Space-Charge Calculations in Synchrotrons

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Abstract

One obvious bottleneck of achieving high luminosity in hadron colliders, such as the Superconducting Super Collider (SSC), is the beam emittance growth, due to space-charge effects in low energy injector synchrotrons. Although space-charge effects have been recognized since the alternating-gradient synchrotron was invented, and the Laslett tune shift usually calculated to quantify these effects, our understanding of the effects is limited, especially when the Laslett tune shift becomes a large fraction of the integer. Using the Simpsons tracking code, which we developed to study emittance preservation issues in proton synchrotrons, we investigated space-charge effects in the SSC Low Energy Booster (LEB). We observed detailed dependence on parameters such as beam intensity, initial emittance, injection energy, lattice function, and longitudinal motion. A summary of those findings, as well as the tracking technique we developed for the study, are presented.

I. INTRODUCTION

One of the challenging issues for proton synchrotrons is to store and accelerate a high brightness beam, that is an intense beam with a very small emittance. In a high energy accelerator complex such as the Superconducting Super Collider (SSC), the luminosity at the final collider directly depends on the brightness in the preceding injector chains. Space-charge effects in the low energy end of an accelerator complex, for instance, the Low Energy Booster (LEB) and possibly the Medium Energy Booster (MEB) at the SSC, could be a potential problem for the emittance and thus brightness preservation.

As a measure of space-charge effects, the Laslett tune shift is usually calculated, that for Gaussian distribution is $\Delta\nu = -(r_p n_t)/(4\pi\beta\gamma^2\epsilon^n B_f)$; where r_p is the classical proton radius, n_t is the total number of particles in a ring, β and γ are the Lorentz factors, ϵ^n is the normalized rms emittance, and B_f is the bunching factor. [1] The conventional design criterion imposes the small Laslett tune shift, such as -0.2, to keep the entire beam stay away from lower order resonances. Although most of existing machines has been designed in that way, the brightness has been increased more than the design value experimentally in some machines. An example would be the AGS of BNL and the PS Booster of CERN, and the maximum tune shift like -0.5 to -0.6 has been achieved. The tune shift criterion such as -0.2 is by no means a fixed number.

More importantly, the Laslett tune shift itself does not predict the emittance behavior. If it is small enough, the beam is, for sure, free from any resonances and no emittance growth is expected. When the tune shift becomes large, say -0.5, some particles in the beam would be at some resonance. Once the amplitude of the particles changes, the particles can escape from the resonances and become stable again. The strong nonlinear nature of the space-charge force, namely detuning, makes it difficult to estimate emittance evolution.

As for a source of resonances, the space-charge force itself, can excite it without lattice imperfections. In late '60s, Montague discussed the importance of zero-th harmonics of the fourth order resonance; $2\nu_x - 2\nu_y = 0$, which is inevitable due to the octupole component of the space-charge force [1]. According to the recent work by Parzen and Machida, non-zero harmonics of the even order resonances should be also avoided because they are excited by the space-charge force coupled with periodicity of lattice functions [2]. One way to weaken those resonances is to make the lattice with higher periodicity. The effect of half-integer resonance excited by lattice imperfections was studied for the coasting beam [2]. A study by Machida shows that maximum tune shift can be more than the distance between the bare tune and the half-integer resonance. The ratio of possible tune shift to the distance depends on the charge distribution and it is about two for Gaussian distribution.

In this paper, we first describe the recent development of the space-charge modelling by multi-particle tracking. Then, we discuss systematic exploration in parameter space; beam intensity, initial emittance, injection energy, lattice function, and longitudinal motion, using the LEB as an example.

II. MODELLING OF THE SPACE-CHARGE IN SYNCHROTRONS

A. *The Simpsons Program*

The Simpsons program has been developed to study emittance preservation issues in proton synchrotrons [3]. The program consists of two major parts. One is a par-

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[1] In this simplified formula, we ignored the effects of image charge and current in surroundings such as magnets and beam pipe.

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ticle tracking part that tracks macro particles in the 6-D phase space with acceleration. Realistic modelling of longitudinal motion is essential for a rapid cycling synchrotron such as the LEB in which the synchronous momentum rises from 1.219 GeV/c to 12 GeV/c within approximately 26,000 turns. The space-charge force is strongly time dependent. The independent variable is time, in which one can obtain a snapshot of macro particle distribution at each time step. All lattice elements including an RF cavity are represented by thin lenses lattice created by the program TEAPOT [4].

