Longitudinal Instabilities in the MEB

John A. Palkovic
Superconducting Supercollider Laboratory *
2550 Beckleymeade Avenue
Dallas, TX 75237

Abstract

In this paper longitudinal instability parameters are calculated for the SSC Medium Energy Booster (MEB). Both single and multiple bunch instabilities are investigated. With a beam intensity of 1.0×10^{10} the longitudinal coupling impedance threshold, $|Z_{\|}(\omega)/n|$, is found to be $\sim 60\,\Omega$ at the injection momentum 12 Gev/c, and $12\,\Omega$ at the ejection momentum, 200 GeV/c. Coupled bunch instability growth rates are calculated for the $\lambda/4$ cavity higher order mode (HOM) spectrum.

I. INTRODUCTION

The MEB is a 200 GeV proton synchrotron which is scheduled to be built at the Super Conducting Super Collider Laboratory (SSCL) with first beam in June, 1996. Some of the longitudinal parameters of the MEB are listed in Table 1. The MEB will deliver beam for colliding beam physics and test beam operations. [1]

First the physics of some of the single-bunch instabilities that can occur in the MEB will be discussed. These instabilities arise from the interaction of the beam with the broadband component of the longitudinal coupling impedance $Z_{\|}(\omega)$, where ω is the angular frequency in *rad/s*. A discussion of the multiple bunch instabilities that occur in the MEB follows. A HOM of an accelerating cavity provides a longitudinal coupling impedance which can cause a multi-bunch instability or coupled bunch mode (CBM) if the beam current is above a threshold value. We have used ZAP [2] to calculate CBM growth rates and frequency shifts for the $\lambda/4$ cavity.

II. SINGLE BUNCH STABILITY

The one-dimensional Vlasov equation which describes the evolution of the longitudinal phase-space distribution is written [3, 5, 6]

$$\frac{\partial\psi}{\partial t} + \dot{\theta}\frac{\partial\psi}{\partial\theta} + \dot{\delta}\frac{\partial\psi}{\partial\delta} = 0, \quad (1)$$

where $\theta = 2\pi S/R$ is the angular position of the particle around the synchrotron, $\psi = \psi(\theta, \delta, t)$ is the longitudinal distribution function, $\delta = \Delta p/p$, and

$$\dot{\delta} = -\frac{e\omega_0 I_1 Z_L(\omega)}{2\pi\beta^2 E} e^{i(n\theta - \Omega t)}$$

*Operated by Universities Research Association for the U.S. Department of Energy under Contract No. DE–AC35–89ER40486.

Table 1
MEB Parameters

Injection momentum	12 GeV/c
Extraction momentum	200 GeV/c
Circumference C	3960 m
Harmonic number	792
Momentum "compaction" α	1.85×10^{-3}
Transition γ	23.28
RF frequency at injection	59.776 MHz
RF frequency at extraction	59.958 MHz
95% bunch area, injection	0.04 eV-s
95% bunch area, extraction	0.1 eV-s
Minimum RF Voltage	170 kV
Maximum RF Voltage	1.6 MV
particles/bunch (collider mode)	1×10^{10}

is the term which describes the interaction of the particle distribution with the coupling impedance $Z_L(\omega)$, where Ω is the angular frequency of the perturbation in the lab. The revolution frequency of the synchronous particle is ω_0, I_1 is the perturbation current, and E is the particle energy.

