Fundamental and Harmonics of Thomson Backscattered X-Rays from an Intense Laser Beam

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Abstract

We have formulated and obtained analytical expressions for Thomson backscattered x-ray radiation for an electron beam interacting with a linearly polarized electromagnetic undulator. The analytical expressions are valid for the fundamental and harmonics with arbitrarily large laser intensities. The formulation includes the effect of small angular misalignment between the laser pulse and the electron beam. This misalignment is found to increase the spectral width and distort the symmetry of the backscattered radiation.

I. INTRODUCTION

Tunable, near monochromatic, high brightness x-rays would be an important tool in research and medical diagnostics. Synchrotron light sources produce useful x-rays for a large user community. In this paper we examine a closely related method of x-ray generation, Thomson backscattering of x-rays from intense laser beams. [1-4] The configuration, shown in Fig. 1, consists of an electron beam intercepting an incoming laser pulse propagating in the opposite direction. Radiation is backscattered at a double Doppler upshifted frequency. The laser pulse acts in the same way as the static magnetic wiggler in synchrotron light sources or free electron lasers. [5-6]

One advantage of using a laser undulator is that the electron beam energy can be much lower than the electron beam energy using static magnetic undulators to obtain the x-ray of identical energy, since the wavelength of the laser may be many orders of magnitude smaller than that of the magnetic wiggler. Thus, this method has the virtue of being extremely compact.

II. FORMULATION

The laser pulse is assumed to be linearly polarized with frequency ω_L. The vector potential of the laser pulse can be separated into fast and slow components, $\mathbf{A}(\eta) = A(\eta)\sin(\eta)\hat{e}_x$, where $\eta = k_L z + \omega_L t$, $k_L = \omega_L/c$. The pulse shape $A(\eta)$ and the wavenumber k_L are assumed to be constants for the interaction time T and $\sin(\eta)$ is a fast oscillating component. We consider an electron with small initial transverse velocity, $\beta_{x0} \equiv v_{x0}/c$, $\beta_{y0} \equiv v_{y0}/c << 1$.

It is possible to separate the energy radiated per unit solid angle ($d\Omega$) per unit frequency ($d\omega$) per electron into two components

$$\frac{d^2 I}{d\omega d\Omega} = \frac{d^2 I_\theta}{d\omega d\Omega} + \frac{d^2 I_\phi}{d\omega d\Omega}. \quad (1)$$

Analytical expressions for each term on the right-hand side of Eq. (1) can be obtained.

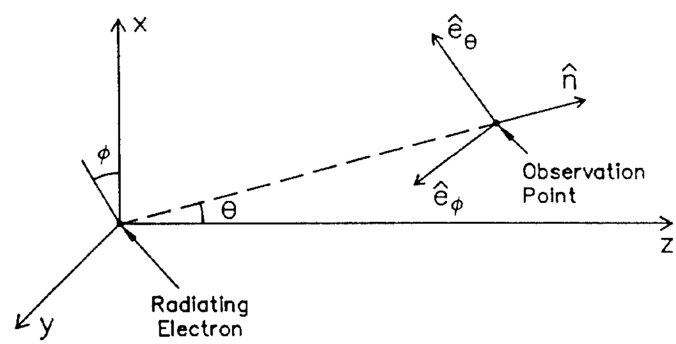

Fig. 1. Schematic of the Thomson backscatter configuration, where an electron beam intersects an incoming laser pulse.

The frequencies of the on-axis radiation associated with the peak intensity are

$$\omega_h = h 4\omega_L \gamma_0^2/(1 + a^2/2), \quad (2)$$

where h is the harmonic number, γ_0 is the relativistic factor, $a = (e/m_0 c^2)A$ and m_0 is the rest mass of the electron.

To obtain the analytical expression for (1), we assumed that the electron transverse motion is small, i.e., $a(\beta_{x0}^2 + \beta_{y0}^2)\gamma_z^2/2 << 1$, the laser intensity is not exceedingly large, i.e., $a << 2\gamma_0/\beta_{x0}$, and $k_\perp \Delta r << 1$, where Δr is the radius of the electron oscillation in the transverse direction driven by the laser and $k_\perp$ is a measure of the transverse wavenumber of the laser field. We find

$$\frac{d^2 I_\theta}{d\omega d\Omega} \simeq \frac{e^2\omega^2}{4\pi^2 c\omega_L^2} \left| \left[g_{0,\theta} I_0 - \frac{\bar{a}^2}{4\gamma_0^2} I_z \sin\theta - \frac{\bar{a}}{\gamma_0} I_x \left(\cos\theta\cos\phi + \bar{\beta}_{x0}\sin\theta\right) \right] \right|^2, \quad (3)$$

