# Electron Beam Diagnostics by Means of Edge Radiation

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*Abstract*

Method for measurement of electron beam divergences and transversal sizes with visible range edge radiation (ER) is discussed. Intensity distribution of the ER generated by an infinitely thin electron beam at two adjacent bending magnet edges in a storage ring represents a system of concentric interference rings. The real beam divergences and transversal sizes smooth off the interference pattern; information on the extent of the smoothing off, being processed numerically, allows to determine the above-mentioned beam parameters. Precise computing technique of the ER intensity distribution with regard to finite beam emittance is presented. The results of simulations illustrating the application of the method for 450 MeV electron storage ring Siberia-1 are exhibited.

## I. INTRODUCTION

Electromagnetic radiation generated by relativistic protons at bending magnet edges in synchrotron was already used for proton beam diagnostics: in Ref. [1] proton beam profile was measured with the visual range ER, the same way as electron beam profile is usually measured with visible synchrotron radiation (SR) [2], [3]. It was possible since in the proton synchrotron the ER intensity at $\lambda << \lambda_c$ (where $\lambda_c$ is critical SR wavelength) greatly acceded the intensity of the standard SR.

It is proposed to use edge radiation for electron beam diagnostics by other means in this paper. It was shown both experimentally and theoretically [4] - [8], that ER intensity distribution in electron storage rings is very sensitive to beam divergences and transversal sizes. Calculations of the ER characteristics in the approximation of infinitely thin electron beam [7], [8] showed the intrinsic parameters of the ER angular distribution at $\lambda >> \lambda_c$ to be the inverse reduced energy $\gamma^{-1}$ and the value $[\lambda/(2l)]^{1/2}$, where $l$ is inter-magnet distance. The first parameter results from angular distribution of single bending magnet ER, whereas the second one arises from the distribution of interfering radiation generated at two adjacent bending magnet edges. If $\lambda$ belongs to visible region, these parameters are comparable with typical angular divergences of electron beam and the ratios of beam transversal sizes to observation distance in storage rings.

Experimentally, the method discussed consists in the measurement of the ER intensity distribution in the area adjacent to straight section axis (Fig.1). Monochromatic

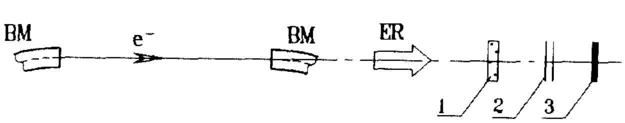


Figure 1. Edge radiation registration scheme.
1- neutral light filters, 2- monochromatic filter, 3- detector.

filter providing sufficiently narrow transparency band ($\Delta\lambda/\lambda \approx 10^{-2}$) should be used, since the radiation in wide spectral region smoothes off the interference pattern, same as beam divergences and transversal sizes do.

The general point of the method is the technique for numerical processing of the measurement results, which allows to determine the parameters of emitting beam from intensity distribution of the ER being registered. This problem consists of two ones. The first is the computation of the ER intensity distribution with due regard for finite beam emittance; the second is the fitting of measurement results over the beam parameters, to be based on successive solution of the first problem. Effective least squares fitting algorithms are well-known; therefore only the method of computing the ER intensity distribution in view of finite beam emittance is discussed in this paper.

## II. METHOD OF COMPUTATION

The expression for Fourier component of electric field emitted by single electron in its motion along the trajectory $\vec{r}(\tau)$ is readily obtainable from Fourier transformations of delayed potentials [9],

$$\vec{E}_\omega = \frac{ie\omega}{c}\int_{-\infty}^{+\infty}\frac{\vec{\beta}-\vec{n}}{R}\exp\left[i\omega\left(\tau+R/c\right)\right]d\tau, \qquad (1)$$

where $\vec{\beta} = \frac{1}{c}\frac{d\vec{r}}{d\tau}$ is relative velocity of electron, $\vec{n} = \vec{R}/R$, $\vec{R} = \vec{r}^{\,*} - \vec{r}$, $R = |\vec{R}|$, $\vec{r}^{\,*}$ denotes observation point position, $\omega$ is radiation frequency; $e$ is the charge of electron; $c$ is the speed of light; $i$ is unit imaginary number. The integration variable is time $\tau$. Eq. (1) is valid at $\lambda/R << 1$.

