Betatron function measurement at LEP using the BOM 1000 turns facility

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Abstract

A new method for measuring the beta function around LEP is presented. The method uses phase difference measurements between three adjacent beam position monitors to obtain the value for the beta function at the monitors and in their neighbourhood, e.g. at interaction points, electrostatic separators etc. The phase differences are obtained from measuring coherent betatron oscillations for 1024 turns at the 504 beam position monitors. After a discussion of the accuracy of the method the measured values for the beta function are compared with the theoretical values for different lattices. In regular parts of the lattice (e.g. arcs) the beating of the beta function measured using this method agrees well with the beating obtained by another method using a fit of the phase over 15 monitors.

I. INTRODUCTION

During 1992 on several occasions the optics mismatch was measured at LEP using the phase difference measurements realised by the BOM (Beam Orbit Measurement system) 1024 turns facility. In this report a new method to obtain the experimental values of the beta and alpha functions from these phase difference measurements is presented. The accuracy of this technique has been studied by obtaining the error in the phase measurement and its good performance proved by comparing the results with other methods. This method is used for checking the machine optics, however a very precise measurement of the beta function will be very helpful at the radiation source for the exact calibration of emittance monitors, at Beam Position Monitors (BPM) for their calibration, at Interaction Points (IP) and at Electrostatic Separators (ES).

II. PHASE MEASUREMENT AT LEP

The 1024 turns beam position measurement at each BPM is used in combination with the LEP Q-meter. The Q-meter measures the fractional part (q) of the betatron tunes by exciting and observing coherent transverse oscillations in the horizontal and vertical planes with a single dedicated beam position monitor [1]. To measure the phase, one specific bunch is excited in one plane (horizontal or vertical) with a frequency close to the betatron tune. The amplitude of the bunch oscillations depends on the proximity of the exciting frequency to the tune and on the strength of the excitation. For a precise measure of the phase of these oscillations, the beam must be excited to high amplitude to gain in signal to noise ratio. The maximum amplitude is limited by the machine aperture but also the presence of non-linear elements (like sextupoles etc.) constrains the maximum due to non-linear fields. These oscillations are then sampled at each BPM for 1024 turns (i.e. during 91 msec) (see fig. 1). When the conditions of the machine are stable, a constant amplitude of a few milimeters is observed in the plane of excitation. Applying harmonic analysis we obtain the amplitude A and the phase μ of these oscillations at each BPM:

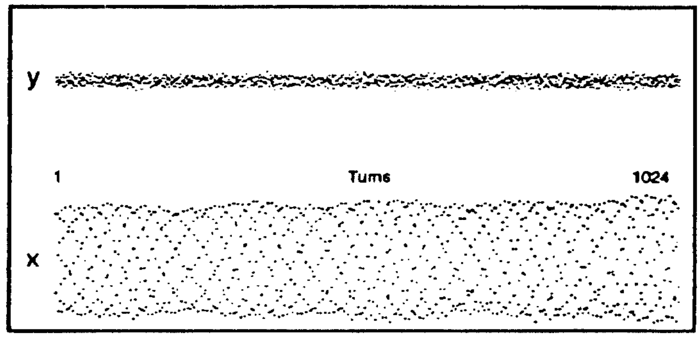

Figure 1: Single BPM recording the excited horizontal beam motion (scale: 8 mm peak to peak, time=88.9 μsec/turn)

$$A = \frac{2\sqrt{C^2+S^2}}{N} \qquad \mu = -\cot(\frac{S}{C})$$

where $N = 1024$ and

$$C = \sum_{i=1}^{N} x_i \cos(2\pi iq) \qquad S = \sum_{i=1}^{N} x_i \sin(2\pi iq)$$

Because the amplitude obtained is proportional to $\sqrt{\beta}$, one can compare the measured values of the amplitudes at the BPMs with the expected ones and deduce β. Unfortunately this method depends on the calibration factor of each BPM. However, the phase differences are measured with high precision and there is no systematic error since the phases are independent of individual monitor calibration errors.

III. PHASE ERROR MEASUREMENT

With constant amplitude, the error of the phase is proportional to the noise of the position signal of the BPM. Using harmonic analysis we obtain the following expression for the error:

$$\sigma_\mu = \frac{1}{A}\sqrt{\frac{2}{N}}\sigma_x \tag{1}$$

with:
N : the number of the samples (1024),

A : amplitude of the signal, and
σ_x : the estimate of the BPM error [3]; it is calculated on the basis that both contributions (the electronic noise and the numerical error of the ADC) are superimposed:

$$\sigma_x \approx \sqrt{\sigma^2_{Num.Error} + \sigma^2_{Electr.Noise}} = 0.07mm$$

