

Beam Emittance From Coherent Cherenkov Radiation in a Solid Dielectric*

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Abstract

We report experimental results of a technique for direct measurement of the emittance in high energy beams. This technique is analogous to the well known "pepper pot" masking approach, but with no upper limit on particle energy. Single shot emittance profiles were obtained on the 10 kA, 4 MeV Sandia National Laboratories' electron Recirculating Linear Accelerator showing agreement with theory and with alternate emittance measurements. Coherent (i.e., not strongly scattered or diffused) Cherenkov radiation from a flat, transparent, range-thin dielectric foil was split by an array of mirrors in order to view the emission profile up to a divergence angle of 70°. The mirrors were imaged by a distant telescope attached to an intensified, 2 ns video framing camera. The relative intensity profiles of the multiple images were unfolded using the properties of classical Cherenkov emission and geometric optics to obtain directly, without precise knowledge of other beam parameters, the transverse velocity distribution in the viewing plane. In our case the rms emittance was directly proportional to the product of the beam diameter and the transverse velocity spread. This and prior research efforts indicate applicability over a wide range of high energy beam parameters.

I. INTRODUCTION AND BACKGROUND

Understanding the transport properties of high current (several kA), high energy (several MeV) electron beams is complicated by difficulties in experimental measurement of the beam phase space parameters. Several techniques¹ based on apertured masks have been developed that work well with repetitively pulsed or cw beam sources. However, the particle range in the mask material must be small compared to the working aperture size and the beam transverse temperature must be low enough to avoid collimation errors.

Our research into Cherenkov radiation based emittance measurements was motivated by the need for a diagnostic which could potentially operate at energies into the 100 MeV range and beyond, provide local measurements of emittance with nanosecond time resolution, and be minimally perturbing to the beam. We also expected to encounter difficult conditions such as plasma backgrounds, rotating hollow beams, and beams with large amplitude collective instabilities. In non-fluorescing materials Cherenkov emission dominates other radiation processes such as bremsstrahlung and transition radiation and has a time

response on the order of the particle transit time through the material. For range-thin converter foils there is minimal perturbation to the beam. Transparent FEP Teflon material was chosen for study due to its excellent performance as an optical Cherenkov radiator, its availability in large area thin films, and its relative ease of mechanical workability. The optical emission in the range of 400-700 nm from 2 mil thick foils exhibited strong peaking at the nominal Cherenkov angle, time dependence that tracked the beam current, and no evidence of fluorescence or radiation darkening. These results were obtained over a range of beam current densities from 1kA/cm² to 20kA/cm² and pulse lengths from 20 to 50 ns.

The emittance measurements were performed on the SNL Recirculating Linear Accelerator (RLA) injector in support of an ion focused regime (IFR) transport experiment. The RLA injector, referred to as IBEX, nominally produced a 10 kA, 4 MeV, 25 ns electron beam with a 1-2 cm radius at the measurement point. Emittance data were collected after 2m and 5m of linear transport along the ion focusing channel. These data were in good agreement with theoretical predictions of the emittance growth and also with emittance estimates obtained from a vacuum expansion technique. While Cherenkov witness plates have been used in other laboratories as optical current meters or as beam profile diagnostics², to the authors' knowledge, this represents the first emittance diagnostic for an IREB based entirely upon the directional properties of Cherenkov radiation.

II. DESCRIPTION OF THE DIAGNOSTIC

A. Essential properties of Cherenkov radiation

For a charged particle traveling at velocity v in a straight line through a dielectric medium with refractive index n , Cherenkov radiation is emitted in a thin cone centered on the trajectory with opening angle θ_c given by $\cos \theta_c = 1/\beta n$, where $\beta = v/c$ is the usual relativistic factor. The basic properties of this radiation are well described in the literature^{3,4}. The radiation power per kiloampere of current per millimeter of dielectric thickness in the visible spectrum $\lambda_1=400\text{nm}$ to $\lambda_2=700\text{nm}$ is found to be $P \approx 58 \text{ kW/kA-mm}$, where βn was chosen to be 1.4. Since this power is radiated into a small solid angle, the observed intensities can be quite strong even with very inefficient light collection optics.

