Multi-Bunch Beam-Break-Up Studies for a SWFEL/TBA*

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Abstract

A set of parameters minimizing BBU is obtained for a high current, low energy "drive beam" in a standing wave free electron laser two-beam acceleration(FEL/TBA). A large reduction in the transverse wake function is obtained by making the cavity-iris junction gradual by means of a cone. BBU is examined under various BBU reduction schemes.

I. INTRODUCTION

Multi-bunch beams have been shown to exhibit unstable transverse beam dynamics due to cumulative transverse wake fields. The purpose of this work is to design a cavity of low transverse wakes and to produce a set of optimized parameters minimizing BBU for a high current, low energy "drive beam" of the SWFEL/TBA[1,2]. The physical set-up is shown in Figure 1.
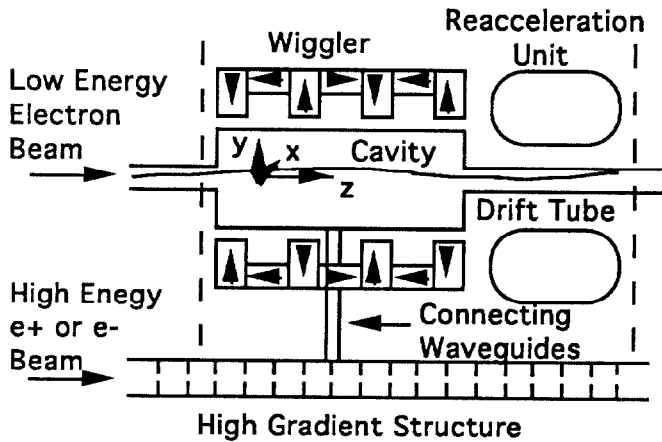

Figure 1. Schematic diagram of SWFEL/TBA

Standing waves can be obtained by making the irises small enough that the FEL mode is below the cutoff frequency of the iris modes. This requires a large change in radius at the junction between the cavity and iris, resulting in large wakes. The wakes can be dramatically reduced by introducing a cone junction (making the junction gradual).

For long cavities the transverse momentum kicks are determined by the junction conditions only, resulting in the same amount of momentum kick for a cavity regardless of the length of the cavity.

BBU calculations were performed, with various BBU reduction schemes, using the parameters in Table 1.

Table 1. Parameters of a SWFEL for *17.1 GHz* with cylindrical cavities

cavity	dimension (radius x length) = *1.5 cm ×88 cm* iris (radius x length) = *0.4 cm × 3 cm* junction = *2 cm* length; cone ~30°
beam	energy $\gamma = 16.6$ average current *I = 1 kA* pulse length = *100 ns*
focusing	focusing length λ_β=52 *cm;* (k_β =0.12 cm⁻¹)

II. BASIC BEAM-BREAK-UP EQUATIONS

A. The Model

Let the planar wiggler magnetic fields lie in the y-z plane as indicated in Figure 1, with the particle wiggle in the x-z plane. These wiggle oscillations in a cylindrical cavity excite the TE_{11p} FEL mode. The beam dynamics in the wiggle plane and that perpendicular to it can be treated separately as a first order approximation. In this paper we consider BBU in the y-direction, as that is the most severe case.

Assuming one macro particle per bunch and equal spacing between bunches the transverse displacements, ξ_j, of the j-th bunch with energy γ_j can be represented in a standard BBU form.

$$\frac{\partial}{\partial z}\left[\gamma_j(z)\frac{\partial}{\partial z}\xi_j\right] + \gamma_j(z)k_{\beta,j}^2(z)\xi_j = \frac{Is_b}{I_0}\sum_{k=0}^{j}W((j-k)s_b)\xi_k,$$

where z is the axial coordinate in cm, s_b is the distance between bunches in cm, W is the transverse wake function in cm^{-3}, I is the average current in z in kA, and $I_0 = mc^3/e = 17.05 kA$.

B. Analytic Estimates

For a purely sinusoidal wake function, $W = W_0 sin(\omega_r s/c)$ the oscillation amplitude of displacements of the j-th bunch can be estimated using the discrete Laplace Transformation as in Reference [3]. If only the first bunch is displaced,

$$\frac{\xi_j(z)}{\xi_0} \approx \left(\frac{1}{5j}\right) A^{1/2} e^A \quad \text{with} \quad A = \left[\frac{I(js_b)}{I_0} W_0 \frac{z}{\gamma k_\beta}\right]^{1/2} .$$

Reduction in BBU can be obtained for smaller wakes, higher $\gamma \, k_\beta$, smaller charge, and shorter device lengths .

The wakes, in reality, are far from a single sinusoidal wave. Any finite quality factor Q for the cavity, or any spread in frequency of the wakes reduces the amplitude for larger s. In this case, we can reduce BBU for the same output energy FEL by making the pulse length longer, since the output energy of FELs depend on the charge of the pulse. If we take into account the quality factor the wake potential is reduced by

$$e^{-(\omega_r / Q)(s / c)}$$

, where ω_r is the wake modes, and the beam displacements are reduced by the same factor. In the presence of frequency spread $\Delta\omega$ of the wake potential, the displacements are also reduced, although depending on the model, roughly by the factor $e^{-(\Delta\omega)(s / c)}$.