Using a perturbation analysis we can linearize the Vlasov equation and find the dispersion relation [5]

$$1 = -i\frac{e^2\omega_0^2 Z_L}{2\pi\beta^2 E}\int \frac{(\partial\psi_0/\partial\omega)}{\Omega - n\omega}d\omega\ . \quad (2)$$

The threshold for longitudinal stability is found by solving Eq. 2 with Ω having a small positive imaginary part, $\Omega = \Omega_r + i\epsilon, 0 < \epsilon \ll 1$. We can evaluate the integral in Eq. 2, solve for the impedance Z_L, and map out the stability boundary in the impedance plane for a given distribution function ψ_0. If the impedance lies inside the boundary, then any disturbance is damped, which is due to a phenomenon known as Landau damping. [4] In general, these stability boundaries have a somewhat complicated shape (Figure 1). However, the stable region can be conservatively approximated by a semi-circle, a result which we write as

$$\left|\frac{Z}{n}\right| \le F\frac{\beta^2 E\,|\eta|}{eI_0}\left(\frac{\Delta p}{p}\right)^2, \quad (3)$$

where $F \approx 1$ is a "form factor" which depends on the radius of the approximating circle, and I_0 is the beam current. This is known as the Keil-Schnell criterion, although an equivalent formulation was published previously by Neil and Sessler. [6] Equation (3) was derived for a

0-7803-1203-1/93$03.00 © 1993 IEEE

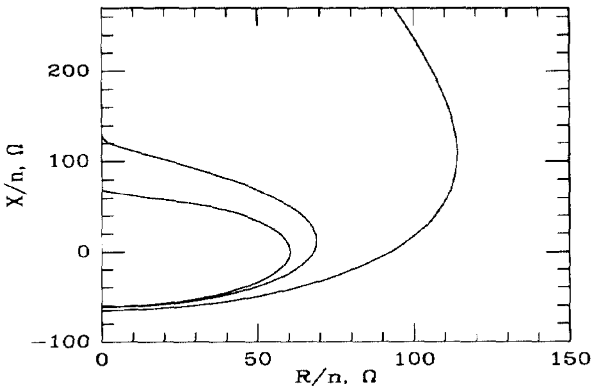

Figure 1. Stability diagrams for the MEB at injection. Positive X/n represents a capacitive impedance. The innermost curve is the boundary for a distribution $f \propto (1-x^2)^{1.5}$; the outermost is for a Gaussian; the intermediate curve is for a quartic, $f \propto (1-x^2)^2$, where x is the normalized rotation frequency.

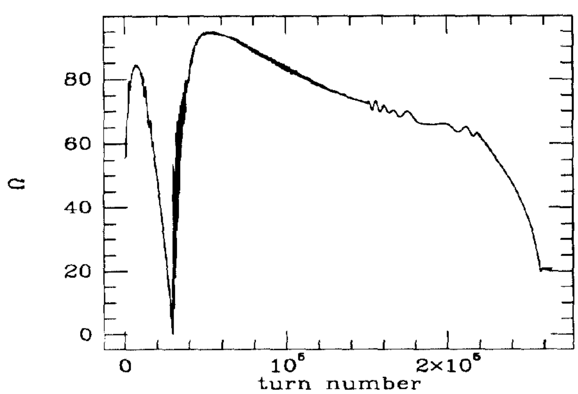

Figure 2. Plot of right hand side of Eq. (3) multiplied by 0.6 vs. turn number in the MEB. The dip in the curve occurs at transition.

coasting beam, but it can be applied to a bunched beam if the growth rate is fast compared to the frequency of synchrotron oscillations, and if the instability occurs at wavelengths short compared to a bunch wavelength. The parameter I_0 is then the peak current in the bunch.

ESME [7], a longitudinal phase-space tracking code, was used to calculate $\Delta p/p$ for an MEB by tracking 2×10^4 pseudo-particles with an amount of charge equivalent to 10^{10} protons/bunch. The ESME output was used to plot Eq. 3 (Figure 2). We see that at injection if, for example, the distribution is similar in shape to a quartic, the MEB bunch will be stable for values of the longitudinal coupling impedance $|Z/n| \leq 60\Omega$. Other than the time spent near transition crossing, it seems unlikely that we will cross the threshold for single-bunch stability, since we expect $|Z/n| \leq 2\Omega$ in the MEB. We will not discuss single-bunch stability at transition crossing.

