Collective Effects of the PLS 2 GeV Storage Ring

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Abstract

Collective effects of the PLS storage ring are discussed. Evaluation of the PLS storage ring coupling impedances is presented. RF cavity Impedances are emphasized. Single-bunch threshold current is studied and longitudinal coupled-bunch instabilties caused by RF narrow-band resonances are analyzed.

I. INTRODUCTION

Charged particles moving very fast feel negligible electrostatic force from other particles. Therefore relativistic bunch of charged particles can maintain its shape and state of motion forever without any disturbance. However, for bunches moving inside a vacuum chamber with finite conductivity and some geometry, the situation is not so perfect. Image charges are induced on the surface of the vacuum chamber and electromagnetic field is generated by the beam bunch and the image charges on the vacuum chamber. This electromagnetic field is called the wake field. Because of the finite conductivity of the vacuum chamber, the wake field is energy consuming and acts as a disturbance to the motion of the bunch. The disturbing effects of the wake field are not only longitudinal but also transversal to the bunched beam motion. To describe these two effects two functions are defined. The longitudinal wake function $W_{||}(s)$ is defined as the amount of energy lost by a unit test charge that follows the wake field generating bunch of unit charge at some longitudinal distance s from a reference position in the bunch. And similarly the transverse wake function $W_{\perp}(s)$ is defined as the total transverse impulse received by a unit test charge that accompanies a bunch of unit charge[1]. Now for the traveling bunches, stable motion is not guaranteed any more. The wake field also causes energy spread of particles in a bunch and increase of the effective emittance of a beam. Since wake fields generated by a bunch do not vanish soon and can spread over a long range, the trailing bunches get influenced. This is why some kinds of coupled bunch instabilities may occur.

Therefore designing accelerators, it is necessary to evaluate these disturbing effects and try to find a way of minimizing them. Sometimes it is useful to work with Fourier transform of wake field to frequency, which is identified as the coupling impedance of the chamber. When the instabilities are described in terms of coherent modes of a beam, the concept of impedance is more convenient.

II. COUPLING IMPEDANCES OF PLS

The frequency description of wake field is given by specifying two impedances, longitudinal impedance $Z_{||}(\omega)$ which is a Fourier transform of $W_{||}(s)$ and transverse impedance $Z_{\perp}(\omega)$ which is a Fourier transform of $W_{\perp}(s)$. All storage ring components are sources of instabilities. In other words they have their own impedances; vacuum chamber resistive wall, RF cavity, bellows, steps, transition pieces, beam position monitors and so on. The total impedance of the storage ring should be as small as possible. In the theories of instabilities, it is not $Z(\omega)$ but $Z(\omega)/n$ that appears, where $n = \omega/\omega_0$ is the frequency divided by the revolution frequency.

The impedance of the resistive wall of the chamber is significant at low frequencies. However at frequencies higher than a few MHz, the impedance of the resistive wall diminishes and effectively negligible. At these high frequencies, RF cavity is the most important one. The cavity has a fundamental resonant mode and higher frequency resonant modes. The first a few resonant modes are very sharp. These narrow band resonances are equivalent to long range wake fields and therefore origins of coupled bunch instabilities. Each of these sharp resonances is significant and so has to be specified. Higher frequency resonances get broader and at sufficiently high frequences, one can assume that the impedance is a continuum.

To evaluate the impedance of the storage ring, we performed simplified model calculation for chamber resistive wall, bellows, transition pieces and so on. However for RF cavity we need more rigorous method. The above mentioned sharp resonances will be searched experimentally. And we used TBCI code to evaluate the broad band impedance of RF cavity, which is a big part of the total impedance budget of the storage ring. We adopted resonator model to approximate the broad band impedance. To determine three parameters of the resonator model, we used longitudinal and transverse loss factor, $k_{||}(\sigma)$ and $k_{\perp}(\sigma)$ defined by

$$k_{||}(\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} W_{||}(s,\sigma)e^{-s^2/2\sigma^2}ds, \qquad (1)$$

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$$k_\perp(\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} W_\perp(s,\sigma)e^{-s^2/2\sigma^2}ds. \quad (2)$$

In the above formulas, σ is the *rms* bunch length. By varying σ and running TBCI, we can obtain graph of k's versus σ. Then we can determine the three parameters of the resonator model by graph fitting.

