Methods of Impedance Calculation*

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Abstract

We present several analytic techniques to calculate the impedance of an obstacle in a beam pipe in a variety of applications.

I. INTRODUCTION

In the present paper we shall review methods of calculating the longitudinal and transverse coupling impedances of an obstacle (e.g. pillbox, hole) in a beam pipe of radius Q for a point charge traveling at ultra-relativistic speeds ($\beta \simeq 1, \gamma \gg 1$). Since the coupling impedance is the frequency domain equivalent of the wakefield written in the time domain, the drive current in the frequency domain has a sinusoidal dependence on z of the form

$$J_z(x,y,z;k) = I_0\delta(x-x_1)\delta(y-y_1)\exp(-jkz), \quad (1)$$

where $k = \omega/c$ and where the time dependence is $\exp(j\omega t)$. Here the point charge travels in the z-direction with constant offset $x = x_1, y = y_1$. The definition of the longitudinal coupling impedance is then

$$\begin{aligned} Z_{\|}(k) &= -\frac{1}{I_0}\int_{-\infty}^{\infty} dz\ E_z(x_1,y_1,z;k)e^{jkz} \\ &= -\frac{1}{|I_0|^2}\int dv\ \vec{E}\cdot\vec{J}^* \end{aligned} \quad (2)$$

where the volume integral is a more general form which will also be used in the transverse impedance. The longitudinal impedance is obtained by setting $x_1 = y_1 = 0$.

We now consider two situations. The first, denoted by the subscript 1, is the lossless pipe and the second, denoted by 2, is the pipe with the obstacle. We then construct

$$|I_0|^2[Z_{\|}^{(2)}(k) + Z_{\|}^{(1)*}(k)] = -\int dv[\vec{E}_2\cdot\vec{J}^* + \vec{E}_1^*\cdot\vec{J}], \quad (3)$$

where $Z_{\|}^{(1)}(k)$ is imaginary. (It actually vanishes in the ultrarelativistic limit.) Using

$$\vec{J} = \nabla\times\vec{H}_{1,2} - j\omega\epsilon\vec{E}_{1,2},\ \nabla\times\vec{E}_{1,2} = -j\omega\mu\vec{H}_{1,2}, \quad (4)$$

Eq. (3) can be converted to a surface integral, leading to[1]

$$|I_0|^2 Z_{\|}(k) = \int_{S_2\neq S_1} dS_2\ \vec{n}_2\cdot\vec{E}_1^*\times\vec{H}_2 = \int_{S_1\neq S_2} dS_1\ \vec{n}_1\cdot\vec{E}_2\times\vec{H}_1^*, \quad (5)$$

where the first integral is over the surface of the obstacle different from the beam pipe and where the second integral is over the surface at $r = a$ which is different from the obstacle. For $x_1 = y_1 = 0$, the solution for $\vec{E}_1$ and $\vec{H}_1$ in the ultrarelativistic limit is

$$E_{1r} = Z_0 H_{1\theta} = \frac{Z_0 I_0}{2\pi r}\exp(-jkz),\ E_{1z} = 0. \quad (6)$$

Thus we need to solve Maxwell's equations for $\vec{E}_2, \vec{H}_2$, with the drive beam given in Eq. (1), and use Eq. (5) to calculate the longitudinal impedance.

The energy loss of the particle traveling past an obstacle can be obtained directly from the real part of the longitudinal coupling impedance. Specifically it is

$$\Delta W = \frac{I_0 I_0^*}{2\pi c}\int_0^{\infty} dk\ Re\ Z_{\|}(k), \quad (7)$$

where we have used $Z_{\|}(-k) = Z_{\|}^*(k)$. Contributions to ΔW can come from wall losses, energy radiation to the outside through the obstacle, and generation by the obstacle of outgoing propagating modes in the pipe.

