Issues of the Transverse Feedback Systems Design at the SSC

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Abstract

The transverse feedback systems are needed at the SSC for several different reasons. The requirements of these systems are analyzed and specified. In addition to the general requirements (power, bandwidth and gain), specific attention is given to the noises in the systems, which need to be controlled in order to keep the emittance growth at a tolerable rate. A quantitative treatment is given to specify the allowable noise level in the feedback systems.

I. INTRODUCTION

The physics involved in the design of the SSC transverse feedback systems has been discussed in detail in Ref [1]. This paper is a brief overview of some selected issues. The feedback systems will serve four different purposes:

1. Correction of the injection errors:

At injection there are errors in the beam position, angle, energy and phase. These errors can lead to beam emittance growth if not corrected. Because the errors are relatively large (\sim mm), the feedback system needs high power.

2. Damping of the resistive wall instability:

The Collider beam tube is made of stainless steel with thin copper coating. At low frequencies (a fraction of the revolution frequency $f_0 = 3.441$ kHz), the skin depth is larger than the coating thickness Δ (~ 100 μ m). In this case, the resistive wall instability growth time is proportional to the product $\sigma_e \Delta$, where σ_e is the conductivity of copper under operation conditions. At the design value $\sigma_e \Delta = 2 \times 10^5 \ \Omega^{-1}$, the growth time is about 110 turns. Therefore, the feedback system needs a large gain.

- 3. Damping of the coupled bunch instability driven by the higher order modes (HOM) of the rf cavities: The PEP 5-cell cavity is the original candidate for the Collider. Its HOM would cause both longitudinal and transverse coupled bunch instabilities. The growth is slow (in the order of seconds). But the feedback system needs a wide bandwidth. (This instability may be avoided if the single-cell cavity with HOM properly damped is to be used [2].)
- 4. Control of continuous emittance growth: Because the radiation damping time of the protons in

the Collider is long (~ 13 hours), external perturbations will be remembered by the beam and lead to eventual emittance growth. A feedback system that keeps the coherent motion of the beam below certain allowable amplitude can effectively reduce the emittance growth rate. This system must be of low noise.

The noise of the feedback systems is a special concern of the SSC, because it may blow up the beam emittance. Previous experiences at the Tevatron and SPS show that the emittance dilution is increased when the transverse feedback system is on.

II. DAMPING PROCESS

Let us define a vector that represents the amplitude and phase of the collective beam oscillation:

$$\eta \equiv \frac{x}{\sqrt{\beta}} + i(\frac{\alpha x}{\sqrt{\beta}} + \sqrt{\beta}x') \tag{1}$$

where α and β are the lattice functions. When a feedback system with gain g is applied, the amplitude $|\eta|$ will be decreased in one turn by:

$$\Delta |\eta| = -g |\eta| \cos^2 \phi_1 \tag{2}$$

where ϕ_1 is the betatron phase at the pickup. Also, the phase angle is changed by:

$$\Delta \phi = -g \sin \phi_1 \cos \phi_1 \tag{3}$$

Note that the fractional decrement in the amplitude is monotonic and on average is equal to g/2, while the change in phase angle oscillates and has a zero average. After N turns, we have:

$$|\eta(N)| = |\eta_0| \exp\left(-g\left(\frac{N}{2} + \frac{\sin\left(2\pi\nu(2N+1)\right) - \sin 2\phi_0}{4\sin 2\pi\nu}\right)\right)$$
(4)

Thus, the collective amplitude damps as an exponential with a characteristic period of 2/g turns, but the exponential also has some minor wiggles.

III. EMITTANCE GROWTH AT INJECTION

A. Emittance dilution due to injection errors

The magnitude of the coherent amplitude x_c due to injection position error δx and angle error $\delta x'$ in, say, the horizontal plane is:

$$|x_c|^2 = \delta x^2 (1 + \alpha_x^2) + 2\alpha_x \beta_x \delta x \delta x' + \beta_x^2 \delta x'^2 \qquad (5)$$

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For the case of an injection energy error $\delta E/E$, the resultant coherent amplitude is:

$$|x_{c}|^{2} = \left(\frac{\delta E}{E}\right)^{2} \left(D_{x}^{2}(1+\alpha_{x}^{2})+2\alpha_{x}\beta_{x}D_{x}D_{x}'+\beta_{x}^{2}(D_{x}')^{2}\right) (6)$$

where D_x and D'_x are the dispersion function and its slope. The eventual fractional emittance increase produced by decoherence is:

$$\frac{\delta \epsilon_x}{\epsilon_x} = \frac{|x_c|^2}{2\sigma_x^2} \tag{7}$$

in which σ_x is the rms beam width.