Inside a linear, isotropic optical medium the angular thickness of the Cherenkov cone is determined mainly by Coulomb scattering and by the optical dispersive properties of the dielectric. Variation of θ_c due to slowing down of the particle may be neglected for range-thin converters. In this case it can be shown that the Cherenkov cone is broadened by less than a few milliradians in the visible spectrum for most optical materials. Now consider the radiation pattern, still within the dielectric, produced by a monoenergetic beam

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having a transverse temperature characterized by a thermal velocity $\beta_{\perp} = v_{\perp}/c$. We will use the following approximations: $\beta_{\parallel} \approx 1$ and $\tan \varphi_i = \beta_{\perp i} / \beta_{\parallel} \approx \varphi_i$, where the component $\beta_{\perp i}$ is the instantaneous transverse speed for the i^{th} particle in the beam as the particle passes through the converter. Hence, there is a one-to-one correspondence between the distribution function of transverse velocities and transverse angles. The accuracy is better than two percent for $\beta_{\perp} < 0.2$. Since the Cherenkov cone angle is the same for all particles, the angular distribution of Cherenkov radiation intensity in a plane perpendicular to the beam cone will be directly proportional to the beam transverse velocity distribution. This simple result is the basis for the emittance diagnostic.

Outside the dielectric the physical and geometric optical properties of the converter will change this simple proportionality relation. The main distortion of the radiation distribution in the laboratory is due to simple geometric optics effects. Figure 1 illustrates the geometry by considering the ray optics of an optically flat slab converter.

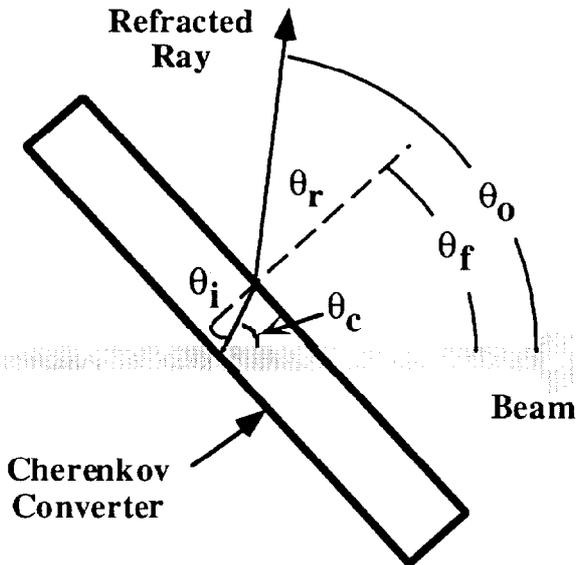


Figure 1. Basic geometry of a slab converter. θ_c , θ_i , θ_r , θ_f , and θ_o are the Cherenkov, internal incidence, external refracted, foil tilt, and observation angles, respectively.

The converter plane is shown tilted with respect to the particle trajectory to include this option explicitly in the analysis. Using Snell's law the observed angle of emission with respect to the beam axis is related to the angle of emission in the dielectric by the expression

$$\theta_o = \theta_r + \sin^{-1} \left[n \sin(\theta_c - \theta_f) \right], \quad (1)$$

where θ_o , θ_f , and θ_c are the observation, converter plane tilt, and Cherenkov angles, respectively. Internal reflection can become significant as the internal angle of incidence approaches the total reflectance angle given by $\theta_{\max} = \sin^{-1}(1/n)$. FEP Teflon has the desirable property of a low index of refraction of 1.345 which yields $\theta_c = 42^\circ$ and $\theta_{\max} = 48^\circ$. For ideal observation conditions the foil is tilted such that $\theta_c = \theta_f$ and the observation angle is equal to the principal Cherenkov angle, and this angle is normal to the foil surface. We use the term principal Cherenkov angle to refer