The other part of the program calculates the space-charge force. It is incorporated as a thin lens kick in each time step and typically 100 to 2000 kicks are applied in one turn. We made three different degrees of approximation to compute the space-charge field. The most advanced one, though it is most time consuming as well, employes the Particle-In-Cell method [5]. Three dimensional grids, in real space, are assigned to enclose the entire beam. At every intersection of the grids, a fractional charge is allocated according to location of macro particles nearby, and then the Maxwell equations are solved in a difference form. A boundary condition is imposed such that the scaler potential is zero at the beam pipe radius, which is constant around the ring. Electro-magnetic fields, at the location of macro particles, are interpolated by the fields at neighboring grids and it changes the momentum. Typically, more than 10,000 macro particles are necessary to represent the charge distribution accurately and it takes about 100 Cray cpu hours to simulate the first 10 msec of the LEB, which is one fifth of a cycle. We call it "strong-strong" approximation, because the emittance and charge distribution are updated continuously and it is expected to be self-consistent.

The second approximation assumes that charge distribution is always Gaussian and the image charge and the longitudinal space-charge forces are negligible. By that assumption, the transverse space-charge force can be computed analytically with a certain truncation [6]. By using approximately 1000 macro particles, the rms emittance calculated at the end of each turn and the space-charge field are computed based on that emittance. Although the charge distribution is assumed to remain Gaussian, it is self-consistent that the space-charge force is a function of the instantaneous emittance. We call it "semi strong-strong" approximation. In the longitudinal plane, the emittance is fixed. The bunch shape is, however, fitted on the matched bucket which is a function of the bending field, its derivative, and RF voltage at each time so that the time dependence of a bunching factor is included. It requires 10 Cray cpu hours when 1000 macro particles are tracked to complete the LEB simulation.

The third approximation, which is the most simplified model, assumes that the charge distribution is always Gaussian and the emittance does not change. The same formula is used to calculate the space-charge force of Gaussian distribution as the "semi strong-strong" approximation without updating the emittance. That approximation should be adequate, when the beam emittance growth is small and when one wants to see either initial behavior of the emittance growth using many macro particles or stability of single particle. We call it "weak-strong" approximation. Most of the following simulation results are based on "semi strong-strong" approximation.

Initial charge distribution was made as Gaussian in both transverse planes, and we examined emittance each turn by two independent definitions. One is the the rms emittance calculated from 1000 to 10000 particles. The other is the emittance by fitting a transverse beam profile to Gaussian. In the following exercise, we found that these emittance definitions agree with each other, implying that the charge distribution remains Gaussian throughout one simulation.

B. *Fermilab Booster Simulation*

A simulation of the Fermilab Booster was performed for comparison with its experimental data that shows the emittance growth as a function of beam intensity [7]. Although our previous study indicated that asymptotic emittance depends on multipoles and misalignment of lattice to some degree [2], we assumed the same magnitude of errors in the Fermilab Booster model as in the LEB because there is no information available on the lattice magnets. (Those are the rms misalignment of 0.4 mm in both horizontal and vertical planes, the rms rotation angle of 1.0 mrad, the rms closed orbit distortion after correction of 1.0 mm, and the same multipole in the dipoles and quadrupoles.) We tested five different lattices with five different seeds to calculate asymptotic emittance of the first 9 msec by "semi strong-strong" and "weak-strong" approximations.

Figure 1 shows the comparison. The error bar of simulation results shows the range due to different seeds. There is apparent discrepancy between the experimental data and the simulation results when the beam intensity is high. The following remarks should be made. First of all, the use of the multipole and misalignment data of the LEB may not be a good model for the Fermilab Booster. Therefore, we do not have rigorous basis for one-to-one comparison. However, a trend such that the emittance start growing when the beam intensity is 1.5×10^{10} per bunch, agrees in the experiment and the simulations. In addition, the simulation shows that the emittance growth lasts about 5 or 6 msec after injection and it agrees with the recent measurement by Graves *et al.*, who took the time dependence of the emittance growth in the Fermilab Booster [8]. The same experimental data also unveils that the emittance evolution has two steps, one is after injection and the other after the transition energy crossing. We may have missed a part of emittance growth at the transition energy crossing which is not included in the simulations. The asymptotic emittance as a sum of the two steps by Graves is more or less equal to experimental data of Figure 1.

