# Loss of precision in resonant beam position monitors due to finite Q

J. P. H. Sladen and W. Wuensch

*CERN, Geneva, Switzerland*

## Abstract

The performance of resonant beam position monitors is limited in part by the presence of a symmetrical (common mode) excitation of the cavity. Due to finite Q, a centred beam will give a non-zero output at the observation frequency. This interference is analysed and general expressions are given for the electrical and magnetic spectral densities, split into the wanted resonant term and the "broad-band" unwanted signal, for a cavity with arbitrary shape. This analysis is particularly relevant to beam position monitor studies for the CERN linear collider (CLIC), where very precise measurement of transverse beam position is required. It is shown that a common mode position error of under 1µm is obtainable from a high Q 33GHz pill-box, even prior to the use of any external difference-taking elements.

## Introduction

The performance of high precision resonant beam position monitors (BPM's) will be limited in part by the off-resonance excitation of common modes ($E_{0ml}$ modes in a cylindrical cavity). Due to finite Q, a centred beam will give a non-zero output at the frequency used for observation and this will remain after narrow-band filtering of the cavity output. The interference from the $E_{010}$ mode in a cylindrical cavity BPM operating in the $E_{110}$ mode has already been treated [1]. Here, the analysis is extended to include all interfering modes and an arbitrary cavity shape. General expressions are given for the electrical and magnetic spectral densities at the BPM operating frequency, split into the wanted resonant term and the "broad-band" interference.

This analysis is particularly relevant to BPM studies for the CERN linear collider (CLIC), where very precise measurement of transverse beam position is required. The BPM that has been proposed is a cylindrical cavity operating in the $E_{110}$ mode at a frequency of 33GHz [1,2]. This case is taken as an example and the position error is calculated for output coupling to the circumferential magnetic field.

Finally, measurements of common mode interference in a prototype BPM are described.

## Cavity spectral density at resonance

The electric and magnetic fields due to a line current $Ie^{j\omega t}$ travelling through a cavity in the z direction with co-ordinates $r_0, \phi_0, z$ can be written as an expansion in terms of orthogonal modes [3,4]. For the electric field, both the zero divergence solenoidal set of modes $\mathbf{E}_n$ and the zero curl irrotational set $\mathbf{F}_n$ are required:

$$\mathbf{E} = \sum_n \frac{-j\omega\mu_0\left(1+\dfrac{1-j}{Q_n}\right)}{k_n^2 - k^2\left(1+\dfrac{1-j}{Q_n}\right)} \int_0^d I\, e^{-jkz} E_{nz}(r_0,\phi_0,z)\, dz\ \mathbf{E}_n - \sum_n \frac{1}{j\omega\varepsilon_0}\int_0^d I\, e^{-jkz} F_{nz}(r_0,\phi_0,z)\, dz\ \mathbf{F}_n \tag{1}$$

where $d$ is the cavity length, $E_{nz}$ and $F_{nz}$ are the z direction components of $\mathbf{E}_n$ and $\mathbf{F}_n$ respectively and $k=\omega/c$. $k_n$ is the $n^{th}$ eigenvalue and $Q_n$ is the quality factor. For the magnetic field, only the solenoidal set $\mathbf{H}_n$ is excited:

$$\mathbf{H} = \sum_n \frac{k_n}{k_n^2 - k^2\left(1+\dfrac{1-j}{Q_n}\right)} \int_0^d I\, e^{-jkz} E_{nz}(r_0,\phi_0,z)\, dz\ \mathbf{H}_n \tag{2}$$

$\mathbf{E}_n$, $\mathbf{F}_n$ and $\mathbf{H}_n$ are real and satisfy:

$$\nabla\times\mathbf{E}_n = k_n\mathbf{H}_n \qquad \nabla\times\mathbf{H}_n = k_n\mathbf{E}_n \qquad \nabla\times\mathbf{F}_n = 0$$

$$\int_V \mathbf{E}_n.\mathbf{E}_n dV = 1 \qquad \int_V \mathbf{F}_n.\mathbf{F}_n dV = 1 \qquad \int_V \mathbf{H}_n.\mathbf{H}_n dV = 1$$