Table I summarizes the longitudinal impedances for PLS storage ring.

Table I PLS Impedance budget

Item	$\lvert Z_\parallel/n\rvert$ (Ω)
RF cavity	0.7
Synchrotron radiation	0.16
Space charge	1.3×10^{-4}
Resitive wall	0.02
Steps and transitions	0.17
Bellows (unshielded)	(1.27)
Bellows (shielded)	0.01
Beam position monitors	0.015
Ceramic chamber coatings	1×10^{-4}
Other components and safety margin	1.0
Total	~2

The corresponding transverse impedance can be obtained by assuming a circular symmetry of the chamber [2]:

$$|Z_\perp| = \frac{2c}{b^2\omega}|Z_\parallel| = \frac{2R}{\beta b^2}|\frac{Z_\parallel}{n}| \quad . \quad (3)$$

For PLS with b=2.5 cm, we get

$$|Z_\perp| = 0.286 \quad \mathrm{M}\Omega/\mathrm{m} \quad .$$

III. THRESHOLD CURRENT FOR SINGLE BUNCH

The peak threshold current for the longitudinal microwave instability (also known as the turbulent bunch lengthening) is given by [3]

$$I_p = \frac{2\pi|\eta|E/e(\beta\sigma_p)^2}{|Z_\parallel/n|} \quad (4)$$

where η is the frequency slip factor ($\eta \approx \alpha$ for a highly relativistic particle and α is the momentum compaction factor), E the energy of a stored beam, $\beta = v/c$, σ_p the relative *rms* energy spread of a beam, and $|Z_\parallel/n|$ is the longitudinal broad-band coupling impedance divided by the mode number.

Threshold current as a function of $|Z_\parallel/n|$ is shown in Fig. 1. Note that the threshold current shown in Fig. 1 is the average beam current I_b. It is related with the peak current via

$$I_b = \frac{\sigma_l}{\sqrt{2\pi}R}I_p \quad , \quad (5)$$

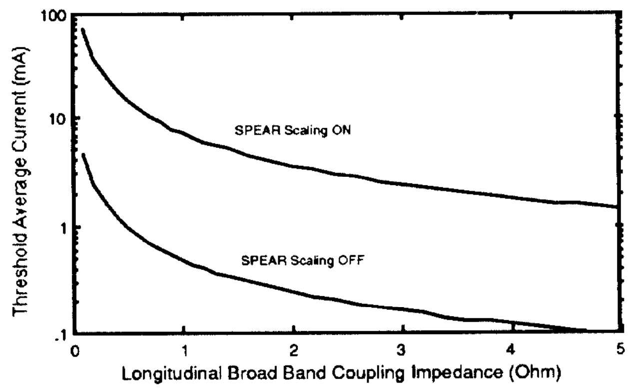

Fig.1 Single bunch longitudinal microwave threshold as a function of coupling impedance

where σ_l is the *rms* bunch length and R is the effective radius of the ring (i.e., $R = C/2\pi$, C=ring circumference). Fig. 1 also compares between with and without SPEAR scaling [4]. It is seen that when the longitudinal broad band impedance is 2 Ω, the threshold average current for a single bunch is approximately 0.24 mA without SPEAR scaling and 3.5 mA with SPEAR scaling. The actual threshold current will be some value in between.

IV. LONGITUDINAL COUPLED-BUNCH INSTABILITIES

As is mentioned in section II, narrow-band resonances of RF cavities are origin of coupled-bunch instabilities. The central equation relating impedance resonances and instabilities is that of complex coherent frequency shift, $\Delta\omega_{s,a}$ for small Gaussian bunches [5]:

$$\Delta\omega_{s,a} = i\frac{\mathcal{I}_0\omega_0^2\eta k_b}{2\pi E_0\omega_s}\frac{(\sigma_\ell/R)^{2(a-1)}}{2^a(a-1)!}(Z_\parallel)^{s,a}_{eff} \quad (6)$$

where

$$(Z_\parallel)^{s,a}_{eff} = \sum_{p=-\infty}^{+\infty}(pk_b+s)^{2a}e^{-(pk_b+s)^2(\sigma_\ell/R)^2} \cdot \frac{Z_\parallel[(pk_b+s+a\nu_s)\omega_0]}{pk_b+s+a\nu_s}. \quad (7)$$

