General Time-Varying State-Space Control Model and its Application for Transient Beam Loading Compensation

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Abstract

A time-varying state-space control model was presented and used to predict the functions required to cure the injection voltage transients. We discuss a novel method to calculate the feedforward functions. Simulation results are shown to validate the method.

I. INTRODUCTION

It is required in the LEB for proper bunching and acceleration that the cavity gap voltage transients are controlled to within specifications. The direct RF feedback and local phase and amplitude loops will no doubt be able to control the injection voltage transients. But before trying the feedback loops it is usual practice to consider some open loop techniques since the loops tend to affect the overall stability of the system. Some rf feedback [1] and feedforward [2] techniques are considered and used in proton accelerators in other Laboratories. Feedforward techniques were generally applied in combination with the local rf feedback loops so that the combined effect would cure the transients. Feedforward techniques require accurate prediction of the functions. In this paper, we have identified the time-varying terms effecting the voltage transients. Later we show a method to predict the compensating terms accurately for the parameters of the Low Energy Booster. We think this technique is applicable to similar proton machines elsewhere. The technique relies on the accuracy of the control model. Our particle tracking studies show that the voltage transients are kept to within specifications when the feedforward functions were realized in the absence of loops.

II. LINEAR STATE-SPACE MODEL

Let x_1= synchronization phase error, x_2= radial position error, x_3 = beam phase error, x_4= cavity gap voltage error, x_5= cavity gap phase error and x_6= tuning error. Then a complete state-space model can be derived for a macro particle using the accelerating cavity as an equivalent RLC circuit. Development of a complete linear time-varying state-space model is well described in Reference 3. We simply reinstate the model below in terms of the system matrix $\underline{A}$, the input matrix $\underline{B}$ and the disturbance matrix $\underline{d}$.

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u} + \underline{d} \qquad (1)$$

For a time-invariant system, the terms in $\underline{d}$ will be zero and the derivative of the synchronous phase, $\dot{\phi}^s$, is negligibly small. Equation 1 agrees with Pedersen's model of [4] and hence satisfies Robinson's stability criteria. For completeness, we include the exact expression of (1) below.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ 0 & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{33} & b_{34} & b_{35} & 0 \\ 0 & 0 & b_{43} & b_{44} & b_{45} & 0 \\ 0 & 0 & b_{53} & b_{54} & b_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\dot{\phi}^s \\ c_4 \\ c_5 \\ c_6 \end{bmatrix}$$

The model was compared with the longitudinal particle tracking code to confirm its validity when the rf frequency and the cavity gap voltage were varied with time during the acceleration cycle. We see that the cavity gap voltage has a large initial voltage transients when there are no feedback loops. In Fig. 1 the cavity gap voltage error is plotted with time for the first 1 ms after injection. The transients have been identified to be due to the high rate of change of voltage at the beginning, just after injection, and due to the sudden appearance of the beam (Fig. 1). We have identified that the terms c_4 and c_5 in the state-space model constitute to the initial voltage transients. This was done by solving for the gap voltage error, x_4, and gap phase error, x_5, from the state-space model by setting terms c_4 and c_5 equal to zero (See Fig. 1). In Pedersen's model due to no direct RF feedback (H=1) and no time varying terms ($\dot{V}$ =0=derivative of the cavity gap voltage) c_4 and c_5 are zero. The terms c_4 and c_5 are shown below to gain some insight into their complexities.

[τ]Operated by the University Research Association, Inc., for the U.S. Department of Energy under contract No. DE-AC35-89ER40486

$$
\begin{aligned}
c_4 = {} & -F_{11}\sigma(H\dot{V} + \omega V \tan\phi_z^0) - F_{12}\omega(\dot{V} + \sigma H V) + \\
& F_{11}\sigma R K_g \omega (I_b \cos\phi^s - I_g \sin\phi_L) + \\
& F_{12}\sigma R K_g \omega (-I_b \sin\phi^s + I_g \cos\phi_L) + \\
& \sigma R K_g \dot{i}_g (F_{11}\cos\phi_L + F_{12}\sin\phi_L)
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
c_5 = {} & -F_{21}\sigma(H\dot{V} + \omega V \tan\phi_z^0) - F_{22}\omega(\dot{V} + \sigma H V) + \\
& F_{21}\sigma R K_g \omega (I_b \cos\phi^s - I_g \sin\phi_L) + \\
& F_{22}\sigma R K_g \omega (-I_b \sin\phi^s + I_g \cos\phi_L) + \\
& \sigma R K_g \dot{i}_g (F_{21}\cos\phi_L + F_{22}\sin\phi_L)
\end{aligned} \tag{3}
$$

F's in (2) and (3) depend on machine parameters.

