Self–Modulated–Laser Wakefield Acceleration*

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Abstract—A new configuration of the laser wakefield accelerator is proposed in which enhanced acceleration is achieved via resonant self-modulation of the laser pulse. This requires laser power in excess of the critical power for relativistic guiding and a plasma wavelength short compared to the laser pulse-length. Relativistic and density wake effects strongly modulate the laser pulse at the plasma wavelength, resonantly exciting the plasma wave and leading to enhanced acceleration.

I. Introduction

Plasma-based accelerators are being widely researched as candidates for the next generation of particle accelerators [1]. One promising concept is the laser wakefield accelerator [2,3] (LWFA), in which a short ($\tau_L < 1$ ps), high power (P > 1 TW) laser pulse propagates in plasma to generate a large amplitude (E > 1 GV/m) wakefield, which can trap and accelerate a trailing electron bunch. In the standard LWFA, efficient wake generation requires $L \simeq \lambda_p/2$, where L is the full-width-at-half-maximum length of the laser intensity profile on axis, $\lambda_p = 2\pi c/\omega_p$ is the plasma wavelength, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ and n_0 is the ambient plasma density. In this case, the peak axial electric field is given by [2,3] $E_z \simeq (\pi^2 m c^2/e) a_0^2/(4\lambda_p \gamma_{\perp})$, where $\gamma_{\perp} = (1+a_0^2/2)^{1/2}$ and $a_0 = eA_0/mc^2$ is the normalized amplitude of the laser vector potential field [4] which is assumed to be linearly polarized throughout this paper.

In this paper, we describe a self-modulated-LWFA[5] in which enhanced acceleration is achieved via resonant self-modulation of the laser pulse. This occurs when a) the laser pulse extends axially over several plasma wavelengths, $L > \lambda_p$, and b) the peak laser power satisfies $P \ge P_c \simeq 17(\lambda_p/\lambda_0)^2$ GW, where P_c is the critical power [6] for relativistic optical guiding and λ_0 is the laser wavelength. At fixed laser parameters, both conditions can be met by choosing a sufficiently high plasma density. Operation in the self-modulated regime could have a dramatic impact on LWFA experiments now being planned.

II. Self-Modulation

In the self-modulated regime, enhanced wakefields are generated, i.e., accelerating fields are more than an order of magnitude greater than those generated by a laser pulse with $L \simeq \lambda_p/2$, assuming fixed laser parameters. Acceleration is enhanced for four reasons. Firstly, since a higher

density is required (assuming L fixed), the wakefield will be increased: $E_z \sim n_0^{1/2}$. Secondly, the resonant mechanism excites a very-high-amplitude wakefield in comparison to the standard LWFA. Thirdly, since $P \ge P_c$, relativistic focusing further enhances the laser intensity, increasing a_0 . Fourthly, simulations show that a portion of the pulse will remain guided over multiple laser diffraction lengths, extending the acceleration distance.

The mechanism can be understood by considering a long laser pulse, $L \gg \lambda_p$, with power $P \simeq P_c$, such that the body of the pulse is relativistically guided [3]. The finite rise-time of the pulse will create a low-amplitude wakefield within the laser pulse. In the wakefield, each region of decreased density acts as a local plasma channel to enhance the relativistic focusing effect, while each region of increased density causes defocusing [7]. This results in a low-amplitude modulation of the laser pulse at λ_p . The modulated laser pulse resonantly excites the wakefield and the process continues in an unstable manner. This instability, which is observed to develop on a time-scale associated with laser diffraction, resembles a highly nonlinear 2-D form of the usual forward Raman scattering instability. It is distinguished by its 2-D nature and by its growth rate, which increases dramatically when $P \geq P_c$.

In the standard LWFA, the acceleration distance is limited by the diffraction length, or Rayleigh length, of the laser pulse: $Z_R = (k_0/2)r_0^2$, where $k_0 = 2\pi/\lambda_0$ and r_0 is the radius of the laser waist. At the high plasma densities and extended laser diffraction lengths associated with the self-modulated-LWFA, single-stage acceleration can be limited by detuning due to the reduced group velocity v_g of the laser pulse, rather than by diffraction. Here, $v_g \simeq c[1-(\lambda_0/\lambda_p\gamma_{\perp})^2/2]$, where $L \gg \lambda_p$ has been assumed. One-dimensional theory indicates that phase detuning limits the maximum acceleration to $\Delta\gamma_{max} \simeq \pi\lambda_p^2 a_0^2\gamma_{\perp}/(2\lambda_0^2)$, assuming fixed a_0 and $\lambda_p a_0^2/\lambda_0 \gg 1$.