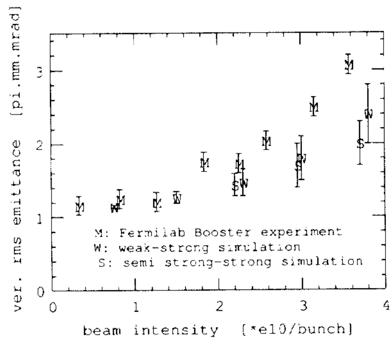

Figure 1. Emittance growth in the Fermilab Booster.

III. SIMULATION RESULTS

A. Beam Parameter Dependence

The design beam parameters of the LEB are 1×10^{10} particles-per-bunch (total number of particles are 1.14×10^{12}) with 0.40 $\pi\cdot$ mm $\cdot$ mrad rms emittance. [2] The beam energy rises from 600 MeV to 11.1 GeV in 50 msec with sinusoidal ramping curve. We explored three parameter space, namely beam intensity, initial emittance, injection energy, keeping other two constant. The asymptotic emittance is defined as the emittance about 15 msec after injection, at that time emittance growth has completed. The model lattice has multipoles, misalignment, and closed orbit errors. We used measured multipole data of the AGS Booster magnets [9]. Before doing systematic parameter search, we made a preliminary run to test five different seeds for lattice randomness and picked up the worst lattice in the following simulations.

Figure 2 shows asymptotic rms emittance as a function of beam intensity. Figures 3 and 4 show the emittance evolution and maximum tune shift. There is slight emittance growth already started when the intensity is 1×10^{10}. Below that intensity, no emittance growth is observed. The asymptotic emittance is almost linear with respect to the intensity when the intensity becomes higher.

Figure 5 shows asymptotic rms emittance as a function of initial rms emittance. Figures 6 and 7 show the emittance evolution and maximum tune shift. When the initial emittance is 0.60 $\pi\cdot$ mm $\cdot$ mrad, there is almost no emittance growth. When the initial emittance is less than that, the emittance growth is inevitable and the asymptotic emittance cannot be less than 0.44 $\pi\cdot$ mm $\cdot$ mrad no matter how small emittance is injected at the beginning. Overshoot phenomena, namely smaller initial emittance ends up with larger asymptotic emittance, is not observed as least within the initial emittance range we explored.

Figure 8 shows the rms emittance as a function of injection energy. Figures 9 and 10 show the emittance evolution and maximum tune shift. If we make the injection energy 1 GeV or higher, almost no emittance growth is observed. Below 1 GeV, less injection energy is taken, more asymptotic emittance is observed as expected.

[2] There is another operational mode called test beam mode, which is supposed to have 5×10^{10} particles per bunch with the rms emittance 4.0 $\pi\cdot$ mm $\cdot$ mrad.

B. Lattice Superperiodicity

The present LEB lattice has three fold symmetry with three long straight sections. If the structure resonance dominates the emittance growth, the higher periodicity lattice should help reducing the emittance growth. By taking only arc part of the present LEB lattice, we made a higher superperiod lattice, 16-fold symmetry with almost same circumference and looked at the emittance growth as a function of the beam intensity. Each lattice element has the similar magnitude of multipoles and misalignment as the three-fold symmetry lattice and the closed orbit are corrected as the same level, namely about 1 mm as rms. As shown in Figure 11, not much improvement was obtained.

C. Synchrotron Oscillation Tune

Synchrotron tune becomes as high as 0.05 at 4 msec after injection and gradually decreased toward the final energy. To look at the dependence of the emittance growth on the synchrotron tune, we fixed the energy and rf voltage and tracked particles under constant synchrotron tune. Keeping the peak intensity constant, we varied synchrotron tune under three different conditions, namely with constant $\Delta p/p$, with constant longitudinal emittance, and with constant bunch length. Figures 12 and 13 show the rms emittance after 6000 turns as a function of synchrotron tune. Since there was no energy ramping and the turn number was not enough to have the emittance saturation, they are not asymptotic in value. The horizontal emittance at $\nu_s = 0.20$ and the vertical emittance at $\nu_s = 0.02$ have a bit larger emittance but the overall synchrotron tune dependence is marginal.