The transverse coupling impedance can be analyzed similarly. Starting with the axial dipole drive current

$$J_z = I_0\delta(y)[\delta(x-x_1) - \delta(x+x_1)]\exp(-jkz), \quad (8)$$

the transverse impedance can be expressed as the limit for small x_1 of

$$Z_x(k) = -\frac{1}{2kI_0x_1}\int_{-\infty}^{\infty} dz\frac{\partial E_z}{\partial x}e^{jkz}. \quad (9)$$

Writing the derivative with respect to x as the difference for $x = \pm x_1$ divided by $2x_1$, we find

$$\begin{aligned} Z_x(k) = & -\frac{1}{4kI_0x_1^2}\int dz[E_z(x_1,0,z) \\ & - E_z(-x_1,0,z)]e^{jkz}. \end{aligned} \quad (10)$$

*Work supported by the Department of Energy

0-7803-1203-1/93$03.00 © 1993 IEEE

Using the drive current in Eq. (8), the transverse impedance can be written in terms of the same volume integral as before, namely

$$Z_x(k) = -\frac{1}{4kx_1^2|I_0|^2}\int dv\vec{E}\cdot\vec{J}^*. \qquad (11)$$

As before the volume integral can be converted to a surface integral, leading to[2]

$$4kx_1^2|I_0|^2 = \int_{S_2\neq S_1} dS_2\vec{n}_2\cdot\vec{E}_1^*\times\vec{H}_2 = \int_{S_1\neq S_2} dS_1\vec{n}_1\cdot\vec{E}_2\times\vec{H}_1^*. \quad (12)$$

In this case, for small x_1,

$$\vec{E}_1 = -\exp(-jkz)\nabla_\perp\phi_1, \quad Z_0\vec{H}_1 = \hat{z}\times\vec{E}_1, \qquad (13)$$

where

$$\phi_1(r,\theta) = \frac{Z_0I_0}{\pi}x_1\cos\theta\left(\frac{1}{r}-\frac{r}{a^2}\right) \qquad (14)$$

satisfies the boundary condition $\phi_1(a,\theta)=0$ at the beam pipe radius $r=a$. Here $\hat{z}$ is a unit vector in the z-direction.

In the sections that follow we will apply the formulation outlined above to a variety of different problems.

II. NUMERICAL CALCULATION

For an arbitrary obstacle, the fields $\vec{E}_2$ and $\vec{H}_2$ can be written as

$$\vec{E}_2 = \vec{E}_1 + \vec{e}, \ \vec{H}_2 = \vec{H}_1 + \vec{h}, \qquad (15)$$

where the fields $\vec{e}$ and $\vec{h}$ now satisfy Maxwell's equations with no drive current, and the boundary condition along the metallic walls of the pipe and obstacle is

$$\vec{n}_2\times\vec{e} = -\vec{n}_2\times\vec{E}_1. \qquad (16)$$

Furthermore one can consider only a finite section of the beam pipe and apply an outgoing boundary condition to $\vec{e}$ and $\vec{h}$ at both ends of the truncated pipe. In this way a mesh code can be constructed with given k and the solution for $\vec{E}_2$ and $\vec{H}_2$ obtained numerically.

The program SUPERFISH[3] has been adapted to the calculation of the longitudinal coupling impedance for an obstacle of azimuthal symmetry[4]. Clearly one can similarly adapt programs like URMEL and MAFIA[5] to calculate the transverse coupling impedance and the impedances of azimuthally asymmetric obstacles if desired.

The above method appears to be somewhat superior to that used in time domain codes to calculate the wakefields, followed by a Fourier transform to obtain the impedances.

III. RESISTIVE WALL IMPEDANCE

Equation (5) can be used directly to calculate both the longitudinal and transverse resistive wall impedances. Specifically, subscripts 1 and 2 denotes the pipe with infinite and finite wall conductivity respectively. Therefore the term in $\vec{E}_1^*$ vanishes, and we can express E_{2z} in terms of $H_{1\theta}$ and the surface impedance. Specifically

$$E_{2z} \simeq -(1+j)(kZ_0\delta/2)H_{1\theta}, \qquad (17)$$

where $Z_0 = 377$ ohms is the impedance of free space and δ is the skin depth of the wall material at frequency $kc/2\pi$. Thus the impedance per unit length is given by

$$|I_0|^2 Z_\parallel^{RW} = (1+j)(kZ_0\delta/2)\oint ds|H_{1\theta}|^2, \qquad (18)$$

where the line integral is over the circumference of the beam pipe. Using Eq. (6) we find for a length of pipe $2\pi R$

$$\frac{Z_\parallel^{RW}}{Z_0} = \frac{(1+j)k\delta R}{2a} = (1+j)\frac{\delta}{2a}n, \qquad (19)$$

where the second form in terms of $n=kR$, the harmonic of the rotation frequency in a circular accelerator, is the one usually used.

