SIMULATIONS OF THE LONGITUDINAL INSTABILITY IN THE SLC DAMPING RINGS*

K.L.F. Bane and K. Oide◇
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309 USA

INTRODUCTION

A longitudinal, single bunch instability has been observed in the SLC damping rings.[1] Beyond a threshold current of 3×10^{10} the energy spread of the beam increases and a "saw-tooth instability" appears. The latter term is meant to describe a rather complicated phenomenon, depending on both current and rf voltage. In one form it describes a cycle that includes a quick increase in bunch length, over a time on the order of a synchrotron period, and then a much slower return to the original length, over a time on the order of a radiation damping time. Although the total relative change in length is only about 10% the resulting unpredictability of the beam properties in the rest of the SLC accelerator makes it difficult, if not impossible, to operate the SLC above the threshold current. With the goal of trying to understand this instability the simulations that are the subject of this paper were begun.

Bunch lengthening calculations have been performed before for the SLC damping rings, to obtain the average bunch shape as function of current.[2] The wakefields of all the important vacuum chamber components were first obtained numerically.[3] The dominant elements were found to be many small discontinuities—bellows, masks, transitions, etc.—elements that are inductive to the beam. Once the total wakefield had been obtained, and the threshold current was known (from measurements), the average bunch shapes were found by means of a potential well calculation. The bunch shapes obtained in this way were found to agree very well with measurement results.[4]

In this paper we investigate the single bunch behavior of the SLC damping rings using time domain tracking and also a Vlasov Equation approach. Since the earlier bunch length calculations the damping ring vacuum chamber has been modified, by sleeving the bellows. Our results will, therefore, include the effects of this modification.

PHASE SPACE TRACKING

The Formalism

We use a now standard tracking method for simulating the effect of the wakefield on the longitudinal phase space of the beam.[5-9] The beam is represented by N_p macro-particles; each particle i has position and energy coordinates (z_i, ϵ_i), with a more negative value of z more toward the front of the bunch. The properties of particle i are advanced on each turn according to the equations:

$$\Delta\epsilon_i = -\frac{2T_0}{\tau_\epsilon}\epsilon_i + 2\sigma_{\epsilon 0}\sqrt{\frac{T_0}{\tau_\epsilon}}r_i + V'_{rf}z_i + V_{ind}(z_i) \qquad (1)$$

$$\Delta z_i = \frac{\alpha c T_0}{E_0}(\epsilon_i + \Delta\epsilon_i) \quad , \qquad (2)$$

with T_0 the revolution period, τ_ϵ the damping time, $\sigma_{\epsilon 0}$ the nominal rms energy spread, V'_{rf} the slope of the rf voltage (a negative quantity), α the momentum compaction factor, and E_0 the machine energy; r_i is a random number obtained from a normally distributed set with mean 0 and rms 1; the induced voltage on any turn is given by

$$V_{ind}(z) = -eN\int_{-\infty}^{z} W(z-z')\lambda_z(z')\,dz' \quad , \qquad (3)$$

with N the bunch population, $W(z)$ the Green function wakefield, and $\lambda_z(z)$ the longitudinal charge distribution. We approximate Robinson damping of dipole oscillations by adding $-2T_0\langle\epsilon\rangle/\tau_d$ on the right of Eq. (1), with $\langle\epsilon\rangle$ the average energy and τ_d the Robinson damping time.[6]

Practical Considerations

Simulations use only a small fraction of the real number of particles in the beam, and numerical noise can suppress real phenomena or generate its own phenomena. This is particularly true with an inductive impedance, such as the SLC damping rings', since then the induced voltage depends strongly on the slope of the charge distribution. To calculate λ_z on each turn we simply bin the macro-particles in z, without smoothing, and count on the use of a very large number of macro-particles to give us a sufficiently smooth distribution.

The wakefield for the ring with bellows sleeves was calculated as before, using the computer program TBCI,[10] with a short, gaussian driving bunch with $\sigma_z = 1$ mm. To make it causal, the part in front of bunch center (at $z = 0$) was reflected to the back (see Fig. 1). We expect to be able to find beam instabilities down to wavelengths of about 1 cm.

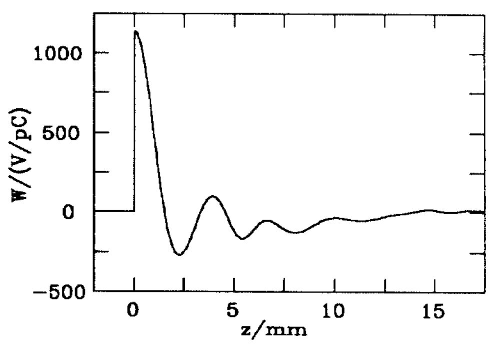

Fig. 1. The Green function wake used to represent the current SLC damping rings.