In the above equation, k_b represents the number of bunches, s the longitudinal mode number $s = 0,1,2\cdots,(k_b-1)$, a the oscillation mode of the bunch shape in phase space, e.g., the dipole mode a=1, the quadrupole mode a=2, etc., η the frequency slip factor, $\mathcal{I}_0$ the average beam current, ω_0 the angular revolution frequency, E_0 the nominal beam energy, ω_s the angular synchrotron frequency, R the average radius of a ring, and σ_ℓ is the rms bunch length in space. At this time of manuscript preparation, the first RF cavity of PLS has been delivered months ago but the full analysis has not been carried out yet. Therefore we used the measured data of Photon Factory, of which cavity is almost the same as that of PLS. To use Eq. (6), the program ZAP has been invoked.

Calculated growth times for the dipole mode (i.e. a=1) for each RF cavity mode are summarized in Table II.

Table II Longitudinal dipole (a=1) mode growth rate for each higher-order mode of the RF cavity

Frequency (MHz), f_{rf}	Growth time (msec)
758.2	230.8
1,048.0	499.6
1,302.1	1161.2
1,328.0	62.6
1,648.4	0.48
1,707.7	9.04
1,860.5	49.76
1,962.4	381.6
2,121.5	43.08
2,167.7	196.24

In obtaining the growth time of Table II, total number of bunches (k_b) are assumed to be 468 (i.e., fill all RF buckets) and the average beam current was taken to be 100 mA. The *rms* bunch length was taken arbitrarily to be 5 mm, which is the natural *rms* bunch length at 2 GeV. The *rms* energy spread was taken to be 6.8×10^{-4}. It is seen that the mode number 6 (i.e, f_{rf}=1,648.4 MHz) gives the worst case. When taking into account the fact that there are total four cavities in the PLS, we expect that the growth times will be smaller than those in Table II. The reason that the mode number 6 gives the smallest growth time is because it is this frequency which is closest to the neighboring coupled-bunch oscillation frequency $(pk_b + s + a\nu_s)\omega_0$, where p=1, s=148 and a=1. Higher-order mode damping or frequency shift of the RF cavity is therefore needed for this mode and others. On the other hand, we believe that the estimation given here is very pessimistic, because in the calculation we have not taken into account the radiation damping and the Landau damping which comes from the frequency spread due to the nonlinear synchrotron oscillation.

The growth times and the tune shifts are tabulated in Table III. Again one RF cavity was assumed in obtaining this table. All the RF buckets were assumed to be filled out and the average beam current was taken to be 100 mA.

Table III shows that for dipole synchrotron mode the coherent growth time increases as the energy increases while for higher synchrotron modes it gets decreasing. This can be easily explained with the help of Eqs. (6) and (7). For dipole mode, the effective impedance decreases as the energy increases because of the increase in bunch length, as seen in Eq. (7). On the other hand, for higher modes

Table III Growth times and tune shifts for the fastest-growing longitudinal coupled-bunch modes for 468 bunches and 100 mA beam current

Energy (GeV)	Mode number (a)	Growth time (msec)	Tune shift
	1	0.34	-2.94×10^{-4}
1.5	2	49.91	-5.86×10^{-5}
	3	1618	-1.98×10^{-5}
	4,···		
	1	0.5	-1.82×10^{-4}
2.0	2	48.7	-3.51×10^{-5}
	3,4···		
	1	0.57	-1.15×10^{-4}
2.5	2	26.5	-2.48×10^{-5}
	3,4···		

(a > 1), the rate of the increase in growth rate rapidly increases as the bunch length increases.

The result indicated that damping mechanism or frequency shift of the RF cavity is required, otherwise the growth time is too small, less than the radiation damping time. It is also seen that dangerous RF modes are mostly located on the high frequency side. The theory employed here has a limitation such that it can not be used for the asymmetric bunch configuration, which will be the actual case for the PLS in order to avoid the ion trapping problem.

V. REFERENCES

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