The corresponding analysis for the transverse impedance for a pipe length $2\pi R$, using Eqs. (12)-(14), leads to

$$\frac{Z_\perp^{RW}}{Z_0} = (1+j)\frac{\delta R}{a^3}. \qquad (20)$$

These results, first obtained by Nielsen, Sessler, and Symon[6], have been extended to pipes of rectangular[7, 2] and elliptical[2] cross section.

IV. IMPEDANCE OF HOLES

Equation (5) is also a natural starting point to calculate the impedance of a small hole in a beam pipe. Taking the integral over the inside surface of the beam pipe we have

$$|I_0|^2 Z_\parallel(k) = -\int_{hole} dSE_zH_{1\theta}^* = -\frac{Z_0I_0^*}{2\pi a}\int_{hole} dSE_ze^{jkz}. \quad (21)$$

For holes whose dimensions are small compared to the wavelength, the integral can be expressed in terms of the fields $E_{1r}, H_{1\theta}$ near the hole and the electric polarizability, χ, and magnetic susceptibility, ψ, of the hole. Specifically, we find

$$\frac{Z_\parallel(k)}{Z_0} = \frac{jk}{8\pi^2a^2}(\psi-\chi)_{inside}, \qquad (22)$$

where ψ and χ here are the "inside" susceptibility and polarizability for a wall of finite thickness[8].

It should be noted that the impedance in Eq. (22) is inductive, implying no energy loss by radiation through the hole. This radiation is proportional to the square of the induced dipole moments of the hole, and therefore to ψ^2 and χ^2. The real part of the impedance of a small hole is therefore much smaller than its imaginary part.

The result for the transverse impedance is obtained in an analogous way, using Eqs. (12)-(14), and is

$$\frac{Z_x(k)}{Z_0} = j\frac{\cos^2\theta}{2\pi^2a^4}(\psi-\chi)_{inside}, \qquad (23)$$

where θ is the azimuth of the hole measured from the x-axis.

V. FIELD MATCHING

The impedance of a pill-box has been calculated by many authors using field maching techniques either at the axial locations of the sides of the pill-box[9] or at the inside radius of the beam pipe[10]. In either case, results are obtained by expanding the fields into a complete set of functions in either z or r, matching coefficients in the two regions, truncating the resulting set of matrix equations, and solving for the coefficients by matrix inversion. Results have been given for a variety of parameters (pill-box radius and length) as a function of frequency.

Similar field matching techniques also work for rectangular irises[11] (pill-boxes extending inside the beam pipe radius). Since the driving current on axis is proportional to $\exp(-jkz) = \cos kz - j \sin kz$, the problem is simplified by obtaining results for an even driving current ($\cos kz$) and an odd driving current ($-j \sin kz$) separately. In each case the fields are expanded with appropriate z symmetry into a set of Bessel functions in both the pipe regions and the iris region. One set of coefficients can be eliminated by matching the fields at the axial location of the end of the iris, and the solution is then obtained by truncating and inverting the resulting matrix equations. (Our experience is that the numerical work is more convergent if the final matrix is obtained in terms of the coefficients in the iris region.)

