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Longitudinal Impedance and Stability Thresholds of the AGS Booster *

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Abstract

The various sources of longitudinal impedance in the AGS Booster are identified and the effects of the total impedance are estimated. In particular, the parasitic effects of the rf cavities during proton acceleration are explored. Dynamics are treated using a first order Vlasov analysis in the weak synchrotron coupling approximation.

I. INTRODUCTION

The AGS Booster is a fast-cycling proton and heavy ion synchrotron. The Booster will accelerate protons from $E_k = 200$ MeV kinetic energy to $E_k = 1500$ MeV in 60 ms. For heavy ions the acceleration time is about an order of magnitude larger with an output kinetic energy $E_k \sim 500$ MeV per nucleon.

This note is concerned with the collective effects that can appear during proton acceleration. Instability thresholds and growth rates are calculated for fixed bunch length, proton energy, etc. These parameters are varied over appropriate ranges, resulting in an overall picture of beam stability. For simplicity, the root mean square radius of the beam is assumed to be 1.5 cm throughout the cycle. In any case, the longitudinal dynamics depend only logarithmically on the beam radius, via the space charge impedance. The parameters which are assumed constant are given in Table 1

Table 1: Unperturbed Beam Parameters

Machine Radius	32.114 m
Harmonic number	3
Number of bunches	3
Momentum compaction	0.0453
Beam radius	1.5 cm
Pipe radius	6 cm
Protons per bunch	5.0×10^{12}
Synchronous RF phase	13°
RF voltage	80 kV

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Table 2: Resonant Impedance Budget

Source	$R(k\Omega)$	f_r(MHz)	Q
Band III rf	40	2.5-4.11	55
Band II rf	5	0.6-2.5	2.5
Kickers	9	60	3
Band III parasitic	5	50	10
PUEs	1	230	5

II. LONGITUDINAL IMPEDANCE

Since the bunches in the Booster are much longer than the radius of the vacuum chamber and the Lorentz factors are small, the impedance above the cutoff frequency of the pipe has a small effect on the general character of the beam dynamics.

For small Lorentz factors, the space charge impedance is important. At injection, the space charge impedance is capacitive with $Z/n = i630\Omega$. The space charge impedance drops as acceleration proceeds and ends up at $Z/n = i80\Omega$ at extraction. The other broad band impedance present is the resistive wall impedance. For these calculations, the resistive wall impedance is negligible. Other than the resistive wall impedance and the space charge impedance, the bulk of the low frequency impedance comes from the rf cavities and, to a lesser extent, modes associated with the kicker magnets and the pick-up electrodes. The resonant impedance budget is summarized in Table 2 [5, 6]. The shunt impedances and quality factors quoted for the band II and band III rf modes are average values, as the cavity is tuned they can change by $\sim 20\%$. The detailed behavior has not been accurately included in the stability calculations and to a large extent has not been measured.

III. STABILITY FORMALISM

Growth rates were calculated using first order pertubation theory on the Vlasov equation in the limit of negligible synchrotron mode coupling with no synchrotron frequency spread [2]. The frequency shifts obtained bear out the assumption of weak coupling. An unperturbed phase space density $\psi_0(r) = K(a^2 - r^2)^{3/2}$ is assumed, where a is the maximum synchrotron amplitude which is also half the bunch length, and K is a normalization factor. It has been noted that actual proton distributions are quite similar to this model [4]. With these assumptions, and taking quantities to vary as $\exp(-i\Omega t)$, the Sacherer integral equation is

$$
\begin{aligned}
(\Omega - m\omega_s)R(r) &= w(r)\int_0^a G_m(r,r')R(r')r'dr' &(1)\\
w(r) &= \frac{15}{2\pi a^4}\left(1-\frac{r^2}{a^2}\right)^{1/2} &(2)\\
G_m(r,r') &= im\kappa\sum_k \frac{Z[(kM+s)\omega_0+\Omega]}{Mk+s} \\
&\times\ J_m[r(Mk+s)]J_m[r'(Mk+s)] &(3)\\
\kappa &= \frac{\omega_0\eta\bar{I}}{\beta^2 E_T\nu_s} &(4)
\end{aligned}
$$

where $\bar{I}$ is the average current, M is the number of bunches, s is the coupled bunch mode number, r is the amplitude of the oscillation in radians of azimuth, and the rest of the notation is the same as in [7]. For $s = 0$, the $k = 0$ term in the sum is neglected to conserve total charge. The calculation proceeds by expanding $R(r)/w(r)$, and $G(r,r')$ in a basis $\{f_\ell(r) : \ell = 1, 2, \ldots\}$ which is orthonormal on $(0, a)$ with weight function $w(r)$. The function $w(r)f_\ell(r)$ is refered to as the ℓth radial mode. This reduces the integral equation to an (infinite) matrix eigenvalue problem. The formulae are rather cumbersome and are given in Satoh's paper [3]; a computer code was written to evaluate the relevant expressions.