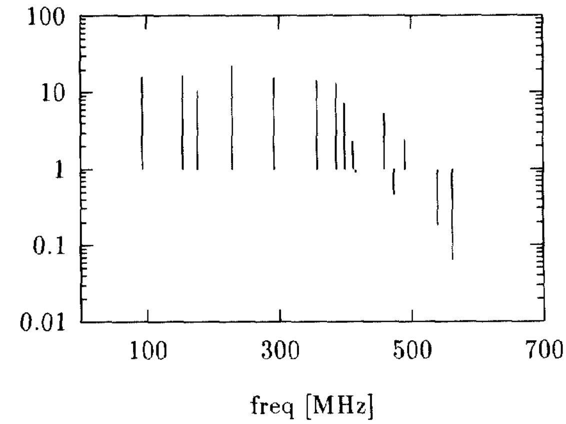

Figure 3. Coupled bunch instability growth rates in s^{-1} at the injection momentum (12 GeV) of the MEB.

III. MULTIPLE BUNCH STABILITY

In Sacherer's formalism [8], the complex frequency shift for coupled-bunch motion is described by

$$\Delta\omega_{m,n} = i\omega_s \left(\frac{m}{m+1}\right) \frac{I}{3B_0^2 hV cos\phi_s} \cdot \sum_p F_m(f_p \tau_l) \frac{Z_L(f_p)}{p} \quad (4)$$

where

$$F_m(f_p \tau_l) = \frac{1}{MB_0} \frac{|\tilde{\lambda}_m(p)|^2}{\sum_p |\tilde{\lambda}_m(p)|^2} \quad (5)$$

is another form factor which weights the contributions from the coupling impedance $Z_L(f_p)$. The Fourier transform of the perturbed line density is denoted by $\tilde{\lambda}_m(p)$, ω_s is the synchrotron frequency, m is the synchrotron mode number, B_0 is the bunch length τ_L/the revolution period T, ϕ_s is the synchronous phase angle which is positive below transition and negative above, and

$$f_P = (n + pM)f_0 + mf_s, \quad -\infty < p < +\infty . \quad (6)$$

M is the number of bunches, which are assumed to populate the ring in a symmetric fashion.

ZAP [2],which implements Eq. 4, was used to calculate coupled-bunch instability growth rates for the MEB. The calculation assumes that the bunches have a Guassian profile in momentum, that the ring is filled with $792 = h$ bunches, and that the beam has an intensity of 10^{10} protons/bunch. The HOM input data was generated with MAFIA, a 3d particle-in-cell code (Table 2). Measured data on a prototype cavity would be preferable, but there is no prototype RF cavity for the MEB as of this writing. The results are plotted in Figures 3 and 4. We see there are many modes with growth rate $> 1s^{-1}$.

Table 2
HOM data for MEB $\lambda/4$ cavity

Frequency MHz	shunt imp. R_s $k\Omega$	quality factor Q
59.8	629	12300
93.0	13	9500
154.7	14	6700
176.5	34	13600
229.2	48	15600
292.3	70	22000
357.1	49	24400
386.5	33	9800
399.3	23	10300
415.6	9	22700
459.3	103	23500
411.6	22	22300
473.4	3	22100
490.2	208	25000
540.1	12	15200
561.8	5	10200

In normal operation, the MEB will be filled with 6 LEB batches approximately once every 8 seconds. The beam will orbit the MEB for $\sim$ 4 seconds. We choose to limit the multi-bunch integrated growth to 4 e-foldings. Hence, we would like for the growth rate $1/\tau$ to be $< 1s^{-1}$. The rates calculated with ZAP almost all exceed this criterion. Therefore the modes must be damped in order to limit the growth. An approximate formula for the growth rate (useful for estimates) due to any particular HOM is

$$\frac{1}{\tau} \sim \frac{\omega_\phi}{2} \frac{p}{h} \frac{I_0 R}{V cos\phi_s}. \tag{7}$$

