Beam-Wave Interaction in a Quasi-Periodic Structure *

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Abstract

An analytic method to analyze a quasi-periodic disk loaded waveguide is presented. We rely on Cauchy residue theorem to formulate the transmission and reflection from a system composed of radial arms and grooves provided that the inner radius is always the same. The quasi-analytical approach is not limitted to slow variations of the geometry.

I. INTRODUCTION

The constraint imposed by the NLC requirements on the output spectrum of an RF source limits the input section of any system to a very good frequency selective device. From this perspective the klystron cavity or a combination of a cavity with a magnetic field as in the case of the Choppertron(1) or an FEL(2), are the natural candidates for the *input section* of any RF system. The *main section* can be a set of isolated cavities as in a klystron, a traveling wave (TW) section or a combination of the two but the breakdown problem will force us to use a TW structure as an *output section* with one or more extraction ports(3−4). A high power traveling wave structure is conceived as a section of a periodic disk loaded structure and its electromagnetic characteristics are determined as if the system was infinitely long. Practically these are a set of cavities which are *coupled* through the disk aperture. At the other extreme, the klystron is a set of a few *isolated* cavities. In the former case the beam interacts with a wave continuously, whereas in the klystron the beam interacts with the field in the close vicinity of the cavity. The interaction in a uniform periodic structure (TWA) or in a few uncoupled cavities (klystron) is relatively well understood. But we lack analytical or even quasi-analytical tools to accurately investigate the interaction in transition region - which is exactly what is required for construction of an adequate output section. For this purpose we have developed an analytical method to investigate the beam-wave interaction in a quasi-periodic structure. The method relies on an arbitrary number of pill-box like cavities of any dimension and an arbitrary number of radial arms. The only constraint is that the radius of the coupling pipe has to be always the same.

II. BOUNDARY CONDITION PROBLEM

The system described above is illustrated in Fig.1. For the purpose of this presentation we shall describe only the system without the beam and we shall indicate where the differences occur when a beam is present. Unlike in a periodic structure where the field in the inner cylinder ($0 < r < R_{int}$) can be represented by Floquet series we have to consider the entire spatial spectrum of waves therefore the magnetic vector potential reads

$$A_z(r,z;\omega) = \int_{-\infty}^{\infty} dk A(k) I_0(\Gamma r) e^{-jkz} \qquad (1)$$

where $\Gamma^2 = k^2 - \omega^2/c^2$, $I_0(x)$ is the modified Bessel function of the first kind and the system is assumed to be in steady state ($e^{j\omega t}$). In the arms or grooves the electromagnetic field should be represented by a superposition of modes which satisfy the boundary conditions on the metallic walls. In principle an infinite number of such modes is required. Our experience indicates that as long as the vacuum wavelength is about 5 times larger than the groove/arm width the first mode (TEM) is sufficient for most practical purposes.

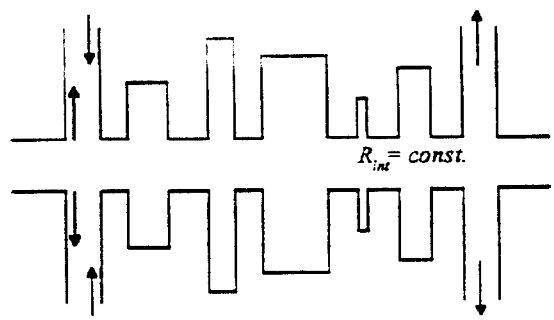

Fig. 1 The schematics of the quasi-periodic system.

