Stability of Gyrotwistrons

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Abstract

The stability of gyrotwistrons against the excitation of parasitic modes is studied. A method is described which can be used to compute the nonlinear saturation amplitude of the parasites when the operating mode is present. An example relevant to driving particle accelerators (near 10 GHz and 500 kV) is discussed.

I. INTRODUCTION

One possible source for driving particle accelerators is the relativistic gyrotwistron,[1] in which the output cavity of a standard gyroklystron amplifier is replaced by a traveling wave section. The primary reason to consider the gyrotwistron over the more conventional gyroklystron amplifier is efficiency: because the interaction region is relatively long, the magnetic field can be tapered to keep the particles in resonance, thereby extracting a large fraction of the available energy. On the other hand, because of the long interaction region gyrotwistrons are highly susceptible to instabilities. In this paper we consider a relativistic gyrotwistron operating in the TE_{01} mode and study its stability with respect to the parasitic TE_{11} mode. Without the operating mode present the TE_{11} mode is unstable when the reflection coefficient at the output cross-section goes above .07%. The question we consider here is: what effect does the operating mode have on the parasite? In the next section, we briefly outline the equations we will use. Section III contains our results for a specific case (425 kV, 160 A, 9.85 GHz, fundamental TE_{01}), and Sec. IV contains our summary and conclusions.

II. EQUATIONS

The equations we use are the Lorentz force equations for the particles and Maxwell's equations for the wave. The electron motion is studied under the following simplifying assumptions:

1. We neglect space charge.
2. We neglect guiding center drift caused by the wave (although we include drift caused by the tapered external magnetic field).
3. We averaged the equations over fast gyrophase.
4. We assume that $k_\perp r_L < 1$ ($k_\perp$ is the perpendicular wave number, r_L is the Larmor radius).

To simplify Maxwell's equations, we average over a wave period as well as the fast gyrophase, and assume that only two modes are present: the operating mode and one parasite. We consider the case where these two modes are incommensurate; i.e. $s_1/s_2 \neq \omega_1/\omega_2$ where s is the harmonic number, ω is the frequency, and the subscript "1" and "2" refer to the operating mode and parasite, respectively. And finally, we work in the steady state regime, so the wave amplitudes are fixed in time (but not space).

What we wish to find in this analysis is the amplitude of the parasite as a function of its reflection coefficient at the output cross-section. In fact, there is generally more than one amplitude for each reflection coefficient, and not all of them are stable. The stable solutions are the ones for which the reflection coefficient increases as the amplitude of the parasite increases. Thus, we need to compute the reflection coefficient of the parasite versus amplitude over a range of amplitudes. This computation must be done at frequencies ranging from cutoff to a large enough frequency that the beam and wave lose resonance.

III. RESULTS

We consider a gyrotwistron relevant to the University of Maryland experiment.[2] This experiment can operate over a range of parameters; for definiteness we consider fundamental operation in the TE_{01} mode at a frequency of 9.85 GHz, a voltage of 425 kV, a current of 160 A, a pitch ratio ($v_{\perp 0}/v_{z0}$) of 1.2 and an RMS axial velocity spread of 6%. These are rather easily attainable values. The output waveguide has an initial radius of 1.825 cm and a 0.5 degree taper; the taper helps stabilize the system by ensuring that no mode can remain near cutoff for too long. Instead of an initial cavity, we start the particles uniformly distributed in phase but with a kick in perpendicular momentum, and then let the particle

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bunch ballistically in the drift section. As a result of the kick, at the entrance to the output waveguide the distribution in phase ψ and normalized perpendicular momentum $u_\perp$ are given approximately by

$$\begin{aligned} \psi &= \psi_0 + \tfrac{q}{K_0}(u_{\perp 0} - 1) + q\cos(\psi_0) \\ u_\perp &= u_{\perp 0} - K_0\cos(\phi_0) \end{aligned}$$

where ψ_0 is uniformly distributed in phase between 0 and 2π, $u_{\perp 0}$ has a top hat distribution in perpendicular momentum, q is the standard bunching parameter, and K_0 is proportional to the kick the particles receive in the first cavity.

We first consider the operating mode. Because both the forward and backward waves are excited, there are boundary conditions at both ends of the circuit: the particle phases and momenta are specified at the beginning of the output waveguide according to the above distribution, while the wave amplitudes are specified at both ends (the initial forward wave amplitude and the final backward amplitude are zero). Thus, a hunt and shoot method must be used to find the correct amplitude and phase of the backward wave at the beginning of the circuit. We chose a relatively small bunching parameter, $q \approx 0.1$. This made it easy to find an equilibrium satisfying the boundary conditions, although a larger bunching parameter would probably have helped suppress the parasites. In future work we will consider such larger values. The magnetic field had a fairly strong taper; it started at 5.6 kG, and then fell to about half that value in 20 cm. The maximum efficiency was 36%, which occured at about 18 cm, but the final efficiency was 28%. We integrate past where the efficiency is a maximum because we use the actual experimental coils to specify the magnetic field, so there are constraints on its profile. We should be able to increase the final efficiency by abruptly decreasing the magnetic field at 18 cm, and we are working in this direction.

Analysis of the parasites proceeds as described above. Typically there are a large number of modes that are capable of interacting with the electron beam at the specified magnetic field: 10 to 20 if only the first and second harmonics are considered; more if the third and higher harmonics are also included. To demonstrate our procedure, here we will consider only the parasitic TE_{11} mode operating at the fundamental cyclotron frequency ($s_2 = 1$). Without the operating mode present, this mode is unstable at 6.4 GHz if the reflectivity at the output cross-section is above .07%. With the operating mode present the TE_{11} mode is still linearly unstable; now near 5.2 instead of 6.4 GHz. However, it has a relatively low saturation amplitude: for reflection coefficients up to 50%, the TE_{11} mode saturates at about 1% of the power of the operating mode. However, even at these low levels it reduces the operating mode efficiency: at 50% reflection, the maximum efficiency falls from 36% to 23%, and the final efficiency falls from 28% to 20%. At lower reflection coefficient, the falloff is similar: at 5% reflection, the maximum efficiency is 26% and the final efficiency is 23%. The parasite was also unstable at other frequencies besides 5.2 GHz, but at those frequencies it had a much lower saturation amplitude.

IV. SUMMARY

For the modes we considered (TE_{01} for the operating mode and TE_{11} for the parasite), we found that with the operating mode present, the power in the parasite saturated at a level about 100 times lower than that of the operating mode. However, significant degradation in the efficiency of the operating mode was observed; its final efficiency dropped from 28% to about 20% at 50% reflection. We suspect that we can improve this result by using a larger bunching parameter so that the operating mode saturates more rapidly, and by decreasing the magnetic field faster to decrease the operating regime of the parasite. One thing we did not consider was the simultaneous existence of parasites over a range of frequencies. Since the unstable spectrum was fairly narrow, we suspect that it is a good approximation to assume that the parasite is excited at a single frequency; nevertheless, this issue deserves further investigation.

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V. REFERENCES

1. G.S. Nusinovich and H. Li, "Theory of the Relativistic Gyrotwistron," *Phys. Fluids B*, **4**, 1058 (1992).

2. W. Lawson, B. Hogan, P.E. Latham, M.E. Read, V.L. Granatstein, M. Reiser, and C.D. Striffler, "Efficient Operation of a High Power X-Band Gyroklystron," *Phys. Rev. Lett.*, **67**, 520 (1991).