B. Decoherence

The decoherence due to the chromaticity ζ and momentum spread σ_p/p and due to the non-linear magnetic fields has been analyzed in Ref [3]. The centroid of the bunch with an initial betatron amplitude a_0 has after N turns the amplitude $a(N) = a_0A(N)$, where A(N) is the decoherence factor. In the chromaticity case, one has

$$A(N) = \exp\left(-2\left(\zeta \frac{\sigma_p}{p} \nu_s^{-1} \sin\left(\pi \nu_s N\right)\right)^2\right) \qquad (8)$$

where ν_s is the synchrotron tune. If ν_s is independent of synchrotron amplitude (linear approximation), the whole bunch decoheres and then perfectly re-coheres every synchrotron period. For the Collider at injection, $\sigma_p/p =$ 1×10^{-4} , $\nu_s = 2.2 \times 10^{-3}$, and for a residual chromaticity of 5, the linear decoherence factor oscillates between 1.0 and 0.90 at the synchrotron period of 455 turns and, therefore, does not significantly affect the feedback requirements.

In the non-linear fields case, simulations have shown for typical magnetic error distributions in the lattice that the horizontal tune is well represented by

$$\nu_x = \nu_0 - \mu x^2 \tag{9}$$

where ν_0 is the tune at zero betatron amplitude, x is the betatron amplitude, and μ is about $1.4 \times 10^{-4} \text{ mm}^{-2}$ [4]. The decoherence factor is:

$$A(N) = (1 + (2\pi\sigma_{\nu}N)^{2})^{-1}$$
(10)

where $\sigma_{\nu} = 2\mu\sigma_x^2$ is the rms tune spread. From Eq. (10), one can define the decoherence time:

$$\tau_d = \frac{T_0}{\sigma_\nu} \tag{11}$$

where T_0 is the revolution time. For the Collider at injection, σ_{ν}^{-1} is 1.8×10^4 turns.

IV. CONTINUOUS EMITTANCE GROWTH

A. Continuous emittance growth without feedback

Consider the beam in a storage ring in which there is a continuous, small emittance growth $\dot{\epsilon}_0$ due to small and random dynamic disturbances, such as, quadrupole motion or power-supply jitter. These disturbances continually produce small-amplitude collective betatron oscillations, which continually smear out through decoherence and so transform into emittance growth. This growth rate can be expressed in terms of an average collective amplitude x_{av} and the decoherence time τ_d :

$$\dot{\epsilon}_0 = \frac{x_{\rm av}^2}{2\beta} \cdot \frac{1}{\tau_d} \tag{12}$$

During the collision period, the decoherence is dominated by the beam-beam interaction, which produces a large tune spread. For a Gaussian bunch, the rms tune spread can be obtained from a numerical integration [5]:

$$\sigma_{\nu} \approx 0.2 \,\xi \tag{13}$$

in which $\xi = N_b r_p/(4\pi\epsilon_N)$ is the beam-beam parameter $(N_b = \text{protons per bunch}, r_p = \text{classical proton radius}, \epsilon_N = \text{normalized rms beam emittance})$. For the Collider in nominal case σ_{ν} is 7.6×10^{-4} , giving a typical decoherence time of 1.3×10^3 turns.

B. Continuous emittance growth with feedback

From Eq. (4) one can define the feedback damping time:

$$\tau_f = \frac{2}{g} T_0 \tag{14}$$

The total emittance growth rate with feedback is [1]:

$$\dot{\epsilon}_f = \dot{\epsilon}_0 \left(\frac{2\sqrt{2}\tau_f}{\tau_d}\right)^2 = \dot{\epsilon}_0 \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2 \tag{15}$$

where $\dot{\epsilon}_0$ is defined in Eq. (12). Therefore, if the feedback gain is big enough such that $g > 4\sqrt{2}\sigma_{\nu}$, we will have $\dot{\epsilon}_f < \dot{\epsilon}_0$, i.e., the feedback will reduce the emittance growth rate.