For the simulations we take $T_0 = 118$ ns, $E_0 = 1.15$ GeV, rf frequency $\nu_{rf} = 714$ MHz, $\sigma_{\epsilon 0} = 0.07\%$. We choose a peak rf voltage $V_{rf} = 0.8$ MV, where the nominal bunch length $\sigma_{z0} = 4.95$ mm, and synchrotron frequency $\nu_{s0} = 99$ kHz. For practical reasons τ_ϵ was reduced by a factor of 10 to 0.17 ms. Therefore there are 1445 turns per damping time, compared to 85 turns per synchrotron period. We take $N_p = 300,000$, and for calculating λ_z we take 100 bins to extend over $10\sigma_z$ of the bunch. We start the program with the potential well bunch distribution and let it run for 3 damping times.

*Work supported by Department of Energy contract DE-AC03-76SF00515.
◇From KEK.

SIMULATION RESULTS

Average Bunch Properties

On each turn we calculate the lower moments of the distributions. By averaging over the last damping time we obtain the "average" properties of the distributions. Fig. 2a displays the average value of the first (the crosses) and the second (the diamonds) moments in z as functions of current. The ring being inductive, the bunch shapes are more bulbous than gaussians, and the bunch length increases with current.

In Ref. 2 the average bunch shapes are obtained by a modified potential well solution: Haïssinski's formula [11] is used to find the bunch shape; above threshold the energy spread, and therefore the natural bunch length used in the formula, are taken to increase as $N^{1/3}$ (since the ring is very inductive). This method applied to the current damping ring, taking the threshold to be 2×10^{10}, are shown by the lines in Fig. 2. This approximate method agrees very well with the tracking results. We should also point out the bunch length for the ring is very similar to that of the old ring, only 10% shorter at 3×10^{10}.

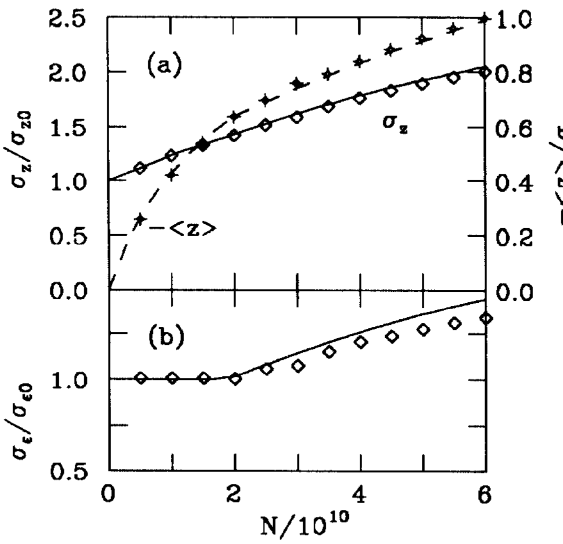

Fig. 2. Average bunch properties *vs* N.

The Threshold Current

In Fig. 2b we plot the average rms energy spread as function of current. Fitting the results to a power law increase above a threshold we find the power law to be 0.28 and the threshold $N_{th} = 2.0 \times 10^{10}$. A confirmation that this is the threshold current is the fact that the unstable mode (discussed below) first appears at this current, with an uncertainty of -0.25×10^{10}.

P.B. Wilson once hypothesized that one criterion for the onset of the instability is that the slope of the total voltage $(V'_{rf} + V'_{ind})$ goes to zero within the bunch.[12] In our case this criterion holds at 1.9×10^{10}; at higher currents, as the bunch lengthens, it continues to hold. A related hypothesis by P.B. Wilson is that the Haïssinski Equation, a transcendental equation of the form $\lambda_z = f(\lambda_z)$, will, when iterated above threshold, asymptotically give two, alternating solutions.[13] In our case this begins at 2×10^{10}.

Repeating the tracking calculation for the old ring (no bellow sleeves) we find a threshold of 1.1×10^{10}; repeating it for a wakefield that represents only the rf cavities (the best impedance we can imagine) we obtain a capacitive wakefield and threshold of 14×10^{10}.

Modes of Instability

Taking a Fourier transform (FT) of one of the moments we find that, beginning at $N = 2 \times 10^{10}$, resonances appear with frequencies above $2.4\nu_s$. Taking the $FT(s_z)$ at 3.5×10^{10}, with s_z the skew in z (see Fig. 3a), we find a very clean signal with only one, very narrow peak (see Fig. 4a). The full width, 1.5%, is given by the limited length of the run and not by any more fundamental limit.

