A Plasma Lens and Accelerator Based Upon Magnetically Driven Charge Separation

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Abstract

An electrostatic lens is described in which a pulsed magnetic field is used to create a radial charge separation in a cylindrical plasma. The radial field can be made strongly focusing for a positively or negatively charged beam passing along the axis. On a longer time scale, the ions initially in the plasma are accelerated to the axis with energies of up to 0.5 MeV. The device may be useful as a generator of neutrons as well as a lens for charged particle beams. Pulsed radial electric fields exceeding 100 MV/m should be possible from modest magnetic compression fields.

I. INTRODUCTION

Several collective accelerators have been described and tested in which ions are trapped in the space-charge electric field of a non-neutral rotating electron ring which is subsequently accelerated by a magnetic gradient[1]. The advantage of such schemes is that the space-charge electric field of the electrons can be stronger than accelerating fields created by other means. In this work, we describe a lens based upon a cylinder of charge-neutral plasma in which there is a radial, space-charge electric field due to a pulsed magnetic field acting unequally on the electrons and ions (Fig. 1). We show that potentials of the order of 0.5 MV can be created. The device is similar to a collective lens which has been experimentally demonstrated[2,3,4,5].

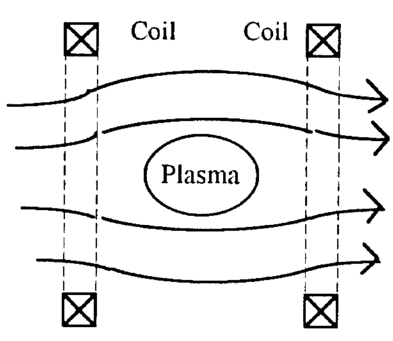

Fig. 1. Schematic diagram of the plasma accelerator. A rapidly rising magnetic field pushes the electrons and ions toward the axis. The difference in masses causes a radial electric field which retards the inward motion of the electrons and which accelerates the ions.

II. EQUATIONS OF MOTION

We assume a cylindrical plasma having a length much longer than its radius so that axial motion can be neglected. We also assume that the plasma is collisionless and that the canonical angular momenta of the particles are conserved. The angular momentum P_θ for the electrons is

$$P_\theta = qrA_\theta + \gamma m_e r^2\dot{\theta} = -er_0 A_{\theta,0} \ , \qquad [1]$$

where q = -e is the electron charge, r is the radius, A_θ is the time dependent vector potential, $A_{\theta,0}$ is the initial vector potential which determines the conserved value of P_θ, γ is the relativistic factor, m_e is the rest mass of the electron, and θ is the azimuthal coordinate. The subscript zero denotes the value at the initial time. For a uniform field the vector potential can be written

$$A_\theta = \Phi/2\pi r = rB_z/2 \ , \qquad [2]$$

where Φ is the flux enclosed at radius r and B_z is the time dependent axial field. The angular velocity is then determined by the change in the vector potential:

$$\dot{\theta} = \frac{-q}{2\gamma m_e}(B_z - \frac{r_0^2}{r^2}B_{z,0}) = \Omega_{L,e} - \frac{r_0^2}{\gamma r^2}\Omega_{L,e,0} \ , \qquad [3]$$

where

$$\Omega_{L,e} = |q|B_z/2\gamma m_e \qquad [4]$$

is the Larmor frequency (half the cyclotron frequency), $B_{z,0}$ is the initial field which determines the initial canonical angular momentum, and $\Omega_{L,e,0}$ is the initial Larmor frequency for which $\gamma = 1$.

The radial equation of motion is

$$\frac{d}{dt}(\gamma m_e \dot{r}) - \gamma m_e r\dot{\theta}^2 = -e(E_r + r\dot{\theta}B_z) \ , \qquad [5]$$

where E_r is the space-charge radial electric field. The angular velocity is known from [3] thus [5] becomes

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$$\frac{d}{dt}(\gamma m_e \dot{r}) + \gamma m_e r\left[\Omega_{L,e}^2 - \left(\frac{r_0^2 \Omega_{L,e,0}}{\gamma r^2}\right)^2\right] = -eE_r. \qquad [6]$$

The radial equation of motion for ions is found in the same way with the simplification that the ions can be treated nonrelativistically:

$$m_i \ddot{r} + m_i r\left[\Omega_{L,i}^2 - \left(\frac{r_0^2 \Omega_{L,i,0}}{r^2}\right)^2\right] = eE_r\ . \qquad [7]$$

The electric field is found from Poisson's equation. If we assume a sufficiently large plasma density, then a small fractional difference in charge density can provide the electric field. This quasineutrality assumption allows us to set the electron and ion densities equal. The continuity equation then requires that the radial velocities be equal. The equations of motion can then be summed to yield a single equation

$$\mathrm{m_i} \ddot{r} = -\gamma m_e r\left(\Omega_{L,e}^2 - \frac{r_0^4 \Omega_{L,e,0}^2}{\gamma r^4}\right), \qquad [8]$$

where we have used that $\gamma m_e << m_i$. This can be further simplified to

$$\ddot{r} = -r\left(\Omega_{L,H}^2 - \frac{r_0^4 \Omega_{L,H,0}^2}{\gamma r^4}\right), \qquad [9]$$

where we have defined a hybrid Larmor frequency

$$\Omega_{L,H}^2 = \frac{e^2 B_z^2}{4\gamma m_e m_i} = \Omega_{L,e}\Omega_{L,i} \qquad [10]$$

and an initial hybrid frequency $\Omega_{L,H,0}$. The electric field is

$$eE_r = -\gamma m_e r\left(\Omega_{L,e}^2 - \frac{r_0^4 \Omega_{L,e,0}^2}{\gamma^2 r^4}\right) - m_e \dot{r}\dot{\gamma}\ . \qquad [11]$$

The last term in the above equation can be ignored because r and γ vary on the hybrid time scale. The electric field rises linearly from the center to the edge of the plasma column.