On 40 occasions with different optics, tunes and excitation amplitude the phase difference was measured and the position data from each BPM recorded. In the following we discuss the results of the phase error obtained for these 40 data sets and compare these results with the error values expected shown above. First of all, the frequency of the oscillations is determined by taking the frequency with the maximum amplitude response using harmonic analysis. A precise value of the oscillation frequency is necessary to compute the phase, because the phase result is very sensible to small changes on the frequency selected for the harmonic analysis.
To calculate the error of the phase the procedure is to repeat the same measurement several times and obtain the sigma. In order to estimate the error in the phase measured at the BPMs, the technique used is to take segments of n points (n smaller than 1024, e.g. 512 or 256) from the first point and sliding it over the entire measurement of 1024 points. Applying harmonic analysis gives $(1025-n)$ phase values. The sigma of these phase results have been found to be 10 or 50 times higher than the expected phase error. In some cases, oscillations of the phase of 50 Hz (probably due to power supplies noise) have been observed in the results.
However, taking two BPM signals from the same data set and calculating the phase difference between them, the dispersion of the results decreases drastically and corresponds to the predicted phase accuracy (see fig. 2). Whatever ef-

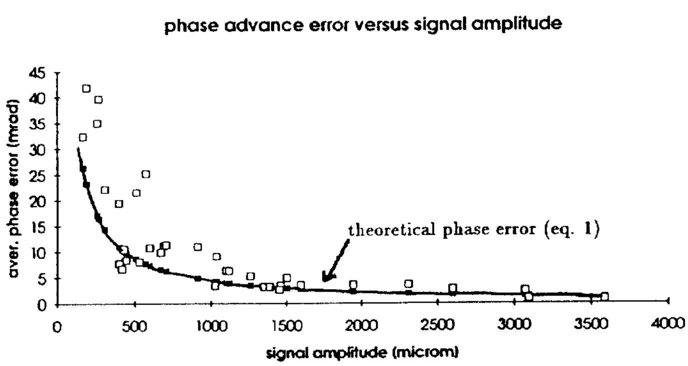

Figure 2: Phase difference error average

fect introduces a change in the oscillations (e.g. 50 Hz noise, changes in the natural tune etc) it is seen by all BPMs, and the phase difference is not very sensitive to beam perturbations. To summarize, a determination of the oscillation frequency with an accuracy less than $\sim 10^{-5}$ and an oscillation amplitude of at least 1 mm are needed to compute, through the Harmonic Analysis, the phase difference with an error of 4-5 mradians.

IV. BETA FUNCTION MEASUREMENT

Once the phase differences have been measured between all BPMs around LEP, the beta and alpha functions are obtained using the following method.
Let M be the transfer matrix of a charged particle from one point s_1 to another s_2:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1 \quad (2)$$

where x is the particle's displacement from the beam center and x' the angle and the elements m_{ij} can be expressed as [2]:

$$\begin{pmatrix} \cos\mu + \alpha_1 \sin\mu & \beta_1 \sin\mu \\ -\gamma_1 \sin\mu & \cos\mu - \alpha_1 \sin\mu \end{pmatrix} \quad (3)$$

where β is the beta function, $\alpha = -\frac{1}{2}\frac{d\beta}{ds}$, $\gamma = (1+\alpha^2)/\beta$ and μ is the phase difference defined as:

$$\mu = \int_{s_1}^{s_2} \frac{ds}{\beta(s)} \quad (4)$$

This matrix M is easily obtained by multiplying successively the transfer matrices for each existing element (drift space or quadrupole) between s_1 and s_2. The matrix elements of the first row

$$\frac{m_{11}}{m_{12}} = \frac{\cos\mu + \alpha_1 \sin\mu}{\beta_1 \sin\mu} \quad (5)$$

show the relationship between the optics parameters β and α at s_1 and μ the phase difference between s_1 and s_2. The phase difference is provided by the BOM 1024 turns facility and the matrix coefficients are calculated from the layout between two BPMs and reading the magnet strengths of the quadrupoles. Consequently, in equation (5) there are two unknown variables: α and β. Therefore, a set of three consecutive BPMs is selected.
Let m_{11} and m_{12} be the elements of the first row of the transfer matrix from monitor 1 to monitor 2 and N_{11} and N_{12} be the similar elements for the transfer matrix from monitor 1 to monitor 3, then the equations are:

$$\beta_1 \frac{m_{11}}{m_{12}} = \cot\Psi_{12} + \alpha_1 \quad (6)$$

$$\beta_1 \frac{N_{11}}{N_{12}} = \cot\Psi_{13} + \alpha_1 \quad (7)$$

with Ψ_{ij} : the phase difference between BPMs j and i.
Finally it yields:

$$\beta_1 = \frac{\cot\Psi_{12} - \cot\Psi_{13}}{(m_{11}\,/\,m_{12}) - (N_{11}\,/\,N_{12})} \quad (8)$$

$$\alpha_1 = \frac{(N_{11}\,/\,N_{12})\cot\Psi_{12} - (m_{11}\,/\,m_{12})\cot\Psi_{13}}{(m_{11}\,/\,m_{12}) - (N_{11}\,/\,N_{12})} \quad (9)$$

Alternatively, knowing the theoretical values of beta and the phase difference (by other programs like MAD), the expression of the experimental $\beta(exp)$ yields as the ratio between the measured $\cot\Psi_{12} - \cot\Psi_{13}$ and the theoretical one:

$$\beta_{1(exp)} = \beta_{1(theo)} \frac{\cot\Psi_{12(exp)} - \cot\Psi_{13(exp)}}{\cot\Psi_{12(theo)} - \cot\Psi_{13(theo)}} \quad (10)$$

and one easily deduces the equivalent expression for alpha. The method is limited for a regular structure such as the FODO cells in the arcs of LEP when the optics has a phase difference of 90 degrees between consecutive BPMs. In this case, $\cot\Psi$ is zero and β can not be calculated.