The growth rate is proportional to R, the shunt impedance of the HOM. For a given HOM that has $1/\tau > 1s^{-1}$, we can limit the beam to 4 e-foldings (1 e-folding/s) if we damp the mode by a factor equal to the growth rate in the MEB. In fact, we may not need to damp this much, since the Landau stability threshold could be crossed. A given CBM will be Landau damped if $|\Delta\omega_{m,n}| \leq m/2 \cdot (m+1)S$, where

$$S = \frac{1}{8}\left(\frac{\sigma_z}{R}\right)^2 h^2 \omega_s \tag{8}$$

is the synchrotron frequency spread within the bunch. [2] For the MEB at injection, we find $S \simeq 15Hz$ and

$$|\Delta f| = \frac{1}{2\pi}|\Delta\omega_{m,n}| \leq S/8\pi \simeq 0.6Hz. \tag{9}$$

is the condition for Landau damping. However, we require $Im(\omega_{m,n}) \leq 1s^{-1}$ to limit the CBM to 4 e-foldings in the MEB. So there is no need to damp the HOM to the Landau threshold at the injection momentum. At extraction, with $\sigma_z \sim 10cm$ we find $S = 0.4$ Hz, so the Landau damping threshold is correspondingly smaller and the same damping criterion applies. If the beam intensity is increased to

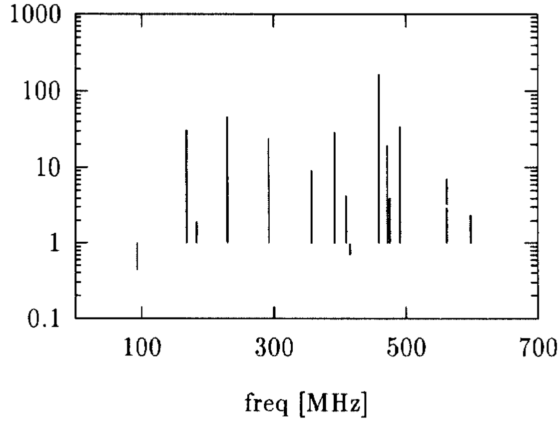

Figure 4. Coupled bunch instability growth rates in s^{-1} at the extraction momentum (200 GeV) of the MEB.

5×10^{10} protons/bunch, then it will be necessary to damp the HOMs by another factor of five; if this is not possible, an active damping system may be required.

IV. ACKNOWLEDGEMENTS

I wish to thank Tai-Sen Wang for the computer program used to calculate singe-bunch stability threshold curves, and Slawomir Kwiatkowski for providing the HOM data for the coupled-bunch calculations.

V. REFERENCES

[1] J.R. Sanford and D.M. Matthews. Site-specific conceptual design report of the superconducting super collider. Technical Report SSCL-SR-1056, Superconducting Super Collider Laboratory, July 1990.

[2] M.S. Zisman et al. "Zap User's Manual." Lawrence Berkeley Laboratory, December 1986.

[3] N.A. Krall and A.W. Trivelpiece. *Principles of Plasma Physics.* San Francisco Press (1986).

[4] L.D. Landau. "On the vibrations of the electronic plasma." *J. Phys. U.S.S.R.* 10:25, (1946).

[5] D.A. Edwards and M.J. Syphers. *An introduction to the physics of high energy accelerators.* Wiley-Interscience, New York, NY (1993).

[6] V. Kelvin Neil and Andrew M. Sessler. "Longitudinal resistive instabilities of intense coasting beams in particle accelerators." *Rev. Sci. Instr.* 36(4):429, (1965).

[7] J. MacLachlan S. Stahl. User's guide to esme. TM 1650, Fermilab, 1990.

[8] F.J. Sacherer. "Bunch lengthening and microwave instability." *IEEE Transactions on Nuclear Science* NS-24(3), (1977).