, $U(\tau) = a\tau^2 + b\tau^3 + c\tau^4$:

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$$a = \frac{\alpha e\omega_g}{2ET_o}(V_{T1}\sin\psi_1 + \nu I F_2 R_2 \sin 2\phi_2) \quad (3)$$

$$b = \frac{\alpha e\omega_g^2}{6ET_o}(V_{T1}\cos\psi_1 - 2\nu^2 I F_2 R_2 \cos^2\phi_2) \quad (4)$$

$$c = -\frac{\alpha e\omega_g^3}{24ET_o}(V_{T1}\sin\psi_1 + \nu^3 I F_2 R_2 \sin 2\phi_2) \quad (5)$$

4. If a is positive and $c < 0.45[a/(\alpha\sigma_E/E)]^2$, then the synchrotron confining potential is mostly quadratic. The synchrotron frequency, bunchlength, and spread in synchrotron frequencies obey:

$$\omega_s = \sqrt{2a} \quad (6)$$

$$\sigma_t = \frac{\alpha\sigma_E}{E\omega_s} \quad (7)$$

$$\sigma_{\omega_s} = \omega_s\sigma_t^2 \mid \frac{3c}{2a} - (\frac{3b}{2a})^2 \mid \quad (8)$$

Otherwise, the confining potential is mostly biquadratic so that the bunchlength obeys:

$$\sigma_t = 0.69(\frac{U_o}{c})^{1/4} \quad (9)$$

where $U_o = \frac{\alpha^2}{2}(\frac{\sigma_E}{E})^2$. The frequency of a synchrotron oscillation of amplitude σ_t obeys:

$$\omega_{\sigma_t} = 1.17(cU_o)^{1/4} \quad (10)$$

The most unstable frequency is $1.72\omega_{\sigma_t}$.

At the dividing line between quadratic and nonquadratic potentials, the bunchlengths determined by the respective formulas are equal.

5. Use the bunchlengths to determine the form factors (for Gaussian bunches): $F_1 = \exp(-\omega_g^2\sigma_t^2/2)$ and $F_2 = \exp(-\nu^2\omega_g^2\sigma_t^2/2)$. Repeat steps 1–5 if the form factors differ greatly from the previous input values. For new input values, use a weighted average of the two most recent calculations of the form factors. After several iterations of steps 1–5, we have quantities calculated using form factors which are consistent with the bunchlength.

6. Determine if the dipole longitudinal coupled bunch instability is damped. For a mostly-quadratic synchrotron potential, the coherent frequency shift for a resonant dipole interaction with a longitudinal cavity mode of impedance $Z(\omega_{C.B.})$ at frequency $\omega_{C.B.}$ is [1]:

$$\mid \Delta\omega \mid^{C.B.} = \frac{eI\alpha\omega_{C.B.}Z(\omega_{C.B.})F_{\omega_{C.B.}}^2}{2ET_o\omega_s} \quad (11)$$

where $F_{\omega_{C.B.}}$ is the bunch form factor at the frequency $\omega_{C.B.}$. To account for radiation damping, subtract τ_L^{-1} from this frequency shift.

The resulting frequency shift may be compared with the calculated synchrotron frequency spread to determine if Landau damping is sufficient to ensure stability. A growing dipole mode will be Landau-damped [6] when the magnitude of the coherent frequency shift is less than $0.78\sigma_{\omega_s}$. For the case of a nonquadratic synchrotron potential, eq. (11) may be used with the most unstable frequency $1.72\omega_{\sigma_t}$ in place of ω_s. Landau damping is sufficient to prevent the coupled bunch instability [7] if the coherent frequency shift is less than $0.3\omega_{\sigma_t}$.

7. Determine if the equilibrium phase instability will occur. Stability is assured if:

$$F_1 I < \frac{V_{T1}\sin\psi_1}{R_1\sin 2\phi_1} \quad (12)$$

8. If the previous inequality is satisfied, calculate the Robinson frequency:

$$\Omega^2 = \frac{e\alpha\omega_g}{T_oE}\{F_1V_{T1}\sin\psi_1 - \frac{R_1F_1^2I}{2}(\sin 2\phi_{1-} + \sin 2\phi_{1+})$$
$$+ \nu R_2F_2^2 I\sin 2\phi_2 - \frac{\nu R_2F_2^2I}{2}(\sin 2\phi_{2-} + \sin 2\phi_{2+})\} \quad (13)$$

This calculation requires iteration, and one can start by evaluating the RHS with zero beam current, and then iterate using a weighted average of the most recently computed value of Ω and the previously computed value.

9. Once the Robinson frequency is known, the Robinson damping rate can be calculated; a positive value gives stability:

$$\alpha_R = \frac{4\alpha eI}{ET_o}[F_1^2R_1Q_1\tan\phi_1\cos^2\phi_{1+}\cos^2\phi_{1-}$$
$$+ F_2^2R_2Q_2\tan\phi_2\cos^2\phi_{2+}\cos^2\phi_{2-}] \quad (14)$$

We evaluate a higher harmonic cavity by performing the above calculation for a sequence of values of ring current and tuning angle. In iterated calculations, the iteration is concluded and a flag is set if convergence does not occur within a reasonable number (~ 500) of iterations.

