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Measurement of the Spin of a Particle Using Undulator Radiation*

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Abstract

Use of many period wigglers to rotate the spin of particles in accelerators has been proposed [1]. An added advantage of this scheme is that a spectrum of synchrotron radiation will be produced that contains a contribution due to the spin, so that the device can be effectively used as spin polarimeter, with advantages compared to Compton backscattering methods. Results are presented for two high energy proton storage rings: RHIC and the SSC.

I. INTRODUCTION

We have proposed to build spin rotators with two transverse wigglers of many poles with fields perpendicular to each other, longitudinally shifted. Wiggler spin rotators can also be used as beam diagnostic tools, allowing measurements of the beam size and spin polarization from observation of the synchrotron radiation spectrum. Beam diagnostics can be done with the SR spectral continuum from bending magnets, however the spectrum of wigglers being the result of interference presents a characteristic line structure and therefore has a higher specific brightness.

Spin polarization can be measured by Compton backscattering of laser light by the particle beam. The physical mechanism of photon production by wiggler radiation and backscattering is the same, and lead to comparable results. However, also laser backscattering produces a continuum and many of the nice properties of the enhanced wiggler spectrum are lost.

II. ELECTRIC AND MAGNETIC DIPOLE SYNCHROTRON RADIATION

A particle of mass m and charge e is accelerated in a magnetic field **B**. The frame of reference is shown in figure 1. The preferred direction of motion is along z (**k**). The radial direction is x (**i**), the vertical direction is y (**j**). In a given point of space the radiated power flux is given by the Poynting theorem [2]

$$\frac{d\mathbf{P}}{d\sigma} = \mathbf{E}\times\mathbf{B} \quad ; \quad \frac{d\mathbf{P}}{d\Omega} = \frac{R^2E^2}{\mu_0 c} \ , \tag{1}$$

with R the distance between charge and field point. If we only retain the "radiation" term in the expression for the field **E** (i.e. the term $\propto 1/R$), it is

$$RE = \frac{\mu_0}{4\pi}ec|\mathbf{A}(t)| \, ; \, 4\pi\frac{dP}{d\Omega} = \frac{\mu_0}{4\pi}e^2c|\mathbf{A}(t)|^2 \ , \tag{2}$$

$$\mathbf{A} = \frac{\mathbf{n}\times\left[(\mathbf{n}-\boldsymbol{\beta})\times\dfrac{d\boldsymbol{\beta}}{dt}\right]}{[1-\mathbf{n}\cdot\boldsymbol{\beta}]^3} \ , \tag{3}$$

where **A** is the vector potential,

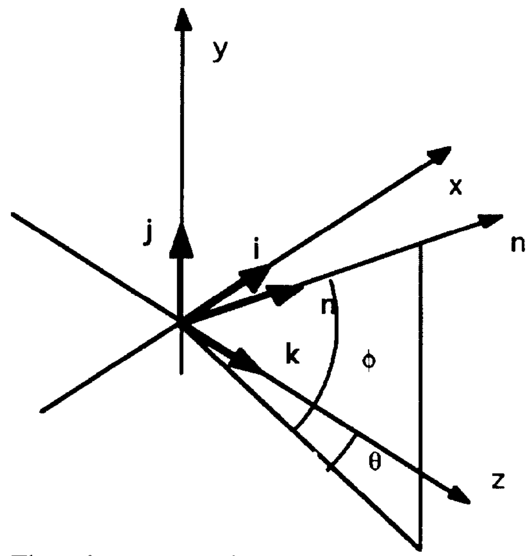

Fig. 1. The unit vector **n** points towards the observation point.

and the energy radiated by the particle in a time interval Δt is

$$4\pi\frac{dW}{d\Omega} = \frac{\mu_0}{4\pi}e^2c\int_{\Delta t}|\mathbf{A}(t)|^2dt \ . \tag{4}$$

To calculate the spectrum, replace t with the retarded time t_R in the integral of Eq. (4) and apply Parseval's Theorem, then, introducing the source current I [amp], obtain for the number of photons emitted per second, per unit solid angle and per unit photon energy interval

$$\frac{d^2n}{d\Omega\, d(\hbar\omega)} = \frac{\alpha I}{2\pi e}|\mathbf{A}(\omega)|^2 \tag{5}$$

(α is the fine structure constant).