$$\frac{d^2 I_\phi}{d\omega d\Omega} \simeq \frac{e^2\omega^2}{4\pi^2 c\omega_L^2} \left| \left[g_{0,\phi} I_0 + \frac{\bar{a}}{\gamma_0} I_x \sin\phi \right] \right|^2, \quad (4)$$

where

$$g_{0,\theta} = (\bar{\beta}_{x0}\cos\phi + \bar{\beta}_{y0}\sin\phi)\cos\theta - \left(\bar{\beta}_1 - \frac{\bar{a}^2}{4\gamma_0^2}\right)\sin\theta, \quad (5a)$$

$$g_{0,\phi} = \bar{\beta}_{x0}\sin\phi - \bar{\beta}_{y0}\cos\phi, \tag{5b}$$

$\beta_{z0} = (1 - \gamma_0^{-2} - \beta_{x0}^2 - \beta_{y0}^2)^{1/2}$ is the initial axial velocity, $\bar{\beta}_1 = (1 - (1 + \gamma_0^2(\beta_{x0}^2 + \beta_{y0}^2))/(\gamma_0^2(1+\beta_{z0})^2))/2$, $\bar{\beta}_{x0} = \beta_{x0}/(1+\beta_{z0})$, $\bar{\beta}_{y0} = \beta_{y0}/(1+\beta_{z0})$ and $\bar{a} = a/(1+\beta_{z0})$. The expressions for I_0, I_x and I_z, written in terms of the harmonic number, are

$$I_0 = 2e^{i\psi_0}\sum_{h=1}^{\infty} i^h p_h \sum_m (-1)^m J_m(d_z) J_{h+2m}(d_x) \tag{6a}$$

$$I_x = -e^{i\psi_0}\sum_{h=1}^{\infty} i^h p_h \sum_m (-1)^m J_m(d_z) [J_{h+2m-1}(d_x) + J_{h+2m+1}(d_x)], \tag{6b}$$

$$I_z = -e^{i\psi_0}\sum_{h=1}^{\infty} i^h p_h \sum_m (-1)^m J_m(d_z) [J_{h+2m-2}(d_x) + J_{h+2m+2}(d_x)], \tag{6c}$$

$$d_x = \frac{\omega}{\omega_L}\left[\sin\theta\cos\phi + \bar{\beta}_{x0}(1+\cos\theta)\right]\frac{\bar{a}}{\gamma_0}, \tag{7a}$$

and

$$d_z = -\frac{\omega}{\omega_L}(1+\cos\theta)\frac{\bar{a}^2}{8\gamma_0^2}. \tag{7b}$$

The form for p_h is $p_h = \pi N_o \sin\chi_h/\chi_h$ and it is peaked at $\chi_h = 0$, where $\chi_h = (d_0 - h)\pi N_o$,

$$d_0 = \frac{\omega}{\omega_L}\left\{1 + \left[-(\bar{\beta}_{x0}\cos\phi + \bar{\beta}_{y0}\sin\phi)\sin\theta - \left(\bar{\beta}_1 - \frac{\bar{a}^2}{4\gamma_0^2}\right)(1+\cos\theta)\right]\right\},$$

and N_o is the number of periods in the laser pulse. See Ref. [3] for the derivation.

The analytical expression for the radiated energy per unit solid angle per unit frequency per electron, given by the sum of the expressions (3) and (4), with definitions given by (5a-b), (6a-c) and (7a-b), are valid for a wide range of values of laser amplitudes. For $a << 1$, only fundamental radiation will be observed. Intensity of harmonic radiation becomes important for $a > 1$.

III. NUMERICAL RESULTS

In this section we present numerical results for the energy radiated per unit solid angle per unit frequency per electron. Three examples are given: 1) no transverse beam velocity, and beam intercepting the laser at a small angle 2) $\beta_{x0} = 0.005$ and 3) $\beta_{y0} = 0.005$. In all cases, the electron beam has $\gamma_0 = 80$. The normalized laser amplitude is $a = 1.0$ and the number of periods in the laser pulse is taken to be 20. The small number of periods is not typical for a laser pulse, but it illustrates the principles, while avoiding difficulties in displaying data with very narrow line widths.