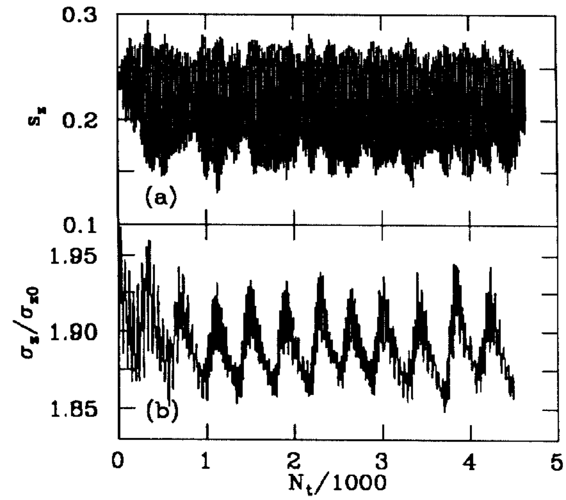

Fig. 3. The turn-by-turn skew when $N = 3.5 \times 10^{10}$ (a), and the rms when $N = 5.0 \times 10^{10}$ (b).

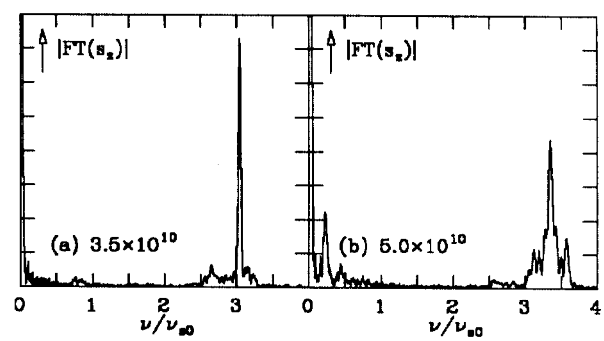

Fig. 4. The absolute value of the Fourier transform of the skew signal for two currents.

At some currents, like 5.0×10^{10}, we find a fairly regular overshoot pattern in the moments as function of time (see Fig. 3b). In this example the bunch length varies by 5% over a cycle: the lengthening time is about $1.5/\nu_{s0}$, the shortening time is maybe twice as long. In the FT we see an extra peak at 22 kHz and sidebands around the instability. At $N = 3.0 \times 10^{10}$ the pattern of the bunch length is more irregular.

Fig. 5 gives two snapshots of the unstable mode when $N = 3.5 \times 10^{10}$. We see that the maximum amplitude of the mode is about 10% and the wavelength about 1.2 cm. We obtain a 3-dimensional mode plot by averaging the distributions at a fixed phase in the oscillation and subtracting from this the average over all phases (see Fig. 6). We see that the mode is a mixture of dipole, quadrupole, and sextupole components. By 5×10^{10} an octupole component can also be seen.

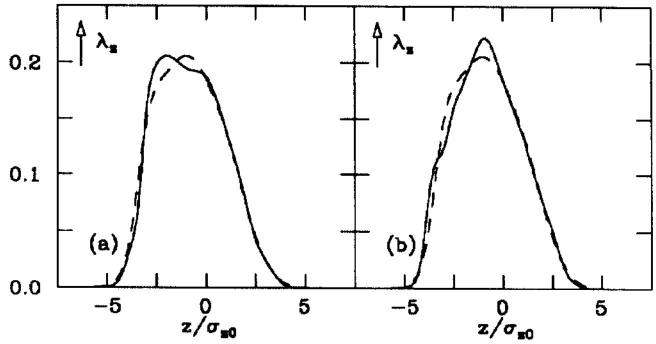

Fig. 5. A snapshot of the beam, at two phases 180° apart, when $N = 3.5 \times 10^{10}$.

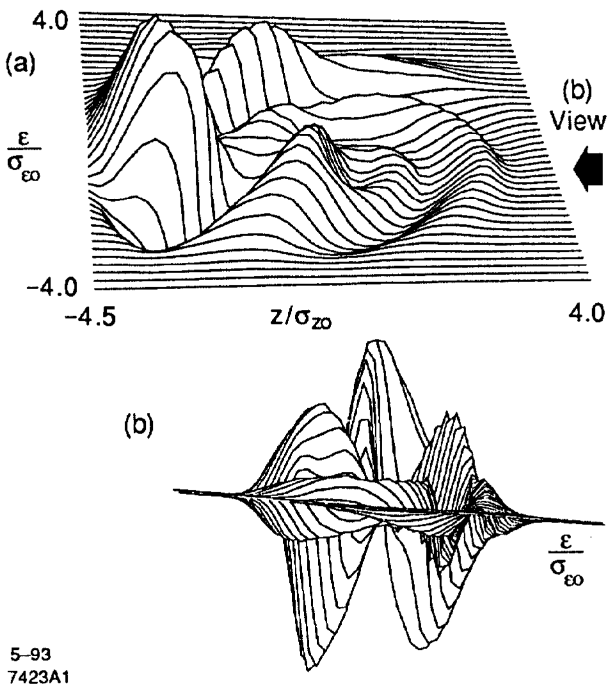

Fig. 6. The shape of the unstable mode from two views at $N = 3.5 \times 10^{10}$.