If we assume that the energy of the particle remains constant, in Eq.(3), the particle velocity β and its time derivative are calculated from the equation of motion

$$\frac{d\boldsymbol{\beta}}{dt} = \boldsymbol{\beta}\times\Omega \quad ; \quad \Omega = \frac{e\mathbf{B}}{m\gamma} \ . \tag{6}$$

In far field (Fraunhofer), the radiation will be calculated along the direction defined by the unitary vector **n**, with angles θ and ϕ. Vector potential components A_p, parallel to the horizontal plane and perpendicular to **n**, and A_σ, perpendicular to **n** and to A_p are

$$A_p = \mathbf{A}\cdot[\mathbf{j}\times\mathbf{n}]; A_\sigma = \mathbf{A}\cdot\{[\mathbf{j}\times\mathbf{n}]\times\mathbf{n}\} \ . \tag{7}$$

Explicit expressions are of the form

$$A_p = \frac{1}{D^3}\mathbf{P}\cdot\Omega \quad ; \quad A_\sigma = \frac{1}{D^3}\mathbf{S}\cdot\Omega \tag{8}$$

with **P** and **S** some vector expressions, and

$$D = 1-\boldsymbol{\beta}\cdot\mathbf{n} \ . \tag{9}$$

The "usual" synchrotron radiation is the radiation of an oscillating electric dipole in the external magnetic field. If the

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particle has a magnetic moment μ proportional to the spin s (in units $\pm 1/2$)

$$\mu = \frac{ge}{mc}\frac{\hbar}{2}\mathbf{s} \tag{10}$$

where s is the particle spin in the particle rest frame (PRF), magnetic dipole radiation will appear.

In the LAB frame, the spin transforms as follows

$$\mathbf{S} = \mathbf{s} + \frac{\gamma^2}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{s})\boldsymbol{\beta}\,. \tag{11}$$

The particle spin, in its precession in an external magnetic field, adds to the radiation field of Eq. (3) a contribution

$$\mathbf{A}^{(s)}(t) = \frac{1}{(1-\boldsymbol{\beta}\cdot\mathbf{n})}\frac{d}{dt}\left[\frac{\mathbf{n}\times[\mathbf{S}+\mathbf{n}\times(\boldsymbol{\beta}\times\mathbf{S})]}{(1-\boldsymbol{\beta}\cdot\mathbf{n})}\right] \tag{12}$$

and the total radiation field will be

$$\mathbf{A}^{(T)}(t) = \mathbf{A}(t) + \eta\mathbf{A}^{(s)}(t)\,, \tag{13}$$

$$\eta = g\frac{\gamma^2}{\gamma+1}\frac{\hbar\omega}{4mc^2} \cong \frac{g\gamma}{4}\frac{\hbar\omega}{mc^2}\,. \tag{14}$$

Eq. (14) shows that at a given photon energy the contribution of the spin to the radiation density is inversely proportional to the mass of the particle.

Perform the time derivative in Eq.(12) and find

$$\mathbf{A}^{(s)}(t) = \frac{D\left[(\boldsymbol{\beta}\cdot\mathbf{s})\mathbf{n}\times\boldsymbol{\beta} + (\boldsymbol{\beta}\cdot\mathbf{s})\mathbf{n}\times\dot{\boldsymbol{\beta}}\right] + (\boldsymbol{\beta}\cdot\mathbf{s})(\dot{\boldsymbol{\beta}}\cdot\mathbf{n})\mathbf{n}\times\boldsymbol{\beta}}{D^3} + \frac{\boldsymbol{\beta}\cdot\dot{\mathbf{s}}}{D^2}\mathbf{n}\times\boldsymbol{\beta} \tag{15}$$

where the first term contains the spin and the second its time derivative.

The spin s precedes in an external magnetic field, according to the BMT equation

$$\frac{d\mathbf{s}}{dt} = C_1\mathbf{s}\times\boldsymbol{\Omega} + C_2(\boldsymbol{\beta}\cdot\boldsymbol{\Omega})(\mathbf{s}\times\boldsymbol{\beta}) \tag{16}$$

$$C_1 = 1+G\gamma \quad ; \quad C_2 = -\frac{G\gamma^2}{1+\gamma} \quad ; \quad G = g-2\,. \tag{17}$$

If only a transverse magnetic field ($\Omega_x = 0$, $\Omega_y \neq 0$) is present and the beam has no emittance, for observation *on axis*

$$\begin{aligned} &A_p = B\Omega_y && A_\sigma = 0 \\ &A_p^{(s)} = 0 && A_\sigma^{(s)} = -F[C_1\theta_b s_x + s_z]\Omega_y \end{aligned} \tag{18}$$

with the positions

$$\begin{cases} F = 1+\gamma^2(\Delta\theta^2+\Delta\phi^2) \\ B = 1-\gamma^2(\Delta\theta^2-\Delta\phi^2) \end{cases};\begin{cases} \Delta\theta = \theta-\theta_b \\ \Delta\phi = \phi-\phi_b \end{cases}. \tag{19}$$

Eq.(18) shows that on axis the spin dependent radiation is polarized in a direction parallel to the magnetic field.