If there is no initial field, [9] is an harmonic oscillator equation at the hybrid frequency. If the magnetic field is instantaneously increased from zero to a value B_z, both the electrons and ions accelerate to the axis in a time $\pi/2\Omega_{L,H}$. The electric field induced by the increasing magnetic field accelerates the electrons azimuthally and they begin to spiral toward the origin. A radial, space-charge electric field develops which prevents the electrons from moving radially more quickly than the more massive ions. If an initial magnetic field is suddenly decreased, the electrons begin to spiral outward which creates a radial space-charge field of the opposite sign.

III. OPERATING LIMITS

A. Upper bound on charge density

The analysis assumes that the azimuthal current induced in the plasma does not reduce the magnetic field at the axis. This assumption places an upper bound on the plasma density. For relativistic electrons, the current density can be estimated by assuming that the electron tangential velocity is the speed of light. From the current density and Ampere's law we find that

$$\Delta B_z / B_z = \mu_0 necr / B_z\ , \qquad [12]$$

where $\Delta B_Z/B_Z$ is the fractional change in the field. Requiring this to be small we find

$$r(\omega_{p,e}/c)(\omega_{p,e}/\omega_{c,e}) << 1, \qquad [13]$$

where $\omega_{p,e}$ is the nonrelativistic electron plasma frequency and $\omega_{c,e} = eB_z/m_e$ is the nonrelativistic electron cyclotron frequency.

In the nonrelativistic limit, the electron angular velocity is given by the Larmor frequency and the fractional change in the field is

$$\frac{\Delta B_z}{B_z} = \frac{1}{2}\mu_0 ne\Omega_{L,e} r^2. \qquad [14]$$

Requiring this to be small we get

$$(\omega_{p,e}^2 / c^2)(\frac{1}{2} r^2) << 1 \qquad [15]$$

which places an upper bound on the product of the density and the square of the radius. This condition is written to show that it corresponds to having the magnetic skin depth $c/\omega_{p,e}$ longer than the radius of the plasma. For example, a plasma density of 10^{12} cm^{-3} results in $c/\omega_{p,e} = 3$ cm.

B. Lower bound on charge density and upper bound on the magnetic field

The assumption of quasineutrality places a lower bound on the plasma density. If we require that

$$(n_i - n_e)/n_e << 1\ , \qquad [16]$$

then Poisson's equation requires

$$\frac{2\gamma\Omega_{L,e}^2}{\omega_{p,e}^2} = \frac{\gamma\omega_{c,e}^2}{2\omega_{p,e}^2} << 1 \,. \qquad [17]$$

This lower bound on charge density has the effect of being an upper bound on the applied magnetic field.

C. Combined limits

The greatest electric field is created by operating at the upper bound on density and the upper bound on magnetic field. We estimate the maximum field by setting $2\omega_{p,e}{}^2 = \omega_{c,e}{}^2$ to approximate the upper bound set by [17], by setting $\omega_{p,e}{}^2 = 2c^2/r^2$ to approximate the upper bound set by [15], and obtain from [11]

$$eE_r = \frac{1}{4}\gamma m_e r\omega_{c,e}^2 = \frac{1}{2}\gamma m_e r\omega_{p,e}^2 = \gamma m_e c^2 / r \,. \qquad [18]$$

This places an upper bound on the electrostatic potential at the surface of $e\phi = \gamma m_e c^2$. Thus the peak potential must be kept below 0.5 MV to avoid violating the assumptions of the derivations.

IV. APPLICATIONS

In a lens of radius 2 cm, for example, it should be possible to create a potential of 0.2 MV which corresponds to a focusing field of 10 MV/m. The focusing electric field has the same radial force on relativistic particles as a magnetic field of strength $E/c \cong 0.03$ T. This is less than the field available from magnetic quadrupoles thus there is no advantage in the lens for relativistic particles. For particles below about 0.03 c, the radial force is greater than can be obtained in quadrupoles and there may be an advantage in the electrostatic lens.

A prototype device with a plasma of radius 3 cm and a density of 10^{11} cm^{-3} will satisfy the upper bound on density. The upper bound on magnetic field is satisfied by a field of 500 G. The hybrid Larmor period for a plasma with barium ions is 161 nsec/rad. The magnetic field should rise to a constant value in a time shorter than this period. The period for a hydrogen plasma is much less and a magnetic field with sufficient risetime would be difficult to create. If the plasma radius is increased to 30 cm, the maximum density falls to 10^9 cm^{-3}, the maximum field falls to 50 G, and the hybrid period for a deuterium plasma is 139 nsec/rad.

As a radial accelerator, the lens may have promise as a source of fusion neutrons due to the line focus of the accelerated ions. The energy which can be imparted to deuterium or tritium ions is of the order of 0.5 MeV which is well above the threshold for fusion reactions. For this application it may be advantageous to locate a solid cylindrical target on axis. If an initial field is used to confine the plasma, this target should have a radius corresponding to the radius where the ion energy is maximized. The fusion yield without a target depends upon the minimum radius to which the plasma is compressed. In the case of no initial field, the minimum radius is determined by the initial angular momentum from the thermal motion of the ions and electrons.

V. REFERENCES

[1]See for example *Collective Methods of Acceleration*, N. Rostoker and M. Reiser, eds., (Harwood Academic Publishers, Chur, Schwiez, 1979).

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