Rf Focusing Effects and Multi-bunch Beam Breakup in Superconducting Linear Colliders

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Abstract

A high gradient standing wave linear accelerator provides axisymmetric transverse focusing due to the presence of strong alternating gradient transverse electromagnetic fields arising from the backward rf wave. This effect is second order in both the field amplitude and in γ^{-1}, so it is of importance only for high gradient, relatively low energy beams. The purpose of the present analysis is to examine the effect of this focusing on multi-bunch beam breakup in a superconducting linear collider, which has both a high accelerating gradient and long bunch train. As an interesting test case, we discuss the beam breakup problem in the TESLA test bed at DESY.

I. RF FOCUSING

The effect of alternating gradient transverse rf fields is to provide net axisymmetric (monopole) focusing to a beam accelerating in a standing wave electron linear accelerator. The focusing strength associated with this effect is, in the smooth approximation,

$$K = \frac{1}{2}\left[\frac{e\overline{E}_{rf}}{\gamma mc^2}\right]^2,$$

where $\overline{E}_{rf}$ is the average accelerating gradient associated with the resonant component of the wave[1]. This focusing effect has the same form as a solenoidal magnetic field, of strength $B_z = \overline{E}_{rf} / \sqrt{2}c$. Note that for a large average accelerating gradient and a low (by the standard of the linear collider final energy) beam that this can be a very strong effect. For TESLA designs, an average accelerating gradient of 20 MeV/m is often assumed, which yields an effective smooth focusing with equivalent beta-function of $\beta_{eq}(\mathrm{cm}) \approx 3.5\gamma$. At the TESLA test bed at DESY, the beam will be injected at low energy (approximately 10 MeV), and thus the initial beta-function will be 0.7 m, which is much stronger focusing than the external quadrupole lenses provide. In addition, if the electron beam at TESLA itself is derived directly from an rf gun[2], the rf focusing will be dominant in the low energy (less than the damping ring energy) section of the linac.

This focusing term can be incorporated into a paraxial ray equation, which has the following simple form in the limit that $\gamma >> 1$,

$$X'' + \frac{3}{4}\left(\frac{\gamma'}{\gamma}\right)^2 X = 0,$$

where $X \equiv x / \sqrt{\beta\gamma}$ is the reduced transverse position (the derivation of the general form of this equation, and the notation, are due to Lawson[3], and $\gamma' = e\overline{E}_{rf} / mc^2$ is the average rate of change of the beam energy in units of rest energy.

In addition, at the entrance/exit of the cavities, the unmatched iris fields is are equivalent to focusing/defocusing lens of focal length $f = 2E_b / E_{rf}$=20 cm[4].

These effects can be incorporated into a single matrix which describes the passage of a particle through a cavity,

$$\begin{bmatrix} \cos(\alpha) - \sqrt{2}\sin(\alpha) & \sqrt{8}\dfrac{\gamma_i}{\overline{E}_{rf}}\sin(\alpha) \\ -\dfrac{3}{\sqrt{8}}\dfrac{\gamma'}{\gamma_f}\sin(\alpha) & \dfrac{\gamma_i}{\gamma_f}\left[\cos(\alpha) + \sqrt{2}\sin(\alpha)\right] \end{bmatrix}$$

where $\alpha = \frac{1}{\sqrt{8}}\ln\left(\frac{\gamma_f}{\gamma_i}\right)$ and $\gamma_{i(f)}$ is the normalized beam energy at the beginning (end) of the cavity.

Using this matrix, plus the linear drift and quadrupole matrices in the intercavity regions of the linac, one can construct a full linear transport matrix for the linac.