In practice, it is impossible to solve the infinite dimensional matrix equations. For the case at hand, it was found that using the first three radial modes sufficed for determining the largest frequency shift.

Synchrotron frequency spread due to the non-linear portion of the rf force was calculated using [8]

$$
\omega_s(r) = \omega_{s,0}\left(1 - \frac{h^2r^2}{16}\,\frac{1+\frac{2}{3}\sin^2\phi_s}{1-\sin^2\phi_s}\right), \qquad (5)
$$

where $\omega_{s,0}$ is the zero amplitude synchrotron frequency, ϕ_s is the synchronous rf phase, and h is the harmonic number. A plot of equation (5) and the synchrotron frequency obtained using a first order symplectic integration are shown in Figure 1.

Stability diagrams were obtained by setting $\omega_s = \omega_s(r)$, and $G_m(r,r') = C_m(rr')^m$ as in Zotter's analysis [8]. Stability diagrams for the dipole and quadrupole coupled bunch modes are shown in Figure 2

Beam stability is estimated using the frequency shift calculated in the absence of frequency spread $\Delta\Omega$, and the synchrotron frequency spread for the mth synchrotron mode $S = m(\omega_s(0) - \omega_s(a))$. The beam is stable if $\Delta\Omega/S$ is to the left of the stability boundary. Of the three frequency shifts obtained for each coupled bunch mode, the most pessimistic was used in the stability analysis.

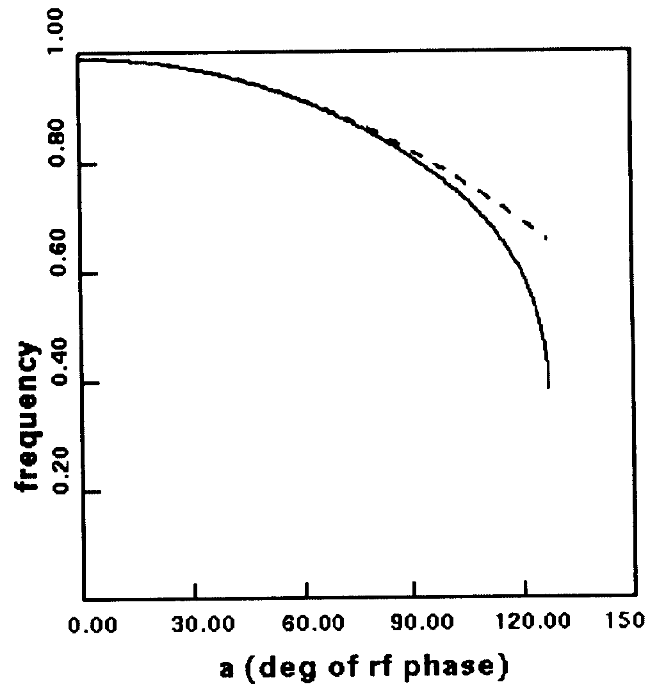

Figure 1: Relative synchrotron frequency for equation (5) (dashed line) and a symplectic integration (solid line) with a synchronous rf phase of 13°

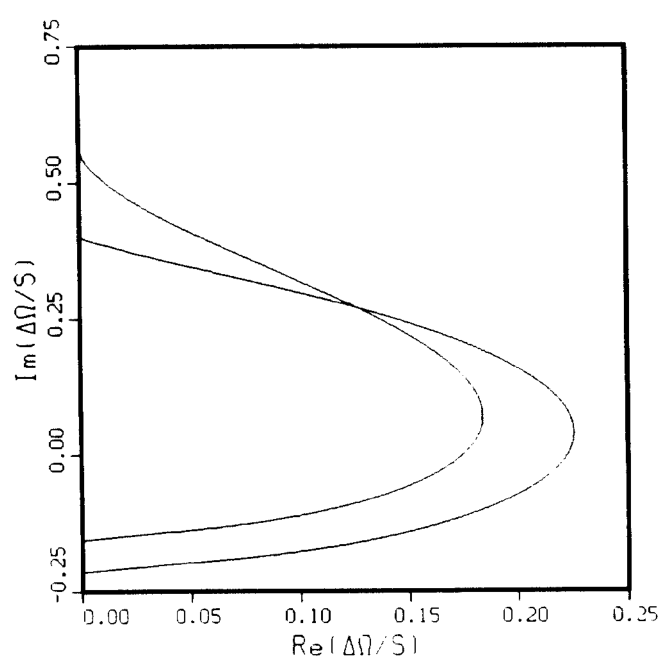

Figure 2: Dipole ($m = 1$) and Quadrupole ($m = 2$) stablity boundaries for $\psi_0(r) \propto (a^2 - r^2)^{3/2}$