This assumption is by no means critical for the present analysis and the arguments are very similar when a larger number of modes is required however we use it since it makes the presentation much simpler. Within the framework of this approximation we can write for the magnetic vector potential in the input arm

$$A_z(r,z;\omega) = A_{in} H_0^{(1)}(\frac{\omega}{c}r) + D_1 H_0^{(2)}(\frac{\omega}{c}r) \qquad (2)$$

where $H_0^{(1)}(x)$ and $H_0^{(2)}(x)$ are the zero order Hankel function of the first and second kind respectively; A_{in} represents the amplitude of the incoming wave and D_1 is the amplitude of the reflected wave which is yet to be determined. In the n^{th} $(1 < n < N)$ groove we have

$$A_z^n(r,z;\omega) = D_n T_{0,n}(\frac{\omega}{c}r) \ , \qquad (3)$$

D_n is the amplitude of the magnetic vector potential, $T_{0,n}(\frac{\omega}{c}r) = J_0(\frac{\omega}{c}r)Y_0(\frac{\omega}{c}R_{ext,n}) - Y_0(\frac{\omega}{c}r)J_0(\frac{\omega}{c}R_{ext,n})$ and

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$R_{ext,n}$ is the external radius of the n^{th} groove; later we shall also use the function $T_{1,n}(\frac{\omega}{c}r) = J_1(\frac{\omega}{c}r)Y_0(\frac{\omega}{c}R_{ext,n}) - Y_1(\frac{\omega}{c}r)J_0(\frac{\omega}{c}R_{ext,n})$. Finally in the output arm

$$A_z(r,z;\omega) = D_N H_0^{(2)}(\frac{\omega}{c}r) \ . \quad (4)$$

In order to determine the various amplitudes we next impose the boundary conditions in a way which is similar to what is being done in the case of a periodic structure. The main difference is that we no longer can look at what happens in a single cell to characterize the entire system but we have to consider each individual region. The boundary condition problem is formulated next in terms of the amplitudes in the grooves and arms in a matrix form:

$$\sum_{m=1}^{N} \tau_{n,m} D_m = S_n A_{in} \quad (5)$$

where

$$\tau_{n,m} = \psi_{1,n}\delta_{n,m} - \psi_{0,m}\chi_{n,m} \ ,$$

$$S_n = -H_1^{(1)}(\alpha)\delta_{n,1} + H_0^{(1)}(\alpha)\chi_{n,1} \quad (6)$$

$$L_n(k) = \frac{1}{d_n}\int_{z_n-d_n/2}^{z_n+d_n/2} dz e^{jkz} \ , \quad (7)$$

$$\psi_{\nu,n} = \begin{cases} H_\nu^{(2)}(\alpha) & n=1 \quad \text{or} \quad n=N \\ T_{\nu,n}(\alpha) & 1<n<N \end{cases} \quad (8)$$

and

$$\chi_{n,m} = \frac{d_m\alpha}{2\pi}\int_{-\infty}^{\infty} dk \frac{I_1(\Delta)}{\Delta I_0(\Delta)} L_n^*(k)L_m(k) \ . \quad (9)$$

With $\nu = 0,1$; z_n is the location of the center of the n^{th} groove or arm and d_n is the corresponding width. Finally $\alpha = \frac{\omega}{c}R_{int}$ and $\Delta = \Gamma R_{int}$.

III. CAUCHY RESIDUE THEOREM

The next step is to evaluate the integral which defines the matrix χ in terms of analytic functions. This is done by using the Cauchy residue theorem. First we substitute the explicit expressions for $L_n(k)$ from Eq.(7). Second, we examine the integrand we observe that there are an infinite set of poles which correspond to $I_0(\Delta) = 0$. Bearing in mind the relation between the modified Bessel function and the regular one($J_0(x)$) we realize that the condition above is satisfied for $k^2 = (\frac{\omega}{c})^2 - \frac{p_s^2}{R_{int}^2}$; here p_s are all the zeros of the zero order Bessel function of the first kind i.e. $J_0(p_s) \equiv 0$. According to the Cauchy's theorem the contribution to the integral will come from the poles of the integrand hence the integral in Eq.(9) reads

$$\int_{-\infty}^{\infty} dk \frac{I_1(\Delta)e^{jk(x_1-x_2)}}{\Delta I_0(\Delta)} = \frac{2}{R_{int}^2}\sum_{s=1}^{\infty}\int_{-\infty}^{\infty} dk \frac{e^{jk(x_1-x_2)}}{k^2+\Gamma_s^2} \quad (10)$$