to the angle of emission for a particle in the beam having $\beta_{\perp i} = 0$. In the converter, by our approximations, the average angle of emission in the plane perpendicular to the beam cone is always the principal angle. The intensities must also be corrected for internal reflection effects. Cherenkov radiation is polarized with the polarization vector normal to the cone surface and to the direction of propagation. Hence, the p-wave transmission reflection coefficient given by

$$R = \frac{\tan^2(\theta_r - \theta_i)}{\tan^2(\theta_r + \theta_i)},$$

where θ_i is the internal angle of incidence and θ_r is the external angle of refraction, is appropriate. Summing the infinite series for total forward transmitted power yields

$$P_{\text{forward}} = \frac{P_0}{1+R}. \quad (2)$$

B. Measurement technique

The method chosen to collect the intensity profiles was to image the converter foils at different angles and use the intensity ratio of the images to unfold the Cherenkov angular distribution inside the dielectric. Stray light sources such as intense sparks can easily be identified in this manner. Data acquisition was via a fast gated, intensified video camera with computer based image analysis. In order to obtain an angular discrimination of a few mrad the camera(s) must be placed at a large distance D from the converter such that $[2a+d]/D < 10^{-2}$, where d is the effective camera aperture size. For our experiments $D=30\text{m}$. The Cherenkov radiation lobe in the viewing plane was split into six different angular slices by using flat mirrors placed near the converter foil. Figure 2 shows the mirror and converter foil layout for the IBEX RLA measurements. The mirror holder consisted of an aluminum block with interference fit grooves machined at 10° intervals to correctly position each mirror in the array. The entire arrangement fitted inside a standard 8 inch vacuum cross. A 10 inch lucite plate served as the vacuum window

Using a 600 mm focal length $f/5.6$ lens on the camera gave an angular resolution for this system of about 5 mrad which is on the order of the diffraction modified Coulomb scattering⁶ limit of the 2 mil FEP Teflon foil and is much less than the expected >200 mrad divergence of the refracted Cherenkov lobe.

C. data analysis

The corrected Cherenkov intensity profile in the dielectric was found, and then the data were fit to a Gaussian transverse velocity distribution. The Gaussian assumption was not necessary, but it was an excellent approximation in our case. Solving equation (1) for the emission angle θ_c^j in the dielectric in terms of the observation angle θ_j yields

$$\theta_c^j = \theta_f + \sin^{-1} \left[\frac{\sin(\theta_j - \theta_f)}{n} \right].$$

The intensity must be corrected for refractive broadening in the viewing plane given by differentiating equation (1)

$$\frac{d\theta_j}{d\theta_c^j} = \frac{n \cos(\theta_c^j - \theta_f)}{[1 - n^2 \sin^2(\theta_c^j - \theta_f)]}.$$

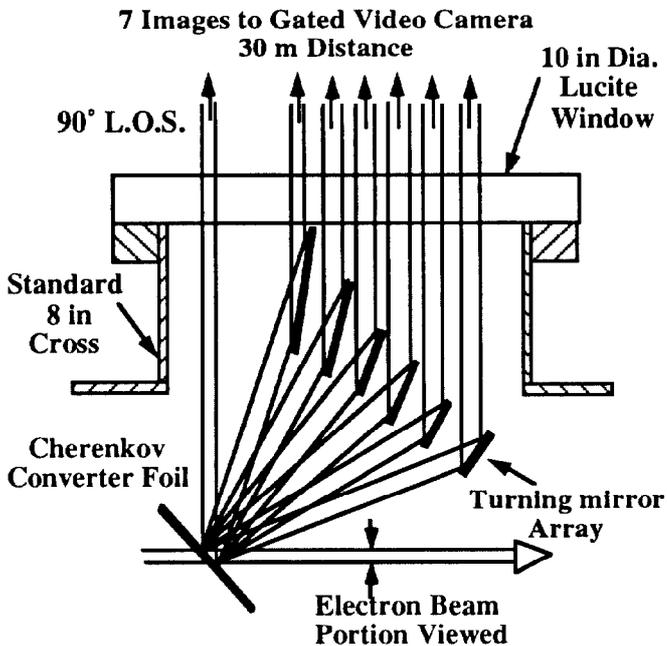


Figure 2. Cherenkov converter foil and turning mirror array arrangement shown as used inside the 8 in vacuum cross. Foil tilt angle was 42° from normal to the beam.