III. TRANSVERSE WAKES

For a Gaussian bunch of 1 Coulomb electrons with width σ, the transverse wake of the dipole($m=1$) mode in an ideal pill-box cylindrical cavity can be evaluated as following.

$$W(r = a, s) = -\left(\frac{1}{\pi\varepsilon_0 L}\right) \alpha^2 e^{-s^2/2\sigma^2} \sum_{n,P} \frac{J_0^2(j_n\alpha) - J_2^2(j_n\alpha)}{J_2^2(j_n)}$$

$$\times \left[1 - (-1)^P \cos(v_{np}L)\right] \frac{1}{2v_{np}a} \text{Im}\left\{w\left(\frac{v_{np}\sigma}{\sqrt{2}} - \frac{is}{\sigma\sqrt{2}}\right)\right\},$$

where $v_{np} = \omega_{np}/c$, ω_{np} being the cavity mode frequencies, $\alpha = a/R$ and j_n are zeros of the Bessel function J_1 and w is the complex error function. For the cavity of Figure 2(a) with $\sigma = 3$ mm, the wake shown in Figure 2(a') can be reproduced with 50 modes.

Wake functions are obtained for a cylinder of radius 1.5 cm and length 87.5 cm using ABCI [4] for the dipole mode $(m=1)$. Transverse wakes for (a) a step junction cavity and (b) a cone junction cavity are shown in Figure 2. By making the junctions gradual, a dramatic reduction in the wakes is achieved for the gradual junction effectively introduces spread in frequencies of wake modes, resulting in similar wake functions as in a de-tuned structure [5]. Smoothing the corners of the step junctions does not adequately reduce the wake. Due to computational limits, we have obtained wakes only up to 3 m. Since with reasonable Q values, the wakes at large s should damp away, we have assumed the wakes beyond 3 m to be negligible.

For long cavities the transverse momentum kicks depend only on the junction conditions and are independent of the cavity lengths. The reason for this is that for long cavities the wakes are superposition of the individual wakes generated due to the two junctions. This has been verified numerically
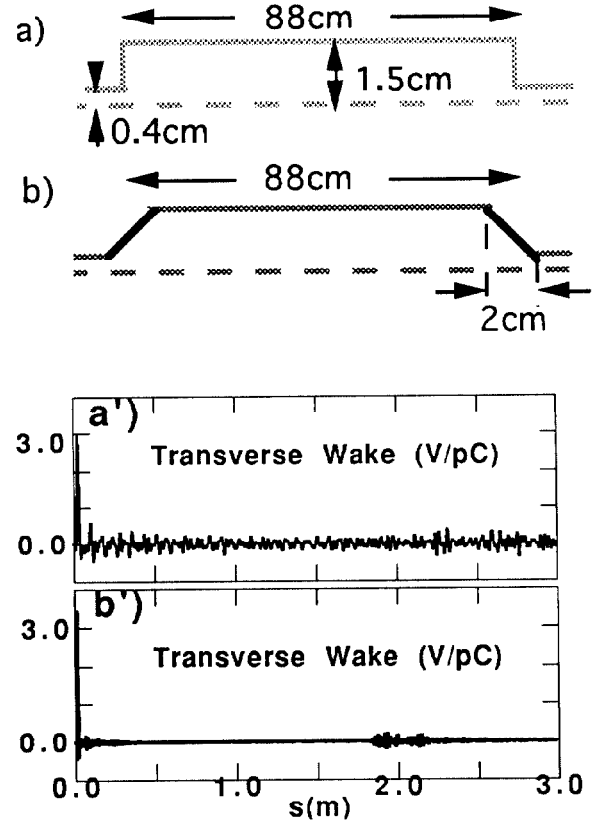

Figure 2. Cylindrically symmetric cavity with abrupt junction (a) and gradual junction with a $2\,cm$ cone (b). Also, the transverse wakes for a Gaussian bunch of $\sigma = 3$ mm are shown with abrupt junctions (a') and gradual junctions (b').

for the monopole $(m=0)$ longitudinal wakes[6]. We believe that the same property applies for the dipole transverse wakes as well. Thus longer cavities reduce the transverse kick per unit length.

A reappearance of the large amplitude wake oscillations near $1.7m$ of Figure 2(b') occurs roughly at twice the length of the cavity. That is likely the first wake generated by the signal charge reflected back to the entrance of the cavity. To push the reappearance of the large oscillations further away in s, one might either make the cavity longer or the cone transition longer. The reappearance of these oscillations are commonly observed with discreteness of frequencies and discontinuities in frequency distribution [5].

IV. NUMERICAL BBU RESULTS

Using the wake functions of Figure 2(b'), we have integrated the basic model equation numerically using the 4-th order Runge-Kutta method. Uniform displacements of the pulse were assumed as an initial condition.