where $\Gamma_s^2 = (p_s/R_{int})^2 - (\omega/c)^2$. The last integral is the Green function of a uniform waveguide and is easily evaluated as $G(x_1|x_2) = \frac{\pi}{\Gamma_s}e^{-\Gamma_s|x_1-x_2|}$. This result permits us to express the matrix χ in terms of analytic functions: for $n = m$

$$\chi_{n,n} = \frac{\alpha}{R_{int}^2}\sum_{s=1}^{\infty}\frac{2}{\Gamma_s^2}\left[1 - e^{-\Theta_{s,n}} sinhc(\Theta_{s,n})\right] \quad (11)$$

and

$$\chi_{n,m} = \frac{\alpha}{R_{int}^2}\sum_{s=1}^{\infty}\frac{d_m}{\Gamma_s}e^{-\Gamma_s|z_n-z_m|} sinhc(\Theta_{s,n}) sinhc(\Theta_{s,m}) \quad (12)$$

otherwise. In this expression $sinhc(x) = sinh(x)/x$ and $\Theta_{s,n} = \Gamma_s d_n/2$. The electromagnetic problem has been now simplified to inversion of a matrix whose components are analytic functions. The transmission pattern of the structure fits well the predictions of the dispersion relation of an infinite structure.

In the presence of the beam, using the fluid model, the denominator in Eq.(9) is a more complex function than the $I_0(\Delta)$, which in addition to the electromagnetic modes it includes the space charge modes. Once the poles are identified the only aspect which remains to be consider is the fact that the space charge waves, unlike the electromagnetic waves, always propagate along the beam.

IV. DISCUSSION

Next we shall illustrate the potential of this method. And the first goal is to determine what should be the location of the arms for adequately feed power into a 9 cell narrow band structure ($R_{ext} = 14.2mm$, $R_{int} = 6.2mm$, $L = 12mm$ and $d = 6mm$). Fig. 2 illustrates the geometry of the narrow band structure with 9 cavities and two arms. In the first case the arms are $6mm$ from the first cells and we observe that the average transmission coefficient is $-20dB$. When the drift region was shortened to $1mm$ the transmission coefficient increases dramatically to an average value of $-3dB$ (this result was qualitatively observed in experiment).

Let us now assume for a moment that we have matched the cold system fo a given frequency i.e the gain in dB, $10log(|D_N|^2 d_N/|A_{in}|^2 d_1)$ is zero. We know that in the narrow band structure very high gradients develop in the interaction process - in particular in the last couple of cells.

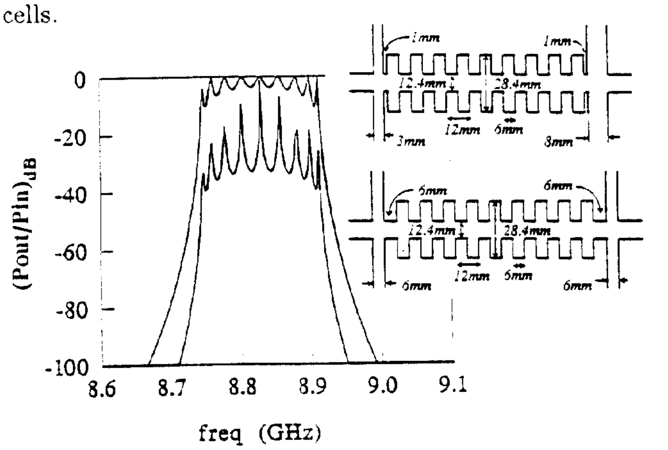

Fig. 2: The transmission coefficient for the two structures shown in the right corner.