V. BEATING OF THE BETA FUNCTION MEASUREMENTS

During last years run, LEP was operated mainly using 90 degrees lattice optics and only a few times in special Machine Development (MD) schedules the 60° lattice optics was used. In the following pictures all the results are shown as the ratio between the experimental value of beta obtained by this method and the theoretical beta calculated with the MAD model [4]. Figure 3 shows the vertical beta function for 60° lattice at the BPMs obtained by this method (line and crosses) compared to the beta beating obtained by fitting [5] the measured phase difference over 15 BPMs (white boxes) in the arc between Interaction Point 2 (IP2) and IP3. The latter method makes the hy-

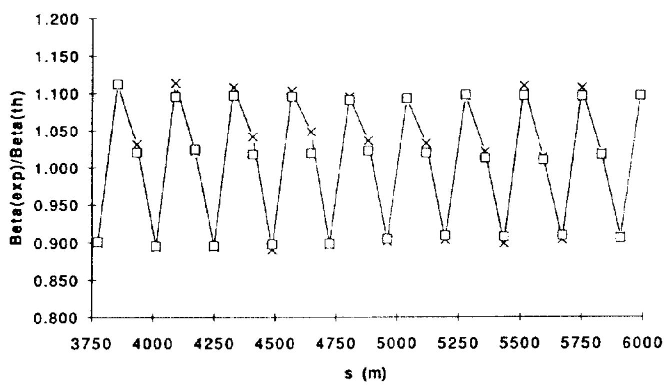

Figure 3: Comparison between beta beating fit method and the value of beta obtained with eq. 8

pothesis that the optics mismatch between the predicted and the measured phase difference is due to beta beating, while the former makes no assumption. The small difference between the results shows that almost all the effect seen is due to beta beating.
Figure 4 shows the beta function calculated at some of the beam instruments at LEP and in fig. 5 at the horizontal electrostatic separators. These values of beta are calculated from the ones obtained at the closest BPM around the instrument.

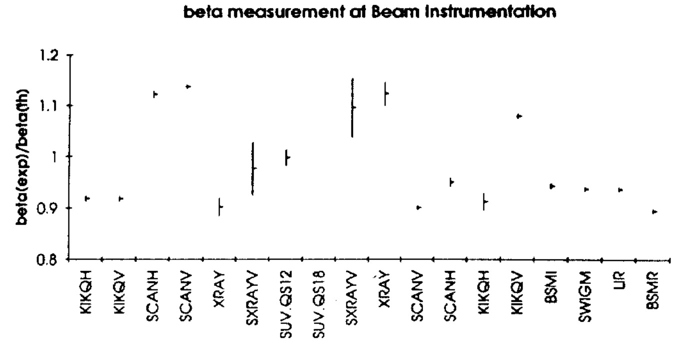

Figure 4: Vertical beta function measured at Beam Instrumentation (90° lattice, 46 GeV)

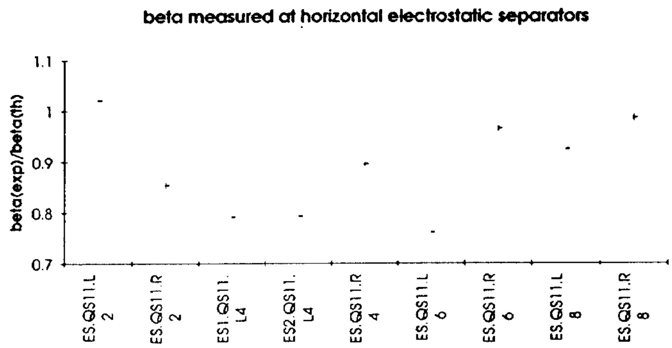

Figure 5: Horizontal beta function measured at Horizontal Separators (90° lattice, 46 GeV)

VI. CONCLUSION

From the BOM 1024 turns measurement the phase error is typically about 4-5 mrad for a signal of 1 mm of amplitude and 1024 points. An algorithm applied to three consecutives BPMs gives the local values of the betatron function assuming no magnetic error between the three BPMs. Apart from when the phase difference between BPMs is 90°, the value of beta is determined with a precision better than 5 % in general. From the same algorithm the alpha function is also calculated and both alpha and beta values can be transported from the BPMs to other points of interest such as at emittance monitors, radiation source instruments, electrostatic separators and interaction points.

VII. REFERENCES

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