At the frequency of the $p^{th}$ resonance, we can split the response into the wanted resonant term and the sum of the interfering modes. Assuming large $Q_n$ and that we are far from resonances $n \neq p$ :

$$\mathbf{E} = -\frac{Q_p}{\omega_p \varepsilon_0} \int_0^d I\, e^{-jk_p z} E_{pz}(r_0, \phi_0, z)\, dz\ \mathbf{E}_p - \sum_{\substack{n \\ n\neq p}} \frac{j\omega_p \mu_0}{k_n^2 - k_p^2} \int_0^d I\, e^{-jk_p z} E_{nz}(r_0, \phi_0, z)\, dz\ \mathbf{E}_n - \sum_n \frac{1}{j\omega_p \varepsilon_0} \int_0^d I\, e^{-jk_p z} F_{nz}(r_0, \phi_0, z)\, dz\ \mathbf{F}_n \quad (3)$$

$$\mathbf{H} = -\frac{jQ_p}{k_p} \int_0^d I\, e^{-jk_p z} E_{pz}(r_0, \phi_0, z)\, dz\ \mathbf{H}_p + \sum_{\substack{n \\ n\neq p}} \frac{k_n}{k_n^2 - k_p^2} \int_0^d I\, e^{-jk_p z} E_{nz}(r_0, \phi_0, z)\, dz\ \mathbf{H}_n \quad (4)$$

These expressions are independent of the Q's of the interfering modes. It should also be noted that all transit time factors are calculated at the frequency $\omega_p$.

It is not possible to give a general expression for the position error resulting from the off-resonance terms since it is very much dependent on the output coupling.

## Pill-box cavity

Equation (4) is now applied to a pill-box cavity operating in the $E_{110}$ mode with output coupling to the circumferential magnetic field, through an iris into rectangular waveguide. Although the summation is in fact a triple series, interference comes only from the $E_{0ml}$ common modes, as only these are strongly excited by a beam near the centre. In addition, we will omit all modes with $l > 0$ since these modes do not couple through the iris to the $H_{10}$ mode in the rectangular output waveguide. For a cavity of radius $a$ and length $d$, we have:

$$E_{110z} = \sqrt{\frac{2}{\pi d a^2 J_0^2(p_{11})}} J_1(p_{11} r/a) \cos\phi \quad (5)$$

$$H_{110\phi} = \sqrt{\frac{2}{\pi d a^2 J_0^2(p_{11})}} J_1'(p_{11} r/a) \cos\phi \quad (6)$$

$$E_{0m0z} = \sqrt{\frac{1}{\pi d a^2 J_1^2(p_{0m})}} J_0(p_{0m} r/a) \quad (7)$$

$$H_{0m0z} = \sqrt{\frac{1}{\pi d a^2 J_1^2(p_{0m})}} J_1(p_{0m} r/a) \quad (8)$$

$$k_{nm0}^2 = (p_{nm}/a)^2 \quad (9)$$

where $p_{nm}$ is the $m^{th}$ root of $J_n$. For $r_0 << a$ and $\phi_0 = \phi = 0$, the circumferential magnetic field given by (4) is:

$$H_\phi = \frac{I}{jk_{110}\pi d a}\left(e^{jk_{110}d} - 1\right)\left(\frac{jQ_{110} r_o}{J_o(p_{11}) a} + \sum_m \frac{p_{0m}}{(p_{0m}^2 - p_{11}^2) J_1(p_{0m})}\right) \quad (10)$$

If we define the position error as the value of $r_0$ for which the magnitudes of the wanted and unwanted terms in (10) are equal, then:

$$\text{error} = \frac{J_o(p_{11}) a}{Q_{110}} \sum_m \frac{p_{0m}}{(p_{0m}^2 - p_{11}^2) J_1(p_{0m})} = 0.305 \frac{\lambda_{110}}{Q_{110}} \quad (11)$$

The error in a 33GHz cavity with a Q of 4000 is thus 0.69μm. For the CLIC BPM, this will be reduced by the symmetry rejection of a pair of diametrically opposed irises feeding a hybrid tee. A reduction by more than an order of magnitude is easily achievable .