For the electron beam without initial transverse velocity, the backscattered radiation is peaked on-axis for the fundamental and the odd harmonics and is null on-axis for even harmonics. Figure 2 is a plot of $d^2I/d\omega d\Omega$ as a function of normalized frequency and angle θ (evaluated in the $\phi = 0$ plane), where ω_1 is the frequency of the fundamental based on Eq. (2). The contour plots of the intensity distribution in the x-y plane are shown in Figs. 3a-f. The intensities are normalized to the peak of the fundamental for (a) $\omega = 0.94\omega_1$ and (b) $\omega = 1.0\omega_1$. The intensities are normalized to the peak of the second harmonic for (c) $\omega = 1.94\omega_1$ and (d) $\omega = 2.0\omega_1$. The intensities are normalized to the peak of the third harmonic for (e) $\omega = 2.94\omega_1$ and (f) $\omega = 3.0\omega_1$. This shows that the fundamental is close to axially symmetric and peaked on-axis. The second harmonic has mirror symmetry with respect to the y-axis and null on-axis. The third harmonic has mirror symmetry with respect to the x-axis and peaked on-axis.

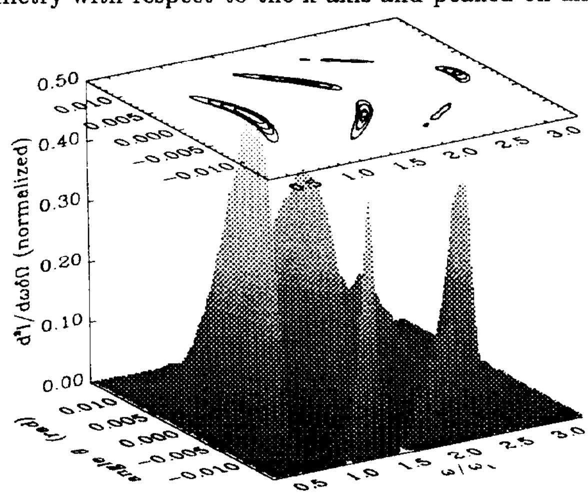

Fig. 2. Plot of normalized $d^2I/d\omega d\Omega$ per electron as a function of normalized frequency and angle θ (evaluated in the $\phi = 0$ plane) for an electron with no initial transverse velocity.

With this formulation, we can examing the effect of misalignment of the laser with respect to the beam. For $\beta_{y0} = 0.005$, all the radiation moves off-axis with the center located to $(x/R = 0.0, y/R = 0.005)$. For $\beta_{x0} = 0.005$, the radiation profiles become distorted.

Figure 4 is a plot of the energy radiated as a function of frequency and angle θ (evaluated in the $\phi = 0$ plane), for $\beta_{x0} = 0.005$. The contour plot of the intensity distribution in the x-y plane are shown in Figs. 5a-f. The intensities are normalized to the peak of the fundamental for (a) $\omega = 0.94\omega_1$ and (b) $\omega = 1.0\omega_1$. The intensities are normalized to the peak of the second harmonic for (c) $\omega = 1.94\omega_1$ and (d) $\omega = 2.0\omega_1$. The intensities are normalized to the peak of the third harmonic for (e) $\omega = 2.94\omega_1$ and (f) $\omega = 3.0\omega_1$. This figure shows that the fundamental is not symmetric and peaked off-axis at $(x/R = 0.005, y/R = 0.0)$. The second harmonic becomes almost axially symmetric peaked at the same position as the fundamental. This is completely different from the perfectly aligned case. The third harmonic also becomes more axially symmetric and the peak is off-axis, as in the case of the fundamental.