The spectrum is obtained by a Fourier transform of the vector potential of Eq. (18). In an undulator with N periods (lines of width $1/N$.), write the magnetic field as

$$\Omega_y = \Omega_0\sin\omega_0 t \quad ; \quad \omega_0 = \frac{2\pi c}{\lambda_0}\,, \tag{20}$$

with λ_0 the undulator period. In this field, an approximate expression for the instantaneous angle of the trajectory is

$$\theta_b = \frac{k}{\gamma}\sin\omega_0 t \quad ; \quad k = \frac{e}{2\pi mc}\lambda_0 B = \frac{\gamma}{c}\lambda_0\Omega_0\,. \tag{21}$$

The first Eq. (18) yields

$$A_p(t) = (1-\gamma^2\theta_b^2)\Omega = \Omega_0(1-k^2\sin^2\omega_0 t)\sin\omega_0 t \tag{22}$$

showing that the spin independent radiation field only contains odd harmonics on axis (in this simplification, only 1 and 3).

The fourth Eq.(13) for the spin dependent radiation field is

$$A_\sigma^{(s)}(t) = -(1+\gamma^2\theta_b^2)[C_1\theta_b s_x + s_z]\Omega\,. \tag{23}$$

After integration of the BMT, e.g. if we assume example that the beam is totally x polarized at the entrance in the magnetic field, we obtain an expression also containing only odd harmonics,.

A first competitor that can make the observation of the spin dependent radiation difficult is the *finite emittance* of the beam. In this case, a contribution A_σ also appears on axis, but in the *even* spectral harmonics. The ratio is

$$\frac{A_\sigma^{(s)}}{A_\sigma} = \frac{2\eta}{\gamma}\sqrt{\frac{\beta_y}{\varepsilon_y}} \tag{24}$$

with ε_y and β_y are the emittance and the twiss function. This may impose a limit on beam emittance, and shows that it is convenient to make the beam vertically parallel and wide.

Another competing effect in the odd harmonics and in the σ polarization is due to undulator field imperfections, since a small $\delta\Omega_x$ field residual component with the *same periodicity* of the main field will again produce an unwanted A_σ on axis. A signal to noise ratio of the order of one is obtained when

$$\frac{\delta\Omega_x}{\Omega_y} \leq \eta = \frac{g\gamma}{4}\frac{\hbar\omega}{mc^2}\,. \tag{25}$$

This is of the order of 10^{-5} for protons, thus to measure the proton spin the contribution to the field errors in phase with the field should be very small. Using the random walk argument through N periods [3], we find an upper limit of the error in phase of the order of

$$\left(\frac{\delta\Omega}{\Omega}\right)_{\text{in phase}} = \frac{1}{\sqrt{N}}\left(\frac{\delta\Omega}{\Omega}\right)_{\text{random}}. \tag{26}$$

III. COMPTON BACK SCATTERING

The fundamental frequency of undulator radiation is

$$\omega_1 = \omega_0\frac{2\gamma^2}{1+\frac{1}{2}k^2+\gamma^2(\Delta\theta^2+\Delta\phi^2)}\,. \tag{27}$$

Compare this with the radiation obtained by scattering of laser light by a charged particle beam (head-on collisions)

$$\hbar\omega_s \cong \hbar\omega_L\frac{2(1+\beta)\gamma^2}{1+2(1+\beta)\gamma\frac{\hbar\omega_L}{mc^2}+\gamma^2\theta_s^2} \tag{28}$$

with ω_s and ω_L the frequency of the scattered and laser radiation, and θ_S the angle of the back scattered photon .

Let us treat the radiation from an undulator as a scattering process of a photon and a charged particle. In the PRF, the undulator is seen as an incoming e.m. wave, since Relativity builds up an electric field from the static magnetic field of the undulator. (Note that this equivalent wave does not travel at the speed of light, but at the lower speed βc).