Let the origin of coordinates be set in the middle of the straight section, $y$-axis be coincident with the straight section axis, $x$ and $z$ be horizontal and vertical ones.

The radiation of ultra-relativistic electron ($\gamma >> 1$) is directed forward with respect to the particle motion; beam transversal dimensions are negligible as compared with observation distance. Therefore one can accept for transversal coordinates of observation point $(x^*, z^*)$: $|n_x| = |(x^*-x)/R| << 1$, $|n_z| = |(z^*-z)/R| << 1$ ($x$, $z$ are transversal coordinates of instantaneous electron position); it takes place in the trajectory region where the observed radiation is generated: $|\beta_x| << 1$, $|\beta_z| << 1$. In view of it, one can obtain the following for the phase in Eq.(1) by the corresponding expansion of $R$ (from here on, the equilibrium trajectory length $s$ is used as the integrating variable instead of $\tau$),

$$\begin{cases} \omega\left(\tau + R/c\right) = \Phi_0 + \Phi(s); \\ \Phi(s) \approx \dfrac{\pi}{\lambda}\left[s\gamma^{-2} + \displaystyle\int_0^s (x'^2 + z'^2)d\hat{s} + \dfrac{(x^* - x)^2 + (z^* - z)^2}{y^* - s}\right], \end{cases} \qquad (2)$$

where $\Phi_0$ does not depend on $s$, $x' = dx/ds \approx \beta_x$, $z' = dz/ds \approx \beta_z$; $y^*$ denotes distance from the origin of coordinates to detector. Eqs. (2) include all the terms that may really contribute to the radiation intensity value in high-energy electron storage rings. It is worth noting that in this approximation the equilibrium trajectory length is coincident with longitudinal Cartesian coordinate of electron and transversal coordinates in the natural (connected with the beam) frame of reference coincide with those of the Cartesian frame.

In the approximation under consideration linear particle dynamics is commonly described by

$$\begin{cases} \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} x_{eq}(s) \\ x'_{eq}(s) \end{bmatrix} + M_x(s) \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix}; \\ \begin{bmatrix} z \\ z' \end{bmatrix} = M_z(s) \begin{bmatrix} z_0 \\ z'_0 \end{bmatrix}, \end{cases} \tag{3}$$

where $M_x(s)$ and $M_z(s)$ are 2x2 matrixes defined by magnet lattice characteristics; $x_0, x'_0, z_0, z'_0$ are initial values of particle trajectory; $x_{eq}(s)$ and $x'_{eq}(s)$ define equilibrium trajectory (supposed to be plane),

$$x'_{eq}(s) \approx -\frac{e}{mc^2\gamma}\int_0^s B_z(\hat{s})d\hat{s}; \; x_{eq}(s) = \int_0^s x'_{eq}(\hat{s})d\hat{s}, \tag{4}$$

where $B_z(s)$ is vertical magnetic field. The ER intensity distribution depends on electron energy [6] - [8], but within small beam chromatisity this dependence is negligible.

In electron storage rings, if longitudinal bunch length is much larger than the wavelength of observed SR, the radiation emitted by different electrons is known to be incoherent. Spectral photon flux density of the radiation generated by the total electron beam may be represented as

$$\left(\frac{dN}{d\Sigma d\omega}\right)_{incoh} = \frac{c^2\alpha I}{4\pi^2 e^3\omega}\int\left|\vec{E}_\omega(\vec{r}^*, x_0, x'_0, z_0, z'_0)\right|^2 F(x_0, x'_0, z_0, z'_0)dx_0 dx'_0 dz_0 dz'_0, \tag{5}$$

where $I$ is electron current, $\alpha$ is the fine structure constant; $\left|\vec{E}_\omega(\vec{r}^*, x_0, x'_0, z_0, z'_0)\right|^2$ is defined by