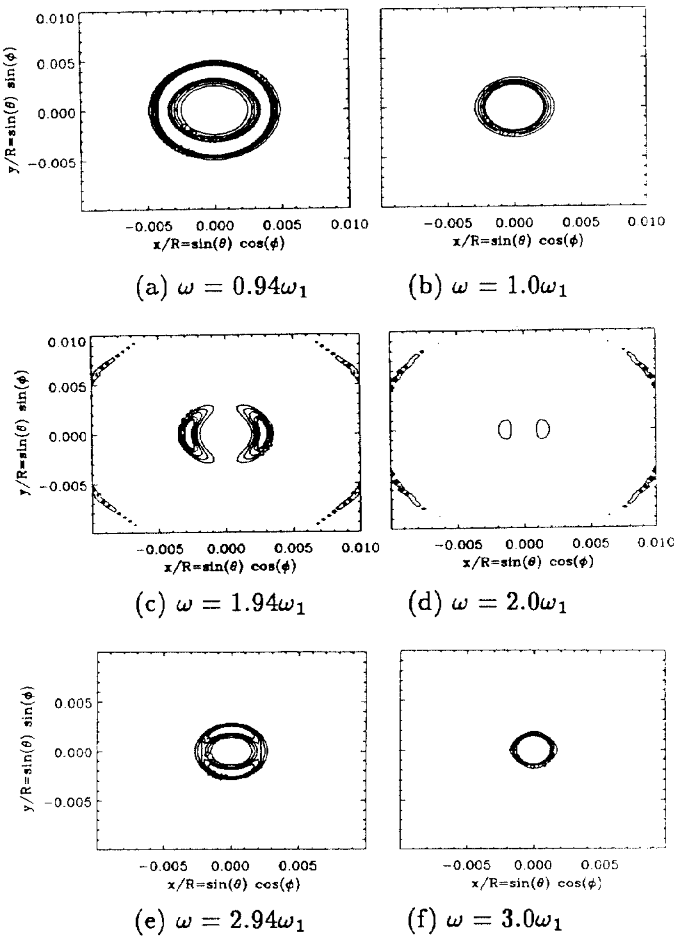

Fig. 3. Contour plots of $d^2I/d\omega d\Omega$ in the x-y plane for an electron beam with $\beta_{x0} = 0.0, \beta_{y0} = 0.0$.

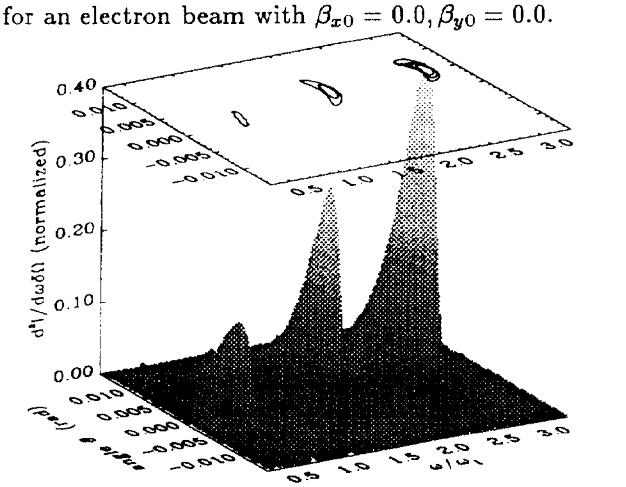

Fig. 4. Plot of normalized $d^2I/d\omega d\Omega$ per electron as a function of normalized frequency and angle θ (evaluated in the $\phi = 0$ plane) for $\beta_{x0} = 0.005$.

IV. CONCLUSION

We have derived an analytic expression for the intensity distribution of Thomson scattered radiation for the case of a linearly polarized laser pulse incident on a counterpropagating electron beam. We have calculated the effects of small initial transverse momentum, including the distortion of the intensity distribution, the reduction in on-axis intensity, and the increase in bandwidth. We are currently extending this work to consider the effects of emittance (for a distribution of electrons) and laser pulse shape on the scattered radiation.

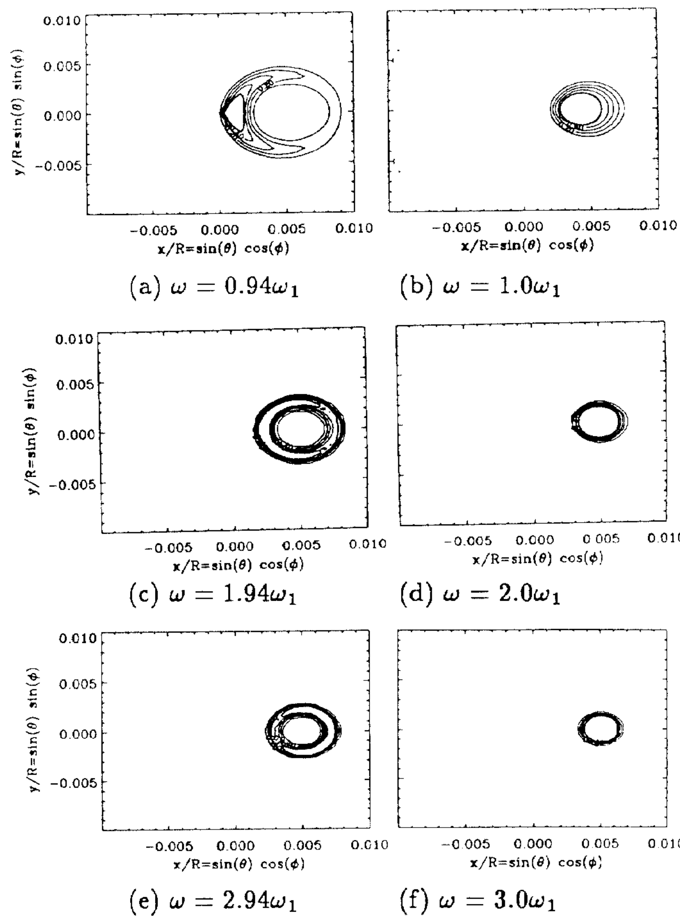

Fig. 5. Contour plots of $d^2I/d\omega d\Omega$ in the x-y plane for an electron beam with $\beta_{x0} = 0.005, \beta_{y0} = 0.0$.

Acknowledgements

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