II. MULTI-BUNCH WAKE FIELDS.

As a bunch which is a member of multi-bunch train passes through a cavity off-axis, it adds a wake-field contribution to the lowest order transverse modes, the dipoles modes. Since the vertical emittance and beam size are much smaller than the horizontal in a linear collider design, we concentrate only on vertical motion, as it will be a more sensitive measure of a wake-field instability. The total vertical kick received at the N-th cavity by a particle in the i-th bunch due to all bunches which precede it is[5]

$$\Delta p_{y,N} = \frac{e^2 N_b}{2} \sum_m \left(\frac{R'}{Q}\right)_m \sum_{j<i} y_{j,N} S_{ij}, \quad \text{where}$$

$$S_{ij} = e^{-\frac{(i-j)\omega_m \tau}{2Q_m}} \sin[(i-j)\omega_m \tau].$$

Here the quantities $(R'/Q)_m$ are the transverse shunt impedances of the m-th dipole mode, which has angular frequency ω_m and a loaded quality factor Q_m. These parameters are taken from the work by Mosnier[5], who has studied this problem for the actual TESLA cavities as they are presently designed.

III. SIMULATION

The beam dynamics of the centroid of each bunch in the train can be simulated by using the matrices describing the rf focusing, acceleration, and drifting, adding an additional kick due to the wake-fields at the exit of each cavity. The input parameters for these studies, corresponding to a TESLA-like bunch train, are given in Table 1. The usual TESLA cavity parameters are adopted: f_{rf}=1.3 GHz, with nine π-mode cells per cavity.

Bunch spacing τ	1 μsec
Number $e^{\pm}$/bunch N_b	5×10^{10}
Number of bunches N	800
Average β-function	83 m
Phase advance/cell	30 degrees
Av. accel. field $\overline{E}_{rf}$	15 MV/m
Cavity misalignments	1 mm (rms)
Beam injection error	50 microns

Table 1: Parameters for TESLA-like multi-bunch beam breakup simulation studies.

In these studies, the average β-function is kept constant at 41 meters, which means that for energies below a half of a GeV or so, the rf focusing will in fact be stronger than the quad focusing for $\overline{E}_{rf}$=20 MV/m. Note that if a damping ring is not employed, that the electron beam must accelerate through this regime of the transverse dynamics. The bunches are all given a 50 micron initial offset. Following Mosnier[5], we assume an rms misalignment of the cavities of 1 mm.

In a TESLA-like machine, with 10,000 cavities, the excitation of multi-bunch BBU by misalignments of cavities in the low energy section of the linac were observed to damp (through the mechanism of adiabatic damping), until the amplitude of the beam oscillation was not significantly different when compared to the case when the rf focusing kicks were turned off. The results were, in general, similar to those found by Mosnier.

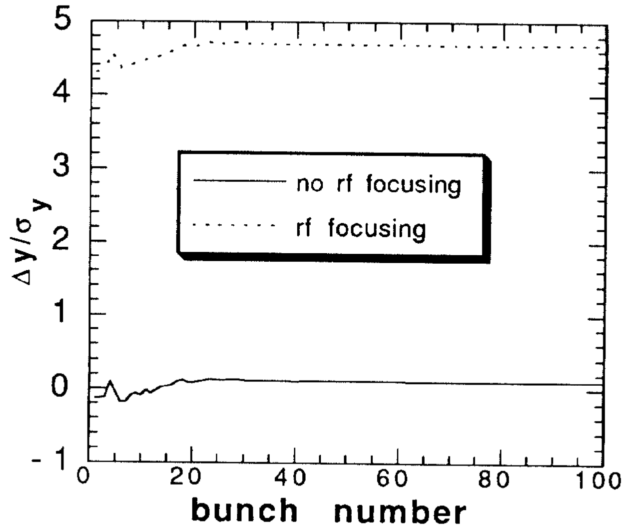

Figure 1: Vertical position as a function of bunch train number , with and without rf focusing for TESLA test bed-like case. The simulation with rf focusing included shows a large, correctable centroid offset.

In the case of the TESLA test bed at DESY, the beam is always in the rf focusing dominated regime, and the results are a bit more interesting. In this case there are only 32 cavities. The first thing that one notices is that the rf focusing kicks move the centroid of the bunch train off axis, as is shown in Figure 1. Since this sort of DC offset is correctable, for the sake of analysis we subtract the leading bunch offset from the coordinates of the remaining bunches, as is shown in Figure 2. The resulting bunch offsets display two notable aspects. First, as expected, the BBU grows faster due to the driving excitation of the rf focusing kicks from misaligned cavities.