The maximum amplitudes of a 100 nsec pulse of 17.1 GHz are shown in Figure 3 with respect to device lengths for the parameters presented in Table 1. Assuming negligible wake beyond 3 m, the beam displacements are small for a 60 m device but become too large for a 100 m device as shown in curve A of Figure 3.
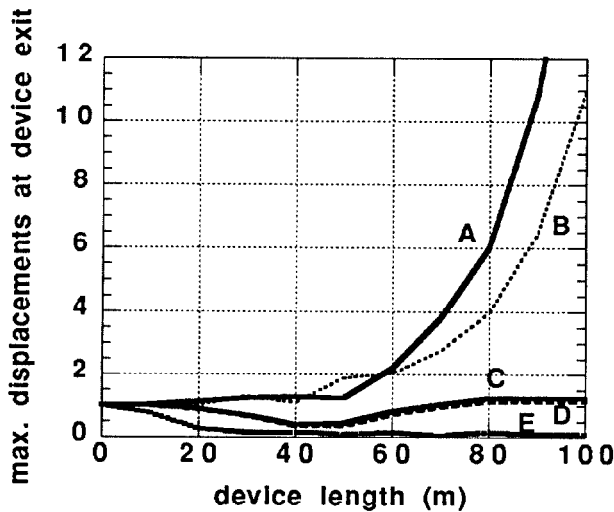
Figure 3. Maximum displacements of a *100 nsec* pulse at the time of exiting the device as a function of device length, for the wakes shown in Figure 2(b'). Shown are the displacements (A) with ignoring wakes beyond $s=3$ m, (B) with BNS damping of a 2% increment in $\gamma(s)$, (C) with ignoring wakes beyond $s=1.74$ m, (D) with $Q=100$, and (E) with 2% spread in k_β.

Now we consider some possible damping mechanisms. First consider BNS damping [8]. The idea is to compensate that wake by increasing or decreasing the energy of the bunch, thus introducing a small change to k_β with s. The sign of the slope, k_β with s, is crucial. However, with either an increment (B in Figure 3) or decrement of $\gamma(s)$, we obtain only a slight reduction in BBU.

Next we assumed that the wake beyond 1.74 m is negligible. When we set the wakes beyond 1.74 m to zero (C in Figure 3), then an external $Q=100$ is introduced instead of zeroing the wakes beyond 1.74 m (D in Figure 3). These gave the same good results indicating virtually no growth for 100 m device. Lastly we introduced some spread in k_β. With 2% spread in k_β, all transverse motion are damped out (E in Figure 3).

V. DISCUSSION

The transverse dynamics of the beam depends on γk_β strongly, yielding smaller BBU for larger γk_β. The presented BBU results are for $\gamma k_\beta = 2.0$ cm^{-1}. On the other hand the fluctuation level of output power amplitude and phase, in beam energy, depends strongly on k_w. For 17.1 GHz the best sensitivity is obtained around $k_w = 0.16$. For a pulse of $1kA$, 100 $nsec$, $a_w = 1.4$ is all that is needed to obtain an output energy of about 10 J/m. With such parameters, and with natural FEL focusing alone, the value $\gamma k_\beta = \bar{a}_w k_w$

(where the bar is used for an rms value,) is too small to reduce BBU to an acceptable level. This dilemma can be eliminated by introducing strong focusing in the drift region to minimize BBU (which does not affect the power extraction performance).

The BBU results presented here can be considered as an upper limit since we did not take into account the wiggler motion. Also, we believe that the BBU can be controlled better with rectangular cavities since we have more freedom to change the junction conditions. Nevertheless, even with the excessive estimate of BBU obtained in this work, we can find suitable SWFEL parameters as can be seen in Reference [7].

Acknowledgments

VI. REFERENCES

[1] A.M.Sessler, in *Laser Acceleration of Particles*, edited by P.J.Channell (AIP, New York,1982), p.154.

[2] A.M.Sessler, D.H.Whittum, J.S.Wurtele, W.M.Sharp, and M.A.Makowski, "Standing-wave free-electron laser two-beam acceleratior", Nucl. Inst. Method in Phys. Res. A306, pp.592-605 (1991).

[3] K.Yokoya, "Cumulative beam breakup in large-scale linacs", preprint DESY 86-084 (1986)

[4] Y.H. Chin, "User's guide for new ABCI Version 6.2", preprint LBL-33091 (1992).

[5] D.Yu and J.S.Kim, "Wakefield suppression using beatwave structures", IEEE Part. Accel. Conf. Proc., p. 1719 (1991).

[6] Y.H. Chin, "New features and applications of ABCI", preprint, LBL-33735.

[7] J.S. Kim, A.M. Sessler, W. M. Sharp, and H. Henke,"Beam Sensitivity to Cavity Geometry and Energy Extraction in a Standing Wave FEL/TBA", This proceding Ja4, (1993).

[8] V. Balakin, A. Novokhatsky and V. Smirnov, "VLEPP: transverse beam dynamics", Proc. of the 12th Int. Conf. on High Energy Accelerators, Fermilab, p.119, (1983).