IV. STABILITY CALCULATIONS

The purpose of the calculations was to determine the beam stabilty during proton operations. Of the impedances in Table 2 the band II and band III radio frequency modes were by far the most important. For clarity, the other resonant impedances were ignored.

The band III impedance is necessary for acceleration, so calculations with it alone were performed. Depending on bunch length, the $s = 2$ dipole ($m = 1$) mode may be unstable, while the $s = 0, 1$ modes are damped. For $E_k = 200$ MeV the minimum bunch length was $\sim 200°$ of rf phase while at $E_k = 1500$ MeV it was 170°. In both cases the magnitude of the frequency shift was dominated by the real part, which is difficult to control. Ignoring instabilities, the bunch length should decrease by a factor of 2 due to adiabatic damping during acceleration. Taking the extraction length of 170° this implies an injection length of 340° which is significantly larger than the 255° length of the accelerating bucket. However, the growth rate of the instability is very small. Taking a bunch length of 180° of RF phase at injection and assuming adiabatic damping of the synchrotron oscillations throughout the cycle results in a peak growth rate for the dipole mode of 10 s^{-1}. The $s = 2$ quadrupole mode is unstable also with a peak growth rate of 4 s^{-1}. Given the 60 ms cycle time neither of these instabilities should be problematic.

When the band II cavity is included in the calculations, the situation becomes more complicated. While the resonant frequency of the band III cavity is tightly constrained by the RF frequency, the first Robinson criterion and the cavity quality factor, the resonant frequency of the band II cavity is a free parameter. Given the results for the band III cavity alone, the analysis was confined to finding the band II resonant frequency which resulted in the smallest growth rates. The calculations assumed a bunch length of 180° of RF phase at injection and adiabatic damping of the synchrotron oscillations. It was found that by tuning the band II resonant frequency, the dipole instability growth rates could be kept below 18 s^{-1} throughout the cycle. The quadrupole growth rates were less than 10 s^{-1} for the same band II impedance. The required band II resonant frequency varies smoothly from 1.14 MHz at injection to 1.83 MHz at extraction. However, the dipole growth rate depends strongly on the resonant frequency of the band II cavity. At some points in the cycle, mistuning the cavity by 50 kHz results in a growth rate of 50 s^{-1}. Since the impedance is not known accurately the optimal band II tune would need to be found empirically.

V. POSSIBLE CURES

A simple cure to the instability problem is to short out the band II cavity during proton operations. The technology is well known and should not present a problem. Alternately, the band II resonant frequency could be tuned throughout the cycle. An interative procedure based on beam measurements would be needed, owing to the high precision required.

It has been suggested [1] that the harmonic number of the AGS Booster be switched from 3 to 2. Such a change would have a profound effect on longitudinal instabilities. Consider equation (3) with $M = 2$ and $s = 1$.

$$G_m(r, r') = im\kappa \sum_{k>0,\text{odd}} \frac{Z[k\omega_0 + \Omega] - Z^*[k\omega_0 - \Omega]}{k} \times J_m[kr]J_m[kr'] \qquad (6)$$

where the $*$ denotes complex conjugate and the sum is over k positive and odd. For the $s = 0$ mode the sum is over k positive and even. For $\Omega \approx m\omega_s$ the imaginary part of equation (6) is essentially determined by the derivative of the real part of the impedance at the revolution lines. As with the first Robinson criterion, it then seems possible that all the coherent modes will have a negative imaginary part. Uncertainties are large, but preliminary calculations show that it should be fairly easy to accomplish this as long as the non-accelerating cavity can be tuned by 1.4 MHz; the revolution frequency at extraction.

VI. CONCLUSIONS

In section IV it was found that the AGS Booster will not be able to operate at 1.5×10^{13} protons per pulse and maintain longitudinal stability with the current machine parameters. The growth rates for band III alone are negligible given the cycle time of the machine. If the beam sees the band II impedance, then careful tuning of the band II resonant frequency will be needed. By changing the harmonic number from 3 to 2, it was found that a suitable tuning current for the parasitic cavity should result in exponential damping of *all* coherent modes.

References

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