IV. DISCUSSIONS

The simulation results described previously indicate that there is a certain limit of the maximum brightness. That can be seen in Figure 14 in which we plotted the brightness defined by the rms emittance divided by beam intensity, as a function of 1) beam intensity, 2) inverse of the initial rms emittance, and 3) the initial $\beta\gamma^2$ which corresponds to injection energy. The horizontal unit is scaled such that the design value of each parameter is one. Figure 14 shows at least two things. One is that the brightness has a certain limit no matter how small initial emittance or intense beam are injected as long as the injection energy is fixed to 600 MeV. That brightness limit is about 10% higher than the design value. The other is that if the injection energy is increased, the brightness can be as high as 20% with the design beam intensity and initial emittance.

A question is then what makes that brightness limit. From Figures 4, 7, and 10, when the tune shift is more than -0.60 or so, beams are not stable. Since the vertical bare tune is 11.80, the maximum loaded tune is about

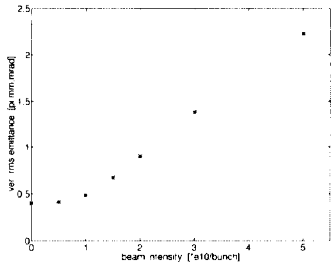

Figure 2. Asymptotic emittance vs. beam intensity. Design beam intensity is 1×10^{10} per bunch.

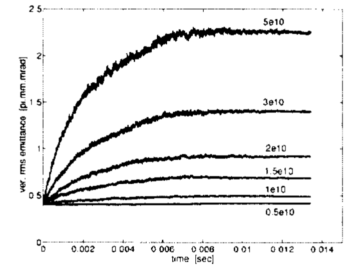

Figure 3. Emittance evolution of Figure 2. Each figure indicates beam intensity.

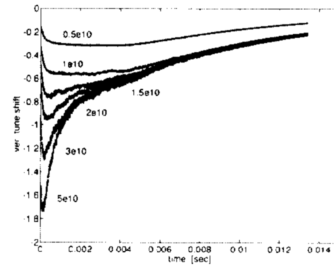

Figure 4. Tune shift evolution of Figure 2. Each figure indicates beam intensity.

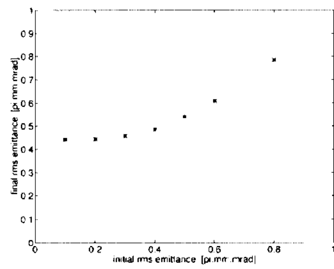

Figure 5. Asymptotic emittance vs. initial emittance. Design initial emittance is 0.4π mm mrad.

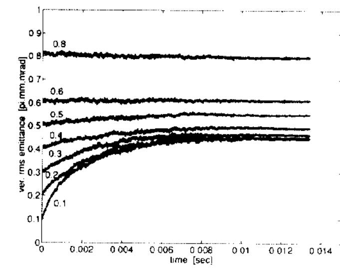

Figure 6. Emittance evolution of Figure 5. Each figure indicates initial emittance in the unit of π mm mrad.

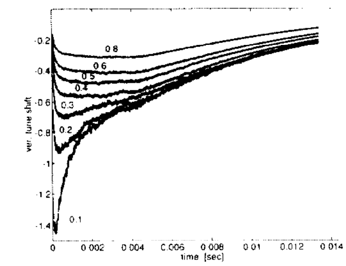

Figure 7. Tune shift evolution of Figure 5. Each figure indicates initial emittance in the unit of π mm mrad.

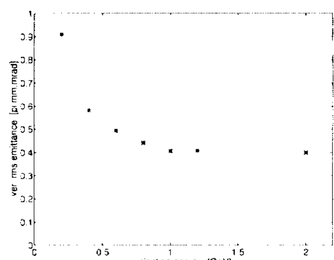

Figure 8. Asymptotic emittance vs. injection energy. Design injection energy is 0.6 GeV.

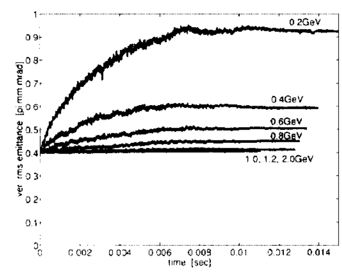

Figure 9. Emittance evolution of Figure 8. Each figure indicates injection energy.