Uncertainty results from the assumption of a resonant coupled-bunch interaction with a parasitic mode, as well as the impedance and frequency of this mode. The bunchlength calculated in the presence of coupled-bunch instability does not include any lengthening resulting from the instability.

3 Application to the SRRC storage ring

To test our algorithm, higher harmonic cavities at MAXlab and BESSY were modeled. The results were in reasonable agreement with experiments when we used a parasitic mode impedance of 0.02 MΩ, which is an order of magnitude below a typical undamped value, consistent with the presence of spurious mode attenuators.

In the 1.3 GeV storage ring being installed at SRRC, the beam pipe aperture and RF-frequency are nearly the same as MAX-lab [8]. Thus, we expect a third harmonic cavity at SRRC would have similar properties to that of MAX-lab, so we used the MAX-lab values $R_2 = 0.6$ MΩ, and $Q_2 = 10,000$. For the parasitic mode driving the coupled bunch instability, we used a frequency 1.5 times the fundamental frequency, and an impedance of 0.02 MΩ. Results are shown in Fig. 1 for the case of two identical passive cavities, which were modeled as a single cavity with $R_2 = 1.2$ MΩ. For a ring current of 60 mA or less, radiation and Landau damping are sufficient to prevent the coupled bunch instability in the absence of a higher harmonic cavity, as shown by the results for passive cavity tuning angles of ± 90 degrees. The coupled-bunch instability can be suppressed at all currents up to the desired maximum current of 200 mA. Similar results were obtained with a single passive cavity.

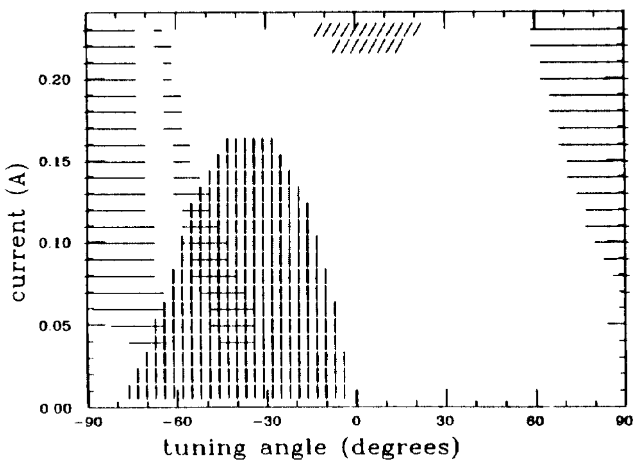

Figure 1. Instabilities are predicted for a range of ring currents and passive cavity tuning angles, for the case of two identical third-harmonic cavities at SRRC.
– : coupled-bunch instability
| : Robinson instability
/ : equilibrium phase instability
\ : there is no equilibrium phase

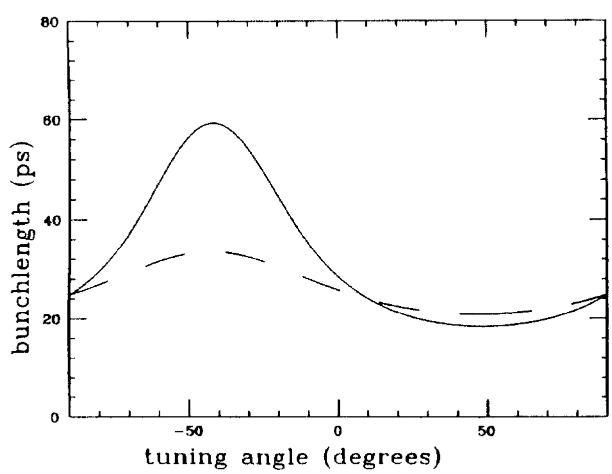

Figure 2: Bunchlength versus passive-cavity tuning angle for a 200 mA current at SRRC. The dashed line shows the results of a single third-harmonic cavity, while the solid line describes two third-harmonic cavities.

In Figure 2, we consider one or two third-harmonic cavities, and show the bunchlength versus the passive cavity tuning angle for a ring current of 200 mA. For stable operation, the bunchlength can be modified in the range of 21–35 ps with a single passive cavity, and 18–60 ps with two identical third-harmonic cavities. With a single passive cavity, a passive cavity power dissipation of 25 kW is required to stabilize the coupled bunch instability. With two passive cavities, about 10 kW per cavity must be dissipated with a tuning angle of –60 degrees. If a tuning angle of –48 degrees is used to maximize the bunchlength, 16 kW per cavity must be dissipated. We estimate that the Touschek lifetime will be proportional to the bunchlength within about 10 percent.

4 Summary

An algorithm has been developed to evaluate instability behavior with a passive higher-harmonic cavity. For the electron storage ring at SRRC, our results support the feasibility of using one or two third-harmonic cavities to suppress coupled-bunch instabilities. Two cavities may be used to substantially increase the bunchlength and the Touschek lifetime.

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