$$\left|\vec{E}_\omega\right|^2 = \frac{e^2\omega^2}{c^4}\left|\int_{-\infty}^{+\infty}\frac{\vec{\beta}-\vec{n}}{y^*-s}\exp(i\Phi)ds\right|^2 \tag{6}$$

and Eqs. (2) - (4). Integration in (5) is over all phase space. $F(x_0, x'_0, z_0, z'_0)$ is initial particle density distribution in phase space; in a much used approximation

$$F(x_0, x'_0, z_0, z'_0) = \left(B_x\Gamma_x - A_x^2\right)^{1/2}\left(B_z\Gamma_z - A_z^2\right)^{1/2}\pi^{-2} \times$$
$$\times \exp\left(-\Gamma_x x_0^2 - 2A_x x_0 x'_0 - B_x x_0'^2 - \Gamma_z z_0^2 - 2A_z z_0 z'_0 - B_z z_0'^2\right), \tag{7}$$

where A,B,Γ are unnormalized parameters of phase ellipse.

If variations of magnetic field across the electron beam dimensions are negligible, then at $A_x=A_z=0$ $|\vec{E}_\omega|^2$ depends only on variable compositions $(x^*-x_0-y^*x'_0)$ and $(z^*-z_0-y^*z'_0)$ [10]. This allows to simplify Eq.(7). Though very useful for understanding the phenomena, this approximation is inapplicable for ER in strong-focusing synchrotrons. First, quadrupole lenses deflect particle trajectories, thus interference conditions for ER emitted at two bending magnet edges are different for different particles. Second, the elements of beam optics are the radiation sources providing unlike emission conditions at different $x_0$, $x'_0$, $z_0$, $z'_0$.

In this paper, an alternative method for computing the incoherent radiation intensity with regard to finite beam emittance is proposed. In view of Eq.(6), relation (5) may be rewritten as

$$\left(\frac{dN}{d\Sigma d\omega}\right)_{incoh} = \frac{\alpha\omega I}{4\pi^2c^2e}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{ds\,d\tilde{s}}{(y^*-s)(y^*-\tilde{s})} \times$$
$$\times\int\left(\vec{\beta}-\vec{n}\right)\left(\tilde{\vec{\beta}}-\tilde{\vec{n}}\right)\exp\left[i\left(\Phi-\tilde{\Phi}\right)\right]F(x_0, x'_0, z_0, z'_0)dx_0 dx'_0 dz_0 dz'_0, \tag{8}$$

where $\tilde{\vec{\beta}}, \tilde{\vec{n}}, \tilde{\Phi}$ depend on $\tilde{s}$, while $\vec{\beta}, \vec{n}$ and $\Phi$ depend on $s$. Eq.(8) means six-fold integration, but in terms of Eqs. (2), (3), (5) and (7) the inner four-fold integration may be done analytically, thus only two-fold integral (over $s$ and $\tilde{s}$) should be computed. This method is valid for insertion devices as well as for any layout of electron beam optics.

## III. COMPUTATION RESULTS

Computations of the ER intensity distribution at different beam divergences and transversal sizes were performed for the Siberia-1 (weak focusing) electron storage ring by the method based on Eq.(8). The following parameters were used in process: $\gamma \cong 881$, $I$=100mA, bending radius $r_0$=1m, $|r_0(\partial B_z/\partial x)/B_z|_{x=0,z=0}$=0.5, inter-magnet distance $l$=63cm; function $B_z(s)$ was determined according to measurements made in the Institute of Nuclear Physics (Novosibirsk).

The computations were done for $\lambda$=600nm. Results are given for the traditional beam parameters $\sigma_x$, $\sigma'_x$, $\sigma_z$, $\sigma'_z$ at $A_x=A_z=0$.

Fig.2 compares the ER intensity distribution of infinitely thin electron beam with that of the beam at expected $\sigma$, $\sigma'$. The detector is offset by $y^*$=572cm. Since $\sigma_x > \sigma_z$, $\sigma'_x > \sigma'_z$, the interference pattern is more smoothed off horizontally.