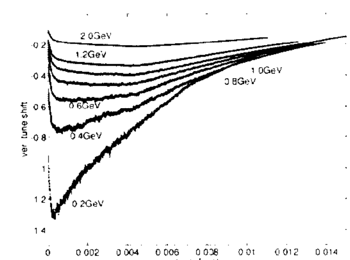

Figure 10. Tune shift evolution of Figure 8. Each figure indicates injection energy.

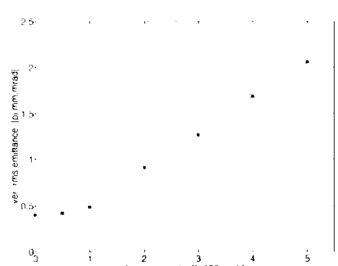

Figure 11. Asymptotic emittance vs. beam intensity in 16-fold symmetry lattice.

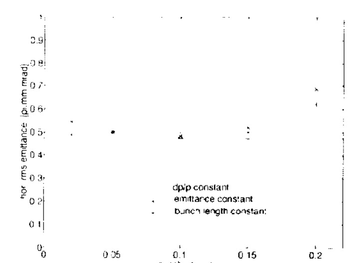

Figure 12. Horizontal emittance after 6000 turns vs. synchrotron tune. There is no energy ramping and the synchrotron tune is kept constant. The horizontal solid line shows the emittance for a coasting beam with the same peak intensity.

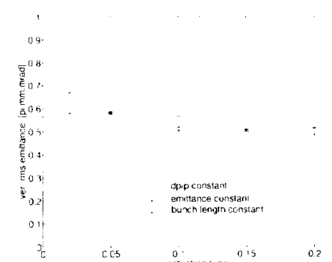

Figure 13. Vertical emittance after 6000 turns vs. synchrotron tune. There is no energy ramping and the synchrotron tune is kept constant. The horizontal solid line shows the emittance for a coasting beam with the same peak intensity.

11.20. According to findings of the previous work, there are two possible mechanism which could explain that limit. One is the structure fourth integer resonance, that is sitting at 11.25. The other is resonances excited by lattice imperfections. Among them, the strongest one is the half-integer resonance at 11.50. By half-integer resonance, emittance growth becomes visible when the maximum tune shift is twice the distance between the resonance and bare tune according to the previous study [2]. Both mechanisms possibly increase the rms emittance because of the superperiodicity of the LEB and lattice imperfections. Similar results in the lattice with 16-fold symmetry implies that the latter is more plausible.

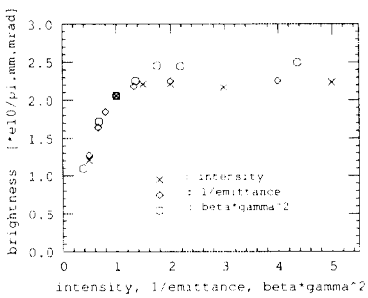

Figure 14. Brightness vs. beam intensity, inverse of initial emittance, and $\beta\gamma^2$.

To confirm that, we looked at the emittance growth in the present LEB and the 16-fold symmetry lattices both without multipole, misalignment, or closed orbit distortion. In these lattice, only the nonlinear elements are chromaticity correction sextupoles. We took 5×10^{10} particles as beam intensity to see the difference if any. Table 1 lists the asymptotic emittance of two different superperiodicity lattices, with and without lattice imperfections. The uncertainty of emittance in the LEB lattice with imperfections indicates the dependence of five seeds. From that table, it is clear that the brightness limit is caused by lattice imperfections in the both lattices. Although the lattice of higher superperiodicity shows the better behavior without lattice imperfections, the difference of the two lattice becomes marginal once the practical imperfections are included.

Table 1
Asymptotic Emittance in the LEB and the 16-fold Symmetry Lattice with and without Lattice Imperfections

	LEB lattice	16-fold lattice
without imperfections	1.2 π mm mrad	0.66 π mm mrad
with imperfections	2.0 $\pm$ 0.2 π mm mrad	2.1 π mm mrad

V. SUMMARY

Using the LEB of the SSC as an example, we explored parameter space and investigated space-charge effects. We found that there is brightness limit. The brightness is bound, no matter how small emittance or intense beam is injected. That limit is independent of the lattice superperiodicity and synchrotron tune. The source of the limit is caused by lattice imperfections of the practical magnitude.

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