VI. INTEGRAL EQUATION METHOD

We here consider an azimuthally symmetric obstacle which does not extend inside the beam pipe radius, and represent the field E_z for $r \leq a$ as

$$E_z(r,z;k) = \int_{-\infty}^{\infty} dq\ e^{-jqz} A(q) \frac{J_0(Kr)}{J_0(Ka)}, \tag{24}$$

where $K^2 \equiv k^2 - q^2$ and where the contour in the q plane goes below any poles on the negative real axis and above any poles on the positive real axis[10]. This choice of contour guarantees that the obstacle will only create outgoing waves in the beam pipe. The driving fields are those in Eq. (1) for $x_1 = y_1 = 0$. It is easy to show that the longitudinal impedance defined in Eq. (2) becomes

$$Z_{\|}(k) = -\frac{2\pi}{I_0} A(k) = -\frac{1}{I_0} \int dz E_z(a,z;k) e^{jkz}, \tag{25}$$

where the second form is obtained from the Fourier transform of Eq. (24) at $r = a$, and where the integral over z extends only over the obstacle (pill-box) region. This equation corresponds directly to the more general result in the second form of Eq. (5).

The azimuthal magnetic field at the pipe radius can be written as

$$Z_0 H_\theta = \frac{Z_0 I_0 e^{-jkz}}{2\pi a} + \int_{-\infty}^{\infty} dq e^{-jqz} \frac{jkaJ_1(Ka)}{KaJ_0(Ka)} A(q). \tag{26}$$

Expanding the ratio of Bessel functions in terms of the residues at the zeros of $J_0(Ka)$, we can write

$$\frac{J_1(Ka)}{KaJ_0(Ka)} = \frac{2}{a^2} \sum_{s=1}^{\infty} \frac{1}{q^2 - b_s^2/a^2}, \tag{27}$$

where $J_0(p_s) = 0$ and $b_s^2 = k^2a^2 - p_s^2$. Writing

$$A(q) = \frac{1}{2\pi} \int dz' e^{jqz'} f(z'), \tag{28}$$

where $f(z') = E_z(a, z'; k)$ is the axial electric field in the opening, we can perform the integral over q by properly closing the contour for $z > z'$ and $z < z'$, and obtain

$$Z_0 H_\theta(a,z;k) = \frac{Z_0 I_0}{2\pi a} e^{-jkz} - \frac{jka}{2\pi} \int dz' f(z') K_p(z,z'), \tag{29}$$

where the pipe kernel is

$$K_p(z,z') = \frac{2\pi j}{a} \sum_{s=1}^{\infty} \frac{e^{-jb_s|z-z'|/a}}{b_s}. \tag{30}$$

When b_s^2 is negative, $b_s = -j\beta_s$, with $\beta_s = (p_s^2 - k^2a^2)^{1/2}$.

We must now write the fields inside the obstacle in terms of E_z at $r = a$. This can be done by expanding the fields in the cavity-like obstacle (shaped like a torus) into a complete set of cavity modes. In this way we find

$$Z_0 H_\theta(a,z;k) = \frac{jka}{2\pi} \int dz' f(z') K_c(z,z'), \tag{31}$$

where the cavity kernel is

$$K_c(z,z') = 4\pi^2 \sum_\ell \frac{h_\ell(z) h_\ell(z')}{k^2 - k_\ell^2}. \tag{32}$$

Here $h_\ell(z) = h_\theta^{(\ell)}(a,z)$ is the normalized azimuthal magnetic field in the ℓ^{th} cavity mode with frequency $k_\ell c/2\pi$. Equating the magnetic field in the opening, we then obtain

$$\int dz' F(z')[K_p(z,z') + K_c(z,z')] = je^{-jkz}, \tag{33}$$

where $F(z) = -ka^2 f(z)/Z_0 I_0$ and

$$Z_{\|}(k)/Z_0 = (1/ka^2) \int dz F(z) e^{jkz}. \tag{34}$$

We therefore need to solve the integral equation [Eq. (33)] for $F(z)$ and obtain $Z_{\|}(k)$ from Eq. (34).

