BREMSSTRAHLUNG BY THE BUNCH OF ULTRARELATIVISTIC CHARGED PARTICLES INTO A THICK TARGET.

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1. INTRODUCTION.

Bremsstrahlung of a system of classically fast charged particles which do not interact with each other but which do undergo multiple elastic scattering by randomly positioned atoms of the medium is studied. We derived the spectrum of the bremsstrahlung of such particles through a systimatic kinetic analysis of radiation process in the medium. It is shown that the spectral distribution of the emission energy of the bremsstrahlung depends significantly on both the characteristics of the scattering of the particles in the medium and the parameters characterizing the minitial set of the particles.

2. STATEMENT OF THE PROBLEM.

We consider the system of charged ultrarelativistic ($E \gg m$) clasically fast ($E \gg \omega$ is a radiation frequency) particles which do not interact with each other (E, m, and e are the energy, the mass and the charge of the particle, $\hbar=c=1$). These particles enter a homogeneous, semi-infinite, amorphous scattering medium. In the initial period, $t=0$, particles are located in the points $\vec{r}_{01}$, $\vec{r}_{02}$, ..., $\vec{r}_{0N}$, and are the velocity $\vec{v}_{01}$, $\vec{v}_{02}$, ..., $\vec{v}_{0N}$, equal to $v_0=[1-(m/E)^2]^{1/2}$ and they are directed under the $|\Delta_\mu| \ll 1$ ($\mu=1,...,N$ - is the number of the particles) angle to the $\vec{e}_z$ vector (vector of the inward normal to the boundary of the medium). Let the characteristic longitudinal size of the beam L_b be such that $L_b v_0^{-1} \ll T$ (the time when the particles move in the medium).

3. SOLUTION OF THE PROBLEM.

The specrtal distribution of the energy emitted by these particles is

$$\frac{d\varepsilon_\omega}{d\omega} = \frac{e^2\omega^2}{2\pi^2} \mathrm{Re} \sum_{\mu,\nu=1}^{N} \int d\Omega_{\vec{n}} \int_0^T dt \int_0^\infty d\tau\, [\vec{n}\times\vec{v}_\mu] \cdot [\vec{n}\times\vec{v}_\nu] \cdot \exp\{-i\omega\tau + i\vec{k}\cdot(\vec{r}_\mu-\vec{r}_\nu + \vec{r}_{0\mu}-\vec{r}_{0\nu})\}, \quad (1)$$

$\vec{k}$ is the wave vector of the radiation field, $d\Omega_{\vec{n}}$ is an element of solid angle in the direction

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$\vec{n}=\vec{k}/k=\vec{k}/\omega$, $\vec{r}_\mu=\vec{r}_\mu(t+\tau)$, $\vec{v}_\mu=\vec{v}_\mu(t+\tau)$, $\vec{r}_\nu=\vec{r}_\nu(t)$, $\vec{v}_\nu=\vec{v}_\nu(t)$ are coordinates and velocities of the particles at the time $t+\tau$ and t respectively, τ is the time scale of the radiation formation (the coherence time), and the t is the time at which the radiation is emitted.

To calculate the observed spectral distribution of the emission energy of the particles in the medium, $dE_\omega/d\omega$, we must average expression (1) over all possible trajectories of the particles in the scattering matter [1]. To solve this problem it is necessary to find [2] two-time distribution function of the particles in a scattering medium. In the case of ultrarelativistic classically fast particles the solution of the problem is determined by the Fourier component of this function $F_k(\vec{v}_\mu, \vec{v}_\nu, t, \tau)$:

$$\frac{dE_\omega}{d\omega} = \frac{e^2\omega^2}{2\pi^2} \mathrm{Re} \sum_{\mu,\nu=1}^{N} \int d\Omega_{\vec{n}} d\vec{v}_\mu d\vec{v}_\nu \cdot \int_0^T dt \int_0^\infty d\tau$$

$$[\vec{n}\times\vec{v}_\mu]\cdot[\vec{n}\times\vec{v}_\nu]\cdot \exp\{-i\omega\tau+i\vec{k}\cdot(\vec{r}_\mu-\vec{r}_\nu+$$

$$+\vec{r}_{o\mu}-\vec{r}_{o\nu})\}\cdot F_k(\vec{v}_\mu, \vec{v}_\nu, t, \tau)$$

