An Optical Approach to Emittance Compensation in FELs*

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Abstract

We present a new approach to compensating for the emittance in very short wavelength Free Electron Lasers (FELs). The idea is based on the realization that the impact of finite emittance is to "wash out" the phase coherence of the electrons after passing some distance through the wiggler. This occurs because the electrons undergo betatron oscillations and those electrons with the largest transverse motion must travel a longer path. The new approach is to compensate for this by introducing an intense optical beam colinear with the electrons. If the beam has a transverse Gaussian profile in the field then the core electrons see on the average a higher field strength and undergo larger oscillations can retard the core electrons sufficiently to allow them to stay in phase with the electrons with large betatron excursions. This paper presents the derivation of this effect, details of the physical interaction and simulation results for sample cases. Limitations as to the practicality of the approach are also discussed.

I. INTRODUCTION

The sensitivity of FEL gain to the electron beam energy spread and emittance is a major limitation especially when wavelengths in the DUV to soft X-ray region are considered. At such short wavelengths the beam emittance and/or energy spread becomes a limiting factor in the performance of most practical devices. Many designs have resorted to very long wigglers or very high peak currents in a MOPA configuration to achieve the required gain since mirrors have limited reflectivity in this region. Early proposals to improve the FEL acceptance for such situations worked with dispersed electrons and involved wiggler modifications to introduce a gradient in the wiggler resonant field. Recent work[1] involves modifications of the electron beam momentum distribution by means of a FODO channel and accelerator cavities operating on the TM_{210} mode to establish a correlation between energy and amplitude of transverse oscillations. These ideas have shown the potential to reduce demands on the accelerator energy and on wiggler length with concomitant cost savings. Presented below is a different idea to accomplish a similar goal, that is to reduce the negative impact of transverse motion of electrons in a wiggler.

The idea of reducing emittance sensitivity is based on the realization that phase coherence is lost because electrons which spend the most time nearest the core are ahead of others after passing some length of the wiggler. (See Figure 1.) The idea of Ref. 1 is to have the electrons on the outside have higher energy so as to better maintain coherence. Our suggestion is the opposite: slow down γ_z for the core e's. This would be accomplished by co-propagating with the e's a non-resonant optical beam of high intensity. (See Figure 2.) Assuming a radial profile to the optical beam, the core electrons wiggle more strongly than those on the outer edges of the distribution the central electrons, take a longer path, are slightly retarded in phase, and therefore remain in resonance longer in terms of the parallel gamma. This increases gain and effectively decreases the influence of emittance.