VII. IMPEDANCE OF A SMALL OBSTACLE

For an obstacle of outer radius b extending from $z = 0$ to $z = g$, with $kg \ll 1, k(b-a) \ll 1$, we can obtain approximate values for $K_p(z,z')$ and $K_c(z,z')$. Specifically, the pipe kernel is

$$K_p \simeq \frac{2\pi j}{a} \left[\sum_{s=1}^{S} \frac{1}{b_s} + j \sum_{s=S+1}^{S_{max}} \frac{1}{\beta_s} \right], \tag{35}$$

where S is the largest value of s for which $p_s \leq ka$, and where $S_{max} \sim a/g$ is a logarithmic cutoff needed in the second sum. The cavity kernel is dominated by the mode with $k_\ell = 0$, for which

$$h_0(z) \simeq [2\pi ga(b-a)]^{-1/2} \tag{36}$$

and is

$$K_c \simeq \frac{2\pi}{ga(b-a)}. \tag{37}$$

Using these values of K_p and K_c, we can obtain $\int dz' F(z')$ from Eq. (33) and are led to the following expression for the *admittance*

$$Z_0 Y_{\|}(k) = 2\pi ka \left[-\frac{j}{k^2 g(b-a)} + \sum_{s=1}^{\infty} \frac{e^{-jb_s g/a}}{b_s} + j\frac{2\ell n 2}{\pi} \right], \tag{38}$$

where the exponential cut-off factor and the third term come from a more careful treatment[12] of the divergent term in Eq. (35). Numerical simulations agree very closely with the result in Eq. (38) for b = 1.1 a, g = 0.05 a.[12] Moreover, the corresponding results for the impedance agree very well with the series calculation of Henke.[10]

The result in Eq. (38) is dominated at low frequency by the first term, which is inductive. Clearly the second and third terms provide both an increasing capacitive term and resistive term as the frequency increases. In fact, the familiar broad resonance occurs when the inductive and capacitive contributions cancel.

Another interesting feature of Eq. (38) is its simplicity when expressed as an admittance. In fact the real part is independent of all features of the pill-box for $g \ll a$. It is not hard to show that this term corresponds to the energy which is lost as the pill-box generates outgoing propagating modes in the pipe. Apparently the reactive part arises from the evanescent pipe modes generated by the pill-box.

VIII. IMPEDANCE AT HIGH FREQUENCY

The high frequency behavior of the impedance has been of concern since Lawson's diffraction calculation[13] suggested a $k^{-1/2}$ behavior which, according to Eq. (7) implied an infinite energy loss. This $k^{-1/2}$ behavior (which does not violate energy conservation when we have an ultrarelativisitic particle of infinite energy) was confirmed by others[14, 15]. The integral equation of Section VI is a convenient starting point for this calculation.

We write Eqs. (33), (34) for a pill-box as

$$\int_0^g dz'\ G(z')[\hat{K}_p(z,z') + \hat{K}_c(z,z')] = 2\pi j/a \tag{39}$$

with $F(z) = (a/2\pi)e^{-jkz}G(z)$, and have

$$Z_{\|}(k)/Z_0 = (1/2\pi ka)\int_0^g dz\ G(z). \tag{40}$$

Here

$$\hat{K}_{p,c}(z,z') = e^{jk(z-z')}K_{p,c}(z,z'). \tag{41}$$

Clearly the impedance arises from the smooth part of $G(z)$, which itself will come from the smooth part of the kernel in Eq. (39). Writing

$$\hat{K}_p(z,z') = 2\pi j/a \sum_{s=1}^{\infty} \exp(j\psi_s)/(k^2a^2 - p_s^2)^{1/2}, \tag{42}$$

where

$$\psi_s = k(z-z') - |z-z'|(k^2 - p_s^2/a^2)^{1/2}, \tag{43}$$

we see that there are rapid oscillations everywhere except near $p_s \ll ka$ for $z > z'$. Expanding $(k^2a^2 - p_s^2)^{1/2}$ for small p_s/ka, and keeping only the lowest order non-vanishing term in the exponent, we obtain for the average of the smoothed pipe kernel

$$< \hat{K}_p(z,z') > \simeq \left\{ \begin{array}{ll} 0\ , & z' > z \\ \frac{j-1}{a}\left[\frac{\pi}{k(z-z')}\right]^{1/2} , & z' < z \end{array} \right\}, \tag{44}$$

where we have converted the sum over s to an integral.