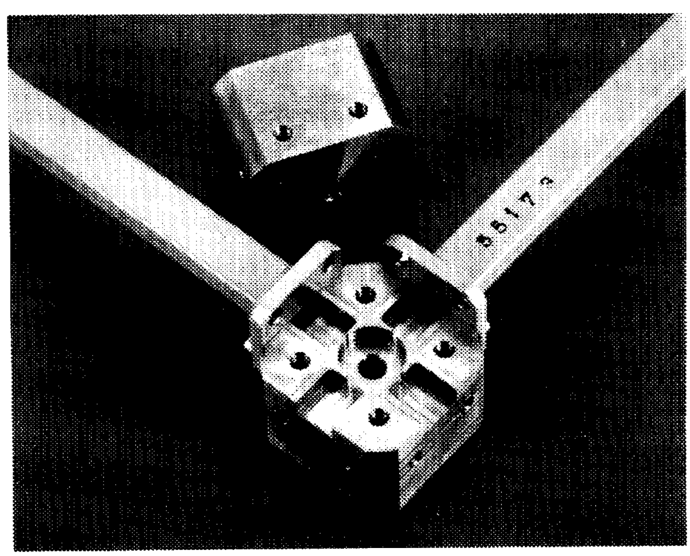

Fig. 1 Prototype dual axis BPM cavity.

## Measurement of position error due to common modes

Common mode interference has been measured experimentally in a prototype 33GHz $E_{110}$ pill-box BPM (Fig. 1). The cavity was excited by a 5mm long antenna placed in the beam hole 0.5mm outside the end-wall and a network analyser was used to measure the transmission through the antenna-cavity-output iris system. The antenna could be moved relative to the cavity using a precision translation stage with a nominal 0.1μm resolution. Fuller details of the experimental configuration have been given elsewhere [5]. For the present measurements, a brazed diamond machined cavity of diameter 10.600mm and with a loaded Q of 4000 was employed. A single cavity output port was used with no external symmetry rejection.

Again, we define the common mode position error as the value of displacement for which the magnitudes of the $E_{110}$ resonant and the quadrature "broad-band" responses are equal. The experimental estimate of this error was 1.0μm. It was found by firstly positioning the antenna at the null of the $E_{110}$ transmission (flat response), and then moving it either side to where the transmission increased by $\sqrt{2}$, at the frequency of the $E_{110}$ resonance. This value was reached at antenna positions of ±1.0μm from the null, as is shown in Fig. 2.

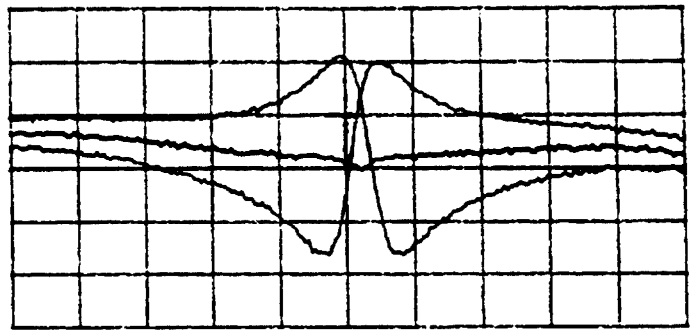

Fig. 2 Transmission through cavity at the $E_{110}$ null and at ±1μm either side. Vertical 2dB/div. Horizontal 10MHz/div, centre 32.8GHz.

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## References

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