C. Noises in the feedback system

If there are noises in the feedback system equivalent to a beam amplitude x_N at the pickup, then there is a contribution $f_0(gx_N)^2$ to the collective amplitude. Thus, Eq. (15) has to be modified and takes the form

$$\dot{\epsilon}_f = \left(\dot{\epsilon}_0 + \frac{f_0 g^2 x_N^2}{2\beta}\right) \cdot \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2 \tag{16}$$

Let ϵ_0 be the initial emittance, one may also define the emittance growth rate as

$$\frac{1}{\tau_{\epsilon}} \equiv \frac{\dot{\epsilon}_f}{\epsilon_0} \equiv \frac{1}{\tau_{\text{ext}}} + \frac{1}{\tau_{\text{noise}}}$$
(17)

in which

$$\frac{1}{\tau_{\text{ext}}} = \frac{\dot{\epsilon}_0}{\epsilon_0} \cdot \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2 \tag{18}$$

is the growth rate due to external sources, and

$$\frac{1}{\tau_{\text{noise}}} = \frac{1}{\epsilon_0} \frac{f_0 g^2 x_N^2}{2\beta} \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2 = 0.64 f_0 \left(\frac{x_N}{\sigma_x}\right)^2 \Delta \nu^2 \quad (19)$$

where we have converted σ_{ν} to the total tune shift $\Delta \nu$ (which equals ξ times total number of interaction points)

Feedback system	Α	В	С
Purpose	Injection errors	Resistive wall instability,	Coupled bunch instability
		Emittance control	
Gain	0.04	0.1	0.02
Damping time	50 turns	20 turns	100 turns
Maximum correction	± 2 mm	$\pm 100 \ \mu m$	$\pm 40 \ \mu m$
Kick angle	$0.27~\mu rad$	$0.04 \ \mu rad$	$0.003 \ \mu rad$
Kicker length	4 m	4 m	4 m
Kicker voltage	1 kV	150 V	150 V
Kicker power	40 kW	0.9 kW	0.9 kW
Bandwidth	500 kHz	500 kHz	$\geq 30 \text{ MHz}$
Noise level	-	$\leq 2 \ \mu \mathrm{m}$	$\leq 1 \ \mu m$
Resolution limit	$0.02~\mu{ m m}$	$0.02 \ \mu m$	$0.16 \ \mu m$

using Eq. (13). It is interesting to note that the emittance growth rate due to the noises in the feedback system is independent of the gain. It is also interesting to compare this result with that presented in Ref. [6], which was obtained using the well established stochastic cooling theory. Eq. (19) above differs from Eq. (8) in [6] by approximately a factor of 2.

V. FEEDBACK SYSTEMS

Three systems are required by the Collider: A. Injection error correction; B. Resistive wall instability and emittance control; C. Coupled bunch instability damping. System A has high power. It is used only during injection. Its bandwidth is determined by the batch spacing $(1.7 \ \mu s)$. System B needs large gain but low power. It has demanding low noise requirement. It's bandwidth is also determined by the batch spacing. System C has a wide bandwidth, which is determined by the bunch spacing $(16.7 \ ns)$. It does not need much power or gain, but the noise level must also be low. (It may be possible to combine B and C into one system.)

All the three systems can share the same pickups. But at least two different kickers are needed — one for A (high power), the other for B and C (low power). Each system has its own signal processor. These systems will be located in the west utility region at the high β -function points. The pickups should avoid the dispersive region. Otherwise the beam loading induced coherent syschrotron oscillation may cause coherent betatron motion through dispersion.

The proposed 2-pickup, 2-turn scheme has certain advantages. By using two pickups, the performance of the systems will be independent of the betatron tune of the machine. By comparing the signals from two (or more) consecutive turns, one can reject the closed orbit signal that is not needed by the feedback systems.

The requirements of the power, bandwidth, gain and noise level are listed in the table above. The noise level is calculated by Eqs. (16)-(19), assuming the allowable emittance growth time τ_{ϵ} is 24 hours, while the growth time τ_{ext} due to external excitations is 0.1 hour.

The theoretical limit of the pickup resolution comes from the thermal and electronic noises. It can be approximately expressed by:

$$\Delta x = \frac{2b}{I_{\rm av}\sin(\frac{\omega\ell}{c})} \sqrt{\frac{k_B T \cdot \Delta f \cdot 10^{NF/10}}{Z}}$$
(20)

in which k_B is the Boltzmann constant, T the temperature, Δf the bandwidth, NF the noise factor (in dB), b the half distance between two pickup electrodes, Z the characteristic impedance, ℓ the length of the electrodes, c the velocity of light, and I_{av} the average beam current. The value of Δx must be smaller than x_N given by Eq. (16) in order to avoid the emittance growth problem.

There are several other error sources that are not included in this analysis but may also put a limit to the pickup resolution. These include the least significant bit (LSB) error if a digital system is used, and the mechanical vibration of the pickup. The LSB error may be significant. As an example, the Tevatron Super-damper utilizes an 8bit digital system for signal processing. The full scale is about 5 mm, which is determined by the residual orbit error. Therefore, the maximum LSB error is about 20 μ m. It is much larger than the theoretical limit Δx and is a possible source of the emittance dilution increase discussed in Section I.

VI. REFERENCES

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