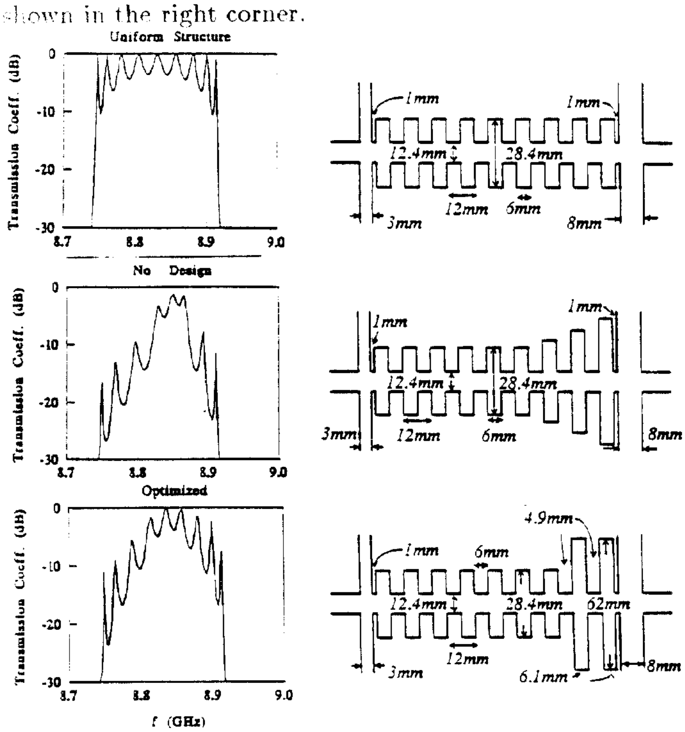

Fig. 3: Transmission coefficient of three structures.

In order to avoid rf breakdown we want to increase the volume where the electromagnetic energy is stored and by that we lower the energy density and consequently the field. We started with a "linear" tapering of the external radius (of the last three cells increases linearly). We have varied the width of these cells and their separation in a wide range of parameters to bring the transmission coefficient to $0dB$ at given frequency and the best we could achieve was $-3dB$ which is not acceptable; see Fig.3. At this stage we returned to the initial geometry only that we have doubled the external radius of the last two cells. These cavities have two (rather than one) resonant frequencies, one of which, is close to that of a cavity in the uniform structure. After some fine tuning we obtained the transmission which is optimized to the required frequency. Fig. 4 illustrates the transmission characteristic of system driven by $1MV$, $1kA$ beam whose radius is $3mm$. The two sections are separated by a $3cm$ long drift region and thus electromagneticaly they are completely isolated. In the left we present the geometry and in the right the transmission coefficient. The location of the drift tube is critical as illustrated. The periodic structure was designed with a phase advance of $2\pi/3$ and we observed that when varying the location of the drift region, each three cells the picture repeats.

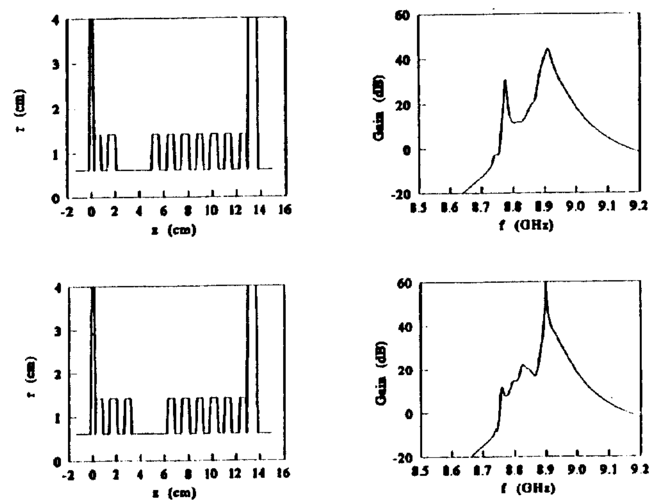

Fig. 4: Transmission coefficient of an active system. The drift region is $3cm$ long.

In conclusion, we presented a method to calculate the electromagnetic characteristics of a quasi-periodic structure which consists of radial arms and a set of coupled disk loaded cells. The main constraint in this method is that the internal radius has to be kept constant. In the simplified version presented here we used only a single mode to represent the field in the grooves and arms this can be extended to a larger number of modes. However this is necessary only for these grooves or arms whose width is more than 1/5 the vacuum wavelength; accordingly the order of the matrix τ increases.

V. REFERENCES

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