The function $F_k(\vec{v}_\mu, \vec{v}_\nu\ t,\tau)$ satisfies to the following equations and the initial condition [2]:

$$\frac{\partial F_k(t,\tau)}{\partial \tau} - i\vec{k}\cdot\vec{v}_\mu(\vec{\eta})\cdot F_k(t,\tau) = \frac{q}{4}\cdot\frac{\partial^2 F_k(t,\tau)}{\partial\vec{\eta}^2}$$

$$\frac{\partial F_k(t,0)}{\partial t} - i\vec{k}(\vec{v}_\mu(\vec{\eta})-\vec{v}_\nu(\vec{\phi}))F_k(t,0) = \frac{q}{4}\cdot\left[\frac{\partial}{\partial\vec{\eta}}+\frac{\partial}{\partial\vec{\phi}}\right]^2 F_k(t,0)$$

$$F_k(0,0) = \delta(\vec{\eta}-\vec{\Delta}_\mu)\cdot\delta(\vec{\phi}-\vec{\Delta}_\nu)\cdot$$

$$\cdot\exp[i\vec{k}\cdot(\vec{r}_{o\mu}-\vec{r}_{o\nu})],$$

where q is the average square of multiple scattering angle per unit of path [3], $\vec{\eta}$ and $\vec{\phi}$ the angle vectors which are connected with $\vec{v}_\mu(\vec{\eta})$ and $\vec{v}_\nu(\vec{\phi})$ by the ordinary formulae of the theory of multiple scattering in a amorphous medium [3].

4. RESULTS

We have constructed a consistent kinetic theory for the radiation emission by a system of classically fast noninteracting charged particles which undergo multiple elastic collisions in a scattering matter. We have found the spectral distribution of the emission energy from such particles. The obtained spectrum depends strongly both on the parameters of the scattering medium, and on the characteristics of the initial system of the particles.

We have studied in detail the emisson by the beam of idetical particles. It is shown in this case that the spectral distribution of the emission energy has at least one extremum in contradiction to the situation of an individual particle [1] when the emission spectrum is a monotonic function from emission frequency. If the pulse beam of identical particles takes place then the extremum is a maximum. Moreover if the width of the initial beam D is such that the condition $qD\xi^{-3} \ll \xi(qT)^{-1/2} \ll 1$ holds, the maximum of bremsstrahlung energy spectrum is a plateau with a width on the order $D^{-1}(qT)^{-1/2}$. The ratio of $(dE_\omega/d\omega)_{max}$ to the background level ($(dE_\omega/d\omega)_o = 2e^2qT/3\pi\xi^3$) is approximately equal to N , the number of emitting particles. As the parameter $qD\xi^{-3}$ increases $qD\xi^{-3} \leq \xi\cdot(qT)^{-1/2} \ll 1$ the plateau convert into " strict " maximum . As before , we have $(dE_\omega/d\omega)_{max}\cdot(dE_\omega/d\omega)_o^{-1} \cong N$. If we have $qD\xi^{-3} \gg 1$, then the quantities $(dE_\omega/d\omega)_{max}$ and $(dE_\omega/d\omega)_o$ become approximately equal to 1.

The bremsstrahlung by a higly anisotropic point source of ultrarelativistic particles is investigated. We show that in this case the spectral disrtibution of the emission energy has a maximum. The last is unique. The shape of the maximum as well as the magnitude of the emission energy depend strongly both on the value $\chi_o \ll 1$ of cone vertex angle which the includes the initial velocities of the particles and on on the scattering properties of a scattering medium.

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