Clearly, the direct rf feedback alone must in theory be capable of reducing the rf voltage transients. This is however insufficient in many machines including the LEB (See Fig. 2) due to the limitation on the direct RF feedback strength, H, (for LEB, H = 20) which is restricted by the loop delays. In theory, the amplitude and phase loops would reduce the transients to zero (Fig. 2) but their gains are again restricted due to the overall stability limits and the cavity tuning conditions. Since the terms responsible for the transients are known we can develop an amplitude and a phase function for the generator current to reduce the voltage transients. Feedback can be applied later, if needed. This is done below.

3. TRANSIENT COMPENSATION WITHOUT FEEDBACK

When we say the transient compensation without feedback we mean without the fast loops such as the direct rf feedback, local amplitude and phase feedback. The cavity tuning loop is, however, a must for the LEB since it affects all other states (see Eqn. 1). To see the compensation method more clearly, at first we extract the cavity voltage and phase equations below.

$$
\begin{aligned}
\dot{x}_4 = {} & a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + a_{46}x_6 \\
& + b_{43}u_3 + b_{44}u_4 + b_{45}u_5 + c_4
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\dot{x}_5 = {} & a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 + a_{56}x_6 \\
& + b_{53}u_3 + b_{54}u_4 + b_{55}u_5 + c_5
\end{aligned} \tag{5}
$$

The control 'u_3' is the frequency shift provided in the global low level rf beam control system which has the effect over the complete ring. Hence, we cannot use this term to generate the feedforward function local to the cavity. Whereas control quantities, u_4-- the generator current amplitude and u_5 - the generator current phase are affected local to each cavity station. Therefore, by rearranging u_4 and u_5 in Eqns. (4) and (5) such that their combined effect nullifies the effect of the terms c_4 and c_5 on x_4 and x_5, we would obtain the compensating function. It is given by the following equation.

The parameters, b's are shown in Reference 3. u_4 and u_5 are calculated from Eqn. (6) for the LEB machine parameters. The computed feedforward functions are

$$
\begin{bmatrix} u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} b_{44} & b_{45} \\ b_{54} & b_{55} \end{bmatrix}^{-1} \begin{bmatrix} -c_4 \\ -c_5 \end{bmatrix} \tag{6}
$$

shown in Figs. 3 and 4. The generator current amplitude function has steps due to the discrete nature of the cavity voltage we used in the program. Fig. 5 shows the gap voltage error with respect to time after applying the compensation. Clearly it is well within 1% of the gap voltage. A disadvantage with this type of compensation is due to the fact that u_4 and u_5 also affect the coherent beam phase oscillations (see Eqn. 1). Also the upper and lower limits of these quantities must be in a realizable form. In the simulation studies using the predicted values of u_4 and u_5, we see a negligible effect on the beam phase oscillations since the cavity voltage amplitude and phase error diminishes with accurate feedforward function. The function u_4 and u_5 look within practical limits and can be realized in practice using function generators with the functions feeding to the same control points (See Fig. 6) where the local amplitude and phase loops were expected.

4. CONCLUSIONS

In this paper we have identified the terms affecting the injection gap voltage transients in proton synchrotrons using the linear time-varying state-space model derived in Reference 3. Mathematical model was derived with the assumption that the ring cavities are lumped and is a simple equivalent RLC circuit. Also, the fundamental of the beam current was considered to include the beam loading effects. Using the model, a method was described to calculate the local feedforward function to cure the voltage transients. We did not see the need of local amplitude and phase loops to cure only predictable voltage transients. However, the presence of direct RF feedback loop for the LEB, although not essential, would benifit in correcting for unpredictable voltage transients occurring in a cycle to cycle basis.

5. REFERENCES

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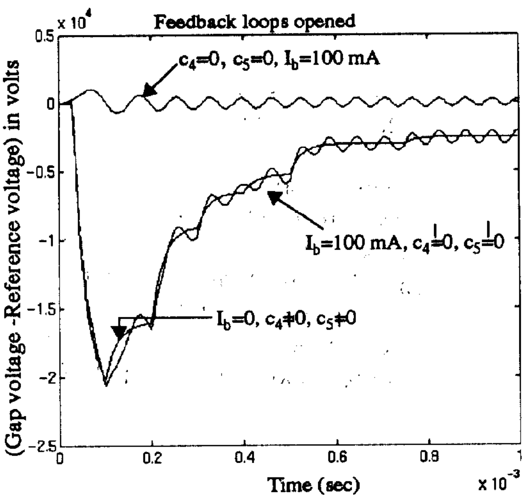