We will illustrate the self-modulated LWFA via two numerical simulations. The first is a standard case which is optimized in the usual sense, with $L = \lambda_p/2$. The second is a self-modulated case, in which the plasma density is increased such that $L > \lambda_p$ and $P > P_c$ are achieved (all other parameters remain unchanged).

III. Model Equations

These simulations were based on the laser-plasma fluid model described in Ref. 7, which utilizes $(r, \zeta = z - ct, \tau =$

^{*}Support by DOE and ONR



Figure 1: Peak accelerating field versus time for the $n_0 = 1.4 \times 10^{17}$ cm⁻³ case (dashed line) and the $n_0 = 2.8 \times 10^{18}$ cm⁻³ case (solid line).

t) coordinates. The laser pulse moves in the positive z direction such that the front of the laser pulse remains near $\zeta = 0$. The physical region of interest extends from $\zeta = 0$, where the plasma is unperturbed, to $\zeta < 0$. The model is valid when $Z_R \gg L$, $Z_R \gg \lambda_p$, $\lambda_0 \ll r_0$ and $\lambda_0 \ll \lambda_p$. To include phase detuning effects, the $\partial^2/\partial\zeta\partial\tau$ term is retained in the wave equation, in contrast to Ref. 7. This model neglects certain laser-plasma instabilities. In particular, Raman side-scattering could limit the effective longitudinal extent of a laser pulse with $P > P_c$ [8].

IV. Simulation Results

In these runs, we will consider a Gaussian laser pulse with $\lambda_0 = 1 \ \mu m$, $a_0 = 0.70$, $r_0 = 31 \ \mu m$ and $L = 45 \ \mu m$ (150 fs), such that $Z_R = 0.3$ cm. Here, we define a_0 to be the amplitude of the laser vector potential \mathbf{a}_f at the point of minimum focus in vacuum. In this case, the peak laser power is $P = 21.5(a_0r_0/\lambda_0)^2$ GW = 10 TW and the energy per pulse is 1.5 J, well within the bounds of present technology [9]. We begin at $\tau = 0$ with the laser pulse outside the plasma. The plasma density is "ramped up" to reach full density at $c\tau = 2Z_R$. The laser pulse is initially converging such that in vacuum it would focus to a minimum spotsize of $r_0 = 31 \ \mu m$ at $c\tau = 3Z_R$. The simulation continues until $c\tau = 10Z_R = 3.0$ cm.

According to standard LWFA theory [2,3], the optimum wakefield will be obtained at a plasma density for which $\lambda_p \simeq 2L = 90 \ \mu\text{m}$, or $n_0 = 1.4 \times 10^{17} \text{ cm}^{-3}$. At this density, $P \ll P_c \simeq 140 \text{ TW}$. The presence of the plasma has little effect on the evolution of the laser pulse, which reaches a peak normalized intensity of $|\hat{a}_f|^2 = 0.56$ at $c\tau = 3Z_R$. This is illustrated in Fig. 1 (dashed line), where the peak accelerating field, plotted versus time, is symmetric about $c\tau = 3Z_R$.

To study the acceleration and trapping of electrons by the wakefield, a particle code is used to accelerate a distribution of 30,000 non-interacting test particles in the



Figure 2: Peak particle energy versus time for the $n_0 = 1.4 \times 10^{17}$ cm⁻³ case (dashed line) and the $n_0 = 2.8 \times 10^{18}$ cm⁻³ case (solid line).

time-resolved electric and magnetic wakefields of the simulation. Here, we consider a continuous electron beam with initial energy of 3.0 MeV and normalized emittance $\epsilon_n = 130$ mm-mrad. The beam is initially converging such that in vacuum it would focus to a minimum RMS radius $r_b = 200 \ \mu\text{m}$ at $c\tau = 3Z_R$. After $c\tau = 10Z_R = 3.0$ cm, a small fraction (0.1%) of the original particle distribution has been trapped and accelerated (simulations show that this fraction can be increased by using a lower emittance beam). At $c\tau = 3$ cm, the peak particle energy is 48 MeV (see Fig. 2, dashed line).