The positions of the major peaks in the spectrum of the skew signal for different currents is shown in Fig. 7. The diamonds show the cases with one narrow spike in the spectrum of s_z, the crosses those with more complicated spectra. We see that the frequency of the unstable mode increases with N; the dashed line has a slope of $0.27/10^{10}$.

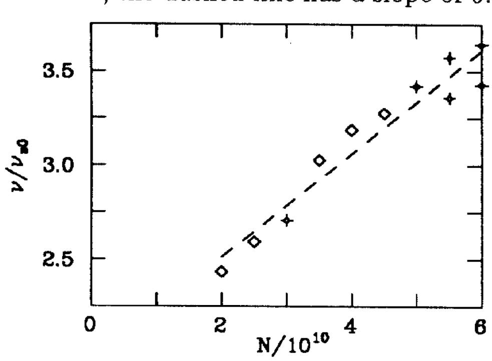

Fig. 7. The positions of the major peaks in the Fourier transform of the skew signal *vs* N.

A Vlasov Equation Calculation

K. Oide and K. Yokoya have written a computer program to solve the time independent, linearized Vlasov Equation including the effects of potential well distortion.[14] Using the wakefield of Fig. 1 we take 6 azimuthal space harmonics and 60 mesh points in amplitude to represent phase space. We find that, due to the potential well distortion, already by 1×10^{10} the large gaps in mode frequencies have disappeared. The first unstable mode is found at 1.9×10^{10} with a frequency of $2.5\nu_{s0}$ (see Fig. 8).

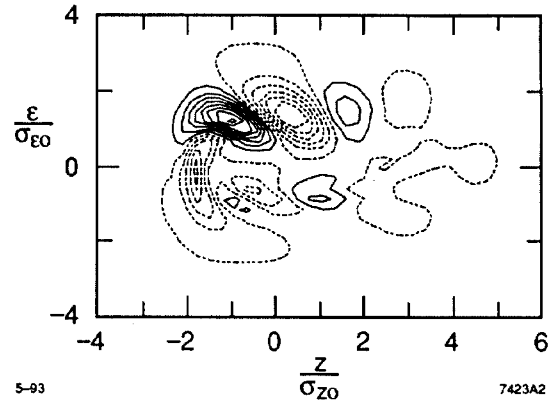

Fig. 8. A contour plot of the unstable mode, obtained by solving the Vlasov Equation.

Comparison with Measurements[1,4]

The agreement with measurements of the average bunch shapes is very good. The calculated threshold currents are about 30% lower than the measurements, which are 3.0×10^{10} in the current ring, 1.5×10^{10} in the old ring. A mode (sometimes call the "sextupole" mode) has been measured above threshold. At 3×10^{10} it has a frequency $2.6\nu_{s0}$; at higher currents the frequency increases at about $0.08/10^{10}$, a much lower slope than calculated here.

Acknowledgements

The authors thank M. Sands and the members of the SLC Damping Ring Task Force for helpful discussions.

REFERENCES

[1] P. Krejcik, *et al.*, "High Intensity Bunch Length Instabilities in the SLC Damping Rings," this conference, Q7.
[2] K. Bane and R. Ruth, Proc. of the 1989 IEEE Particle Acc. Conf., Chicago, 1989, p. 789.
[3] K. Bane, Proc. of the 1st European Particle Acc. Conf., Rome, 1988, p. 637.
[4] L. Rivken, *et al.*, Proc. of the 1st European Particle Acc. Conf., Rome, 1988, p. 634.
[5] A. Renieri, Frascati, Report No. LNF-75/11R, Feb. 1976.
[6] P.B. Wilson, *et al.*, *IEEE Trans. Nucl. Sci.*, **NS-28**, 2525 (1981).
[7] T. Weiland, DESY 81/088, Dec. 1981.
[8] R. Siemann, *Nucl. Instr. Meth.*, **203**, 57 (1982).
[9] D. Brandt, CERN-ISR-TH-89-09, May 1982.
[10] T. Weiland, *Nucl. Inst. Meth.* **212**, 13 (1983).
[11] J. Haïssinski, *Il Nuovo Cimento*, **18B**, No. 1, 72 (1973).
[12] P.B. Wilson, private communication.
[13] P.B. Wilson, PEP-232, Feb. 1977.
[14] K. Oide and K. Yokoya, KEK-Preprint-90-10, 1990.