Including the effect of internal reflection from equation (2) the relative intensity I_j at the emission angle ϕ_j is related to the measured intensity F_j at the corresponding observation angle by

$$I_j = F_j \frac{d\theta_j}{d\theta_c} (1 + R_j) .$$

Fitting a Gaussian distribution to the I_j data yields β_{\perp} .

Transverse emittance ϵ_x may be defined to be $1/\pi$ times the area in xx' phase space, where x is a transverse coordinate and $x'=dx/dz$ is the conjugate angle. Using the definition of rms emittance⁵ it can be shown, for beams of radius a having azimuthal symmetry, that $\epsilon_{rms} = 2a\beta_{\perp}$.

III. EXPERIMENTAL RESULTS

Intensity profiles were taken after various lengths of IFR transport on IBEX. The Gaussian fit results are shown in Figure 3 for both 2m and 5m. The values of β_{\perp} obtained were 0.11 and 0.17, respectively. The combined error in this measurement technique is less than ten percent. For comparison the two intensity curves are normalized to the same peak value. Measurements based upon a vacuum expansion technique⁷ gave $\beta_{\perp}=0.15$ after 5m of transport. Calculations using the beam envelope equation⁸ with the measured plasma channel densities and beam radii gave $\beta_{\perp}=0.18$ for the 5m case.

The main problem encountered in this diagnostic was background light in the chamber due to diffusely reflected Cherenkov light seen as higher than baseline intensities between the mirrors. As the emittance became larger this problem became more noticeable. While not tested on our experiment, it should be straightforward to reduce the background levels using baffles and low reflectance coatings.

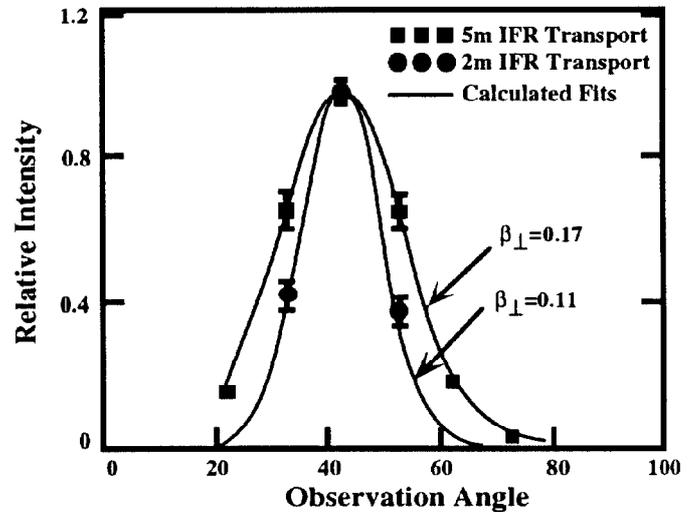


Figure 3. Gaussian fit to Cherenkov intensity profile data for the cases of 2m and 5m IFR transport.

IV. CONCLUSIONS

The angular dependence of Cherenkov radiation from range-thin, transparent, flat dielectric foils can be used to measure the transverse velocity distribution, and therefore the emittance, of a warm IREB. This diagnostic is capable of giving the local beam temperature for each point on the beam/foil intersection plane to an accuracy better than ten percent without detailed models or simulations of the beam transport conditions. Time resolved data is generally necessary. There is essentially no upper limit to the beam particle energy in this application. We have completed a basic feasibility study and a series of experiments culminating with the test of a fully operational emittance meter on the IBEX RLA experiment. Results are in agreement with theoretical predictions and an alternate measurement technique. There are few difficulties involved in fielding Cherenkov diagnostics on IREB machines. The fundamental limit to the angular resolution is diffraction modified Coulomb scattering.

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