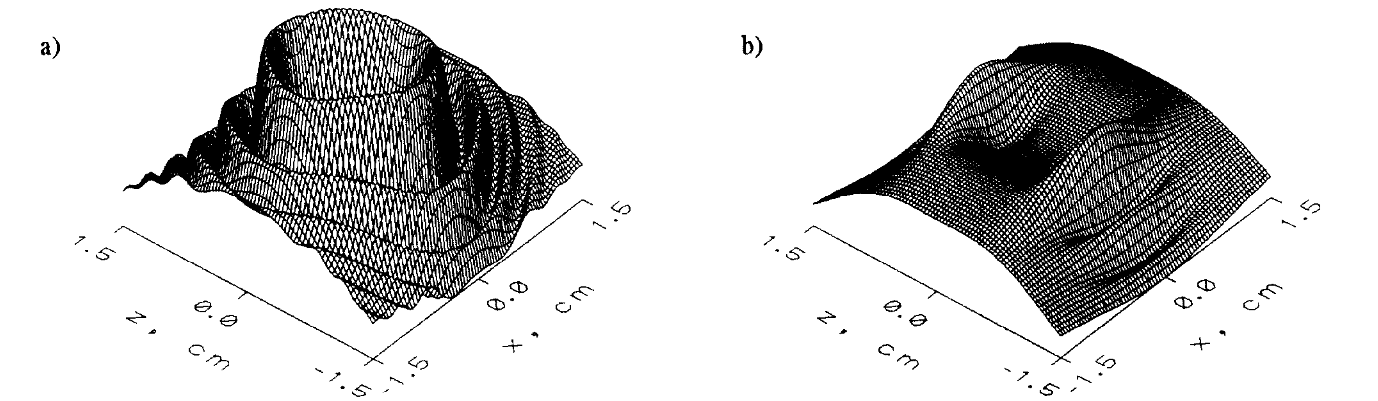


Figure 2. ER intensity distribution in detector plane: a) $\sigma_x=\sigma'_x=\sigma_z=\sigma'_z=0$; b) $\sigma_x$=1.62mm; $\sigma'_x$=0.65mrad; $\sigma_z$=0.13mm; $\sigma'_z$=0.08mrad.

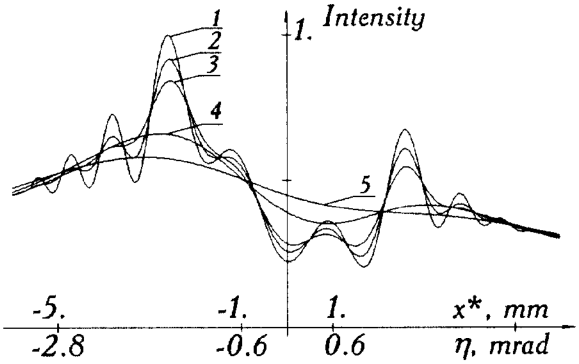


Figure 2. ER intensity distributions in median plane; $\eta$ is horizontal angle respective to straight section axis.
1- $(\sigma'_x+\sigma_x/y^*)=0$; 2- $(\sigma'_x+\sigma_x/y^*)=0.17$mrad; 3- $(\sigma'_x+\sigma_x/y^*)=0.28$ mrad; 4- $(\sigma'_x+\sigma_x/y^*)=0.91$mrad; 5- $(\sigma'_x+\sigma_x/y^*)=1.54$mrad.

The ER sensitivity to transversal dimensions and divergences of the beam is also clearly illustrated by Fig.2, where the intensity distributions in the median plane at different values of $\sigma'_x$ and $\sigma_x$ are exhibited ($\sigma'_z=0.08$mrad, $\sigma_z=0.13$mm and $y^*=181.5$cm for each curve). Since the computations performed refer to the case of weak-focusing synchrotron, the effective parameters of the ER intensity distribution are the compositions $(\sigma'_x+\sigma_x/y^*)$ and $(\sigma'_z+\sigma_z/y^*)$ (similar to those discussed in previous chapter). Therefore one should make measurements at different distances $y^*$ to distinguish contributions of $\sigma'$ and $\sigma$ (large $y^*$ is preferable for distinct measurement of $\sigma'$).