The salient feature of the bunch train offset profile is that after an initial transient, the bunches follow each other - they "lock-on" to the position of the bunches which precede. This effect is explained theoretically in the case of a nearest neighbor (daisy chain) model developed by Adolphsen[6]. In this model, only the wake-field from the bunch immediately preceding is included in the calculation. The assumption is that the damping of the mode due to its loaded Q is several e-folds in the time interval between bunches.

While this is not quite the case here, the behavior can be seen to be qualitatively the same.

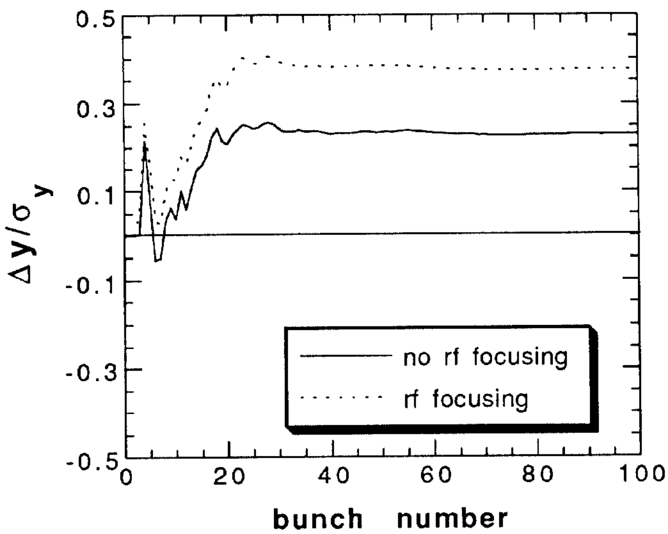

Figure 2: Vertical position as a function of bunch train number, with and without rf focusing for TESLA test bed-like case, with the initial bunch offset subtracted.

It is also clear that the final lock-on offset seen in Fig. 2 is slightly larger for the case which includes the rf focusing kicks. In the daisy chain model the oscillation amplitude is proportional to the average beta-function. Thus a likely explanation for the calculated amplitudes in Fig. 2 is that total amplitude is made smaller by the presence of the additional rf focusing, an effect which cannot, however, overcome the added growth of the instability due to the presence of rf kicks, which provide an additional inhomogeneous driving term in the equations of motion. In both cases, however, the instability amplitude is a relatively small fraction of the bunch height $\sigma_y = 200\ \mu$m. This fraction, as stated above, tends to decrease with additional acceleration in a TESLA-type machine, especially in our case, where the beta-function remains constant.

IV. CONCLUSIONS

The effect of alternating gradient rf focusing has a large effect on the focusing of electrons at low energy in a standing wave linac. This undoubtedly will have some impact on the way one implements external focusing and trajectory correction in an SRF linear collider.

On the other hand, we have found no serious deleterious effects of including rf focusing in a multi-bunch BBU in a long machine. In a low energy machine such as the DESY test-bed, however, the additional effect of the rf kicks should be observable.

The idea behind pursuing this study was that the rf focusing kicks would contribute a forcing of the motion that is correlated to the wake-fields, since both the kick received by the bunch and the wake-field excited by the bunch are proportional to the bunch offset from the cavity center. The bunches affected by the wake-field, however, do not in general execute subsequent motion which is correlated to the cavity offsets, except to the extent that a dipole mode frequency is commensurate with the bunch spacing frequency. This special case is to be avoided, and in fact the most desirable choice of dipole mode frequency places the zero crossings of the dipole modes at the following bunches.[7] Thus the multibunch BBU is not drastically exacerbated by the presence of rf kicks in cases where the bunch train would be otherwise stable, as can be seen in Figure 2.

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