Fig. 1 : Cavity gap voltage error with time

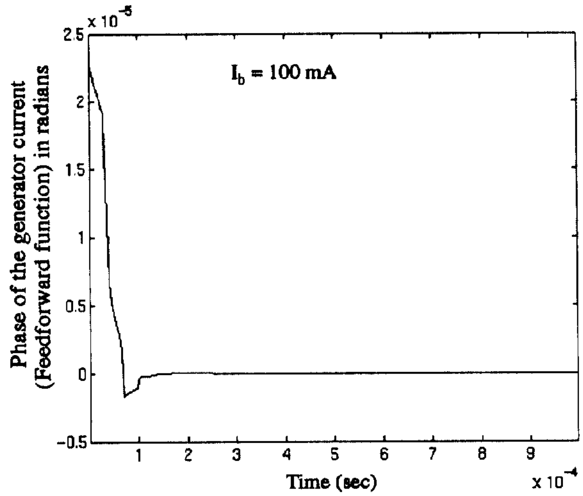

Fig. 4 : Generator current phase (Feedforward function)

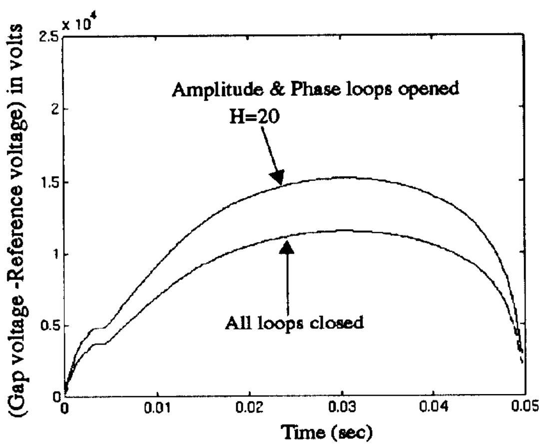

Fig. 2 : Cavity gap voltage error with time

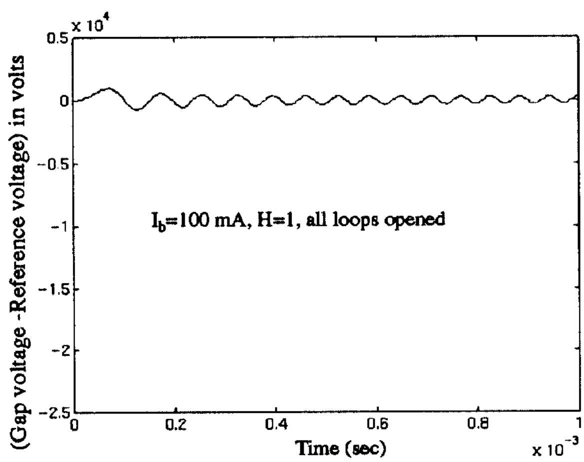

Fig. 5 : Cavity gap voltage error with feedforward correction

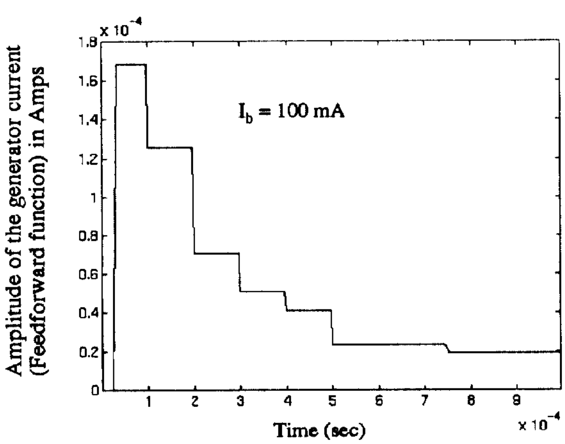

Fig. 3 : Generator current amplitude (Feedforward function)

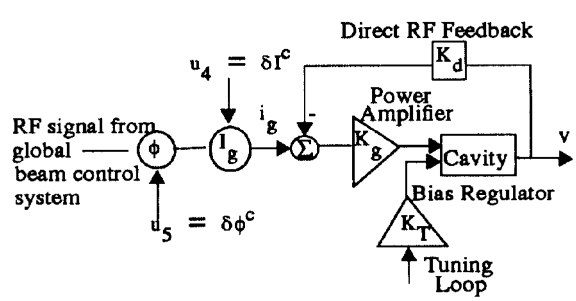

Fig. 6 Schematic representation of the control points for feedforward compensation system.