The evaluation of the cavity kernel for large ka depends on the cavity geometry, but in the form in Eq. (32), the sum over modes can be approximated by an integral over mode number. This has been done for a pillbox, as well as for several obstacles of triangular cross section and, the results suprisingly depend only on $(z - z')$. In fact the cavity kernel contribution turns out to be exactly the same as that in Eq. (44) for the pipe kernel. Thus the integral equation reduces to

$$\int_0^z \frac{dz' G(z')}{\sqrt{z-z'}} \simeq \frac{(1-j)\sqrt{\pi k}}{2}, \tag{45}$$

whose solution is

$$G(z) \simeq \frac{1-j}{2a}\sqrt{\frac{k}{\pi z}}, \tag{46}$$

leading to the impedance

$$\frac{Z_{\|}(k)}{Z_0} \simeq \frac{1-j}{2\pi a}\sqrt{\frac{g}{\pi k}}. \tag{47}$$

Numerical results for the impedance are consistent with the $k^{-1/2}$ average behavior of the impedance, but show a persistent oscillation with frequency as well, suggesting some sort of resonant field behavior within the pillbox.

IX. IMPEDANCE OF MANY OBSTACLES AT HIGH FREQUENCY

We now apply Eq. (39) to a beam pipe containing a large number of obstacles, and assume that they are all identical and separated from each other (center to center) by a constant distance L. Specifically we write the coupled system of integral equations

$$\sum_m \int dz'_m\ G(z'_m)[\hat{K}_p(z_n, z'_m) + \delta_{mn}\hat{K}_c(z_n, z'_m)] = j, \tag{48}$$

where we recognize that only the mth cavity contributes to the cavity kernel for the purpose of matching the magnetic field at the mth cavity.

If we use the asymptotic values for K_p and K_c obtained in the last section and approximate $K_p(z_n, z'_m)$ for off-diagonal ($n \neq m$) terms as its value when z_n, z'_m correspond to the center of the relevant gaps, we can solve Eq. (48) and obtain the following expression for the *admittance* per cavity[16]

$$NZ_0 Y_{||}(k) \simeq F_0(k) + \alpha\sqrt{N-1}\tan^{-1}(\alpha/2\sqrt{N}), \quad (49)$$

where $F_0(k)/Z_0$ is the admittance of a single cavity and

$$\alpha = (1+j)a\sqrt{\pi k/L}. \quad (50)$$

Once again we have a term independent of the cavity parameters $(g, b-a)$ which is added to the admittance.

In the limit of large k with finite N, we find

$$NZ_0 Y_{||}(k) \simeq (1+j)\pi a\sqrt{\pi k/g}[1+\sqrt{gN/4L}] \quad (51)$$

suggesting that the impedance is proportional to $N^{1/2}$ (rather than N) for $Ng \gg L$. This shadowing effect was first suggested by Palmer[17].

If we take the limit for large N with finite k, we find

$$NZ_0 Y_{||}(k) \simeq F_0(k) + j\pi a^2 k/L. \quad (52)$$

which is the result for a periodic structure. For large k the imaginary second term dominates. Using the single pillbox impedance in Eq. (47) we obtain an approximate result which shows that real part of the impedance goes as $k^{-3/2}$, a result also obtained by others[18].

The result in Eq. (52) can also be shown to apply to the case of a small obstacle for the parameter range $ka \gg 1, kL \gg 1$, as long as $kg \ll 1, k(b-a) \ll 1$. Specifically we use the result in Eq. (38) for $F_0(k)$, the admittance of a single obstacle. We also believe, although it has not yet been proved, that the same result holds for a periodic array of holes distributed uniformly in azimuth at axial positions separated by L. In this case $F_0(k)$ in Eq. (52) is to be taken as the reciprocal of Eq. (22). Equation (52) is then expected to be valid as long as the wavelength is small compared with a and L, and large compared with the hole dimensions. In all likelihood, it would be valid for wavelengths comparable with or smaller than the hole dimensions if we used a single hole impedance valid in this parameter range.

X. SUMMARY

We have outlined several alternate methods of calculating the impedance of an obstacle (pillbox, hole) in a beam pipe and illustrated the techniques in several applications. The selection is naturally guided by personal taste. Nevertheless there are other techniques often used which offer comparable insights and results.

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