Fig.3 shows the intensity distributions in median plane computed at different $y^*$ for finite beam parameters as well as for infinitely thin beam. The distribution at larger $y^*$ is evidently less smoothed off (in consequence of the lesser contribution of transversal size); also, it is more symmetric. The distribution asymmetry is explained by the comparability of $y^*$ and inter-magnet distance.

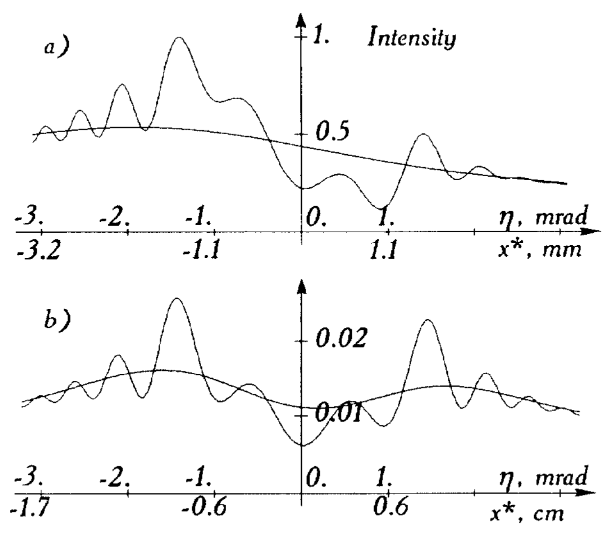


Figure 3. ER intensity distribution in median plane at $\sigma_x=1.62$mm, $\sigma'_x=0.65$mrad compared to one at $\sigma_x=\sigma'_x=0$:
a) $y^*=75$cm; b) $y^*=572$cm.

## IV. SUMMARY

The effective computing the edge radiation with regard to finite beam emittance is supposed to allow the measurement of beam divergences and transversal sizes by means of the visible ER in electron storage rings. The method proposed, being very simple experimentally, may beneficially supplement the existing technique of electron beam diagnostics.

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## V. REFERENCES

[1] R.Bossart, J.Bosser *et al*, "Proton beam profile measurements with synchrotron light", *Nucl. Instr. and Meth.*, Vol.184, Nos.2-3, 349 (1981).

[2] A.P.Saberski, "Monitoring the beam in SPEAR with synchrotron light", *IEEE Trans. Nucl. Sci.*, NS-20, 638 (1973).

[3] J.S.Mackay, "Electron beam profile, position systems and measurements on the Daresbury SRS", Preprint Daresbury Laboratory DL/SCI/P591A (1988), *Proceedings of the 1988 European Particle Accelerator Conference.*

[4] M.M.Nikitin, A.F.Medvedyev, M.B.Moiseyev, "Interference of synchrotron radiation", *Sov. Phys. JETP*, Vol.52, 388 (1980).

[5] M.M.Nikitin, A.F.Medvedyev, M.B.Moiseyev, "Synchrotron radiation from ends of straight-linear interval", *IEEE Trans. Nucl. Sci.*, Vol. NS-28, No.3, 3130 (1981).

[6] Yu.A.Bashmakov, "Synchrotron radiation of electrons in the edge magnetic fields of storage rings", *Rev. Sci. Instrum.*, Vol.63, No.1, 343 (1992).

[7] O.V.Chubar, N.V.Smolyakov, "VUV range edge radiation in electron storage rings", *J. Optics (Paris)*, Vol.24, No.1 (1993).

[8] O.V.Chubar, N.V.Smolyakov, "Generation of intensive long-wavelength edge radiation in high-energy electron storage rings", these proceedings.

[9] L.D.Landau, E.M.Lifshits, *The Classical Theory of Fields*, Pergamon, Oxford (1971).

[10] K-J.Kim, "A new formulation of synchrotron radiation optics using the Wigner distribution", *SPIE* Vol.583, 2 (1985). *The 1985 International Conference on Insertion Devices for Synchrotron Radiation Sources.*