We now consider a self-modulated-LWFA simulation with parameters nearly identical to those considered above. Here, the plasma density has been increased to $n_0 =$ 2.8×10^{18} cm⁻³ ($\lambda_p = 20 \ \mu$ m). This reduces the critical power to $P_c = 6.8$ TW, such that $P \simeq 1.5P_c$. As the laser parameters have not been changed, the laser pulse now extends over several λ_p . In this run, one might expect phase detuning to limit the acceleration to $\Delta \gamma_{max} \simeq 340$ (170 MeV). However, we will see below that self-focusing enhances the laser intensity by a large factor (> 10) such that much higher electron energies can be obtained.

Figure 3 shows the normalized laser intensity at (a) $c\tau = 2Z_R$ (just as the laser enters the full-density plasma) and (b) $c\tau = 3.2Z_R$ (just beyond the vacuum focal point). The axial electric field at $c\tau = 3.2Z_R$ is shown in Fig. 4. The laser pulse has been modulated (three peaks are observable in Fig. 3(b), separated by $\simeq \lambda_p$) and the plasma wave is highly nonlinear. In addition, relativistic and density wake effects have focused the laser to a much higher intensity than was observed in the previous simulation. Figure 1, which shows the peak accelerating field versus time, indicates that the laser pulse is optically guided over $5.5Z_R$. Note that the leading portion of the "beamlet" structure, with length $< \lambda_p/(2\gamma_{\perp})$, diffractively erodes [3,8]. However, the extreme focusing of the laser pulse increases γ_{\perp} such that the erosion is minimized. In addition, the group velocity within the modulated pulse varies



Figure 3: Laser intensity $|\hat{a}_f|^2$, sampled over a coarse grid (the numerical grid is much finer), at (a) $c\tau = 2Z_R$ and (b) $c\tau = 3.2Z_R$. The laser pulse is moving towards the right.

locally with laser intensity and electron density, further distorting the pulse profile. The laser beamlets continue to distort and erode until $c\tau \simeq 7Z_R$, at which time the laser pulse disintegrates entirely.

Figure 1 (solid line) also shows that as the pulse becomes fully modulated, the amplitude of the peak accelerating field saturates. We have performed further simulations that show that as the wake amplitude increases to the point that the plasma electrons are expelled entirely from the axis of the simulation, the growth of the instability slows. In addition, the 2-D nature of the instability requires that regions of focusing and defocusing occur within the pulse. We find that when defocusing is inhibited, the growth of the instability diminishes.

As before, a beam of noninteracting test particles is injected into the time-resolved wakefield, with approximately 2% of the particles being trapped and accelerated. The peak particle energy of 430 MeV is observed at $c\tau = 1.8$ cm = $6Z_R$. At $c\tau = 3.0$ cm = $10Z_R$, however, the peak particle energy has dropped to 290 MeV due to the reduced group velocity of the laser pulse, which causes the electrons to slip out of phase with the wakefield and become decelerated. Figure 2 (solid line) shows acceleration to 430 MeV over 1.8 cm.

V. Conclusions

We have proposed a new configuration of the LWFA in which enhanced acceleration (by a factor > 10) is achieved via resonant self-modulation of the laser pulse (this concept is also discussed in Ref. 10, which was only recently brought to our attention). The self-modulation mechanism requires $P \ge P_c$ and $L > \lambda_p$. We have demonstrated, via simulation, the dramatic advantages of the self-



Figure 4: Axial electric field E_z versus ζ plotted at $c\tau = 3.2Z_R$.

modulated-LWFA[5] relative to the standard LWFA [1-3]. We have further demonstrated the feasibility of the selfmodulated case by confining our simulations to currentlyavailable laser parameters. It is a notable aspect of these simulations that, by increasing only the plasma density, one can test both the simple linear theory and the highly nonlinear, self-modulation regime described here.

Acknowledgements—The authors thank G. Joyce, G. Mourou and D. Umstadter for enlightening discussions.

VI. References

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