The intensity of the back scattered radiation is related to the luminosity of the process as

$$n_s = \frac{N_p N_L}{\Sigma} f \sigma \qquad (29)$$

with n_s the number of back scattered photons per unit time, N_p and N_L the number of particles and primary photons contained in the volume of interaction, Σ the common cross section of the particle and laser beams, f the frequency of encounters, and σ the scattering cross section.

The radiation cone has a half-aperture defined by

$$\gamma\theta \approx 1 . \qquad (30)$$

If the particles have a spin, the scattering cross section can be written as follows [4]

$$\sigma = \sigma_0 \pm \sigma_1 P_L P_p \cos\phi \qquad (31)$$

where σ_0 is the scattering cs for the unpolarized beam, σ_1 the spin dependent scattering cs, P_L the polarization of the laser , P_p the particle spin and ϕ the scattering angle relative to P_p.

It can be shown that the unpolarized cross section is symmetric around the axis of scattering with a maximum there

$$\sigma_0\big|_{max} = 2r_0^2 \quad ; \quad r_0 = \frac{\mu_0}{4\pi}\frac{e^2}{m} \qquad (32)$$

(r_0 the classical radius of the proton), and that the polarized cross section is anti symmetric about the axis of scattering with maxima at angles determined by

$$\gamma\theta_s = \tfrac{1}{\sqrt{3}} . \qquad (33)$$

The ratio between maximum polarized cross section to maximum unpolarized is

$$\frac{\sigma_1\big|_{max}}{\sigma_0\big|_{max}} = \tfrac{3\sqrt{3}}{16}\xi = \tfrac{3\sqrt{3}}{4}\gamma\frac{\hbar\omega_L}{mc^2} . \qquad (34)$$

A comparison of the ξ parameter of Eq. (34) for laser scattering with the η parameter of Eq. (14) shows that the relative intensity of the polarized to unpolarized radiation is of the same order, as it should.

In the scattering of virtual undulator photons, the number of primary photons to enter in Eq. (29) can be calculated by dividing the magnetic energy by the energy of a photon

$$\frac{\varepsilon_0 c^2 B_0^2}{2\hbar\omega_0} V \qquad (35)$$

with V the volume of the particle beam bunch. Both for undulator radiation and BS, the intensity of the observed radiation is inversely proportional to the square of the mass of the particle.

IV. CONCLUSIONS

The spin polarization of the beam can be measured by synchrotron radiation or by Compton backscattering of laser light. The physical principles are similar for the two modes, with some important differences.

Undulator radiation shows the line structure of a diffraction pattern, with characteristic polarization properties. Compton BS produces a wide spectrum of radiation, correlated to the observation angle. If the undulator radiation is interpreted as a scattering of virtual photons, the expression for the luminosity shows that the number of undulator photons in the interaction region can be very large, since each photon (of frequency ω_0) is much smaller than a laser photon (of frequency ω_L).

Tables 1 and 2 show the order of magnitude of some of the quantities in selected machines. We have considered two proton machines: RHIC and SSC.

Table 1. Undulator and Laser Parameters.

mc^2 [MeV]	938
g	5.58
r_0^2 [m^2]	2.36 10^{-36}
undulator	
λ_0 [m]	0.20
B_0 [T]	3.2
N	20
$k =$	$0.05086\lambda_0 B_0 = 0.0326$
$\hbar\omega_0$ [eV]	6.20 10^{-6}
Laser scattering	
ND-Yag, 10 mJ, 1 KHz, $\lambda_L = 532$ nm	
$\hbar\omega_L$ [eV]	2.33

Table 2. Proton Colliders.

	RHIC	SSC
γ ($\approx$ GeV)	270 (250)	2.1 10^4 (20 TV)
emittance [10^{-9} π m-rad]	9	0.05
current	10^{11}/bunch	7.3 10^9/bunch
beta [m]	100	100
undulator radiation		
$\hbar\omega_1$ [eV]	4.4 (5th harm)	5.6 10^3
$\frac{d^2n}{d\theta d\phi}$ [1/sec-rad^2]	6.5 10^{22}	4.1 10^{26}
η , Eq. (31)	1.8 10^{-6}	0.17
backscattering		
$\hbar\omega_s$, max	66 MeV	4.2 GeV
ns [1/sec]	0.030	7.2 10^3
ξ , Eq. (81)	2.6 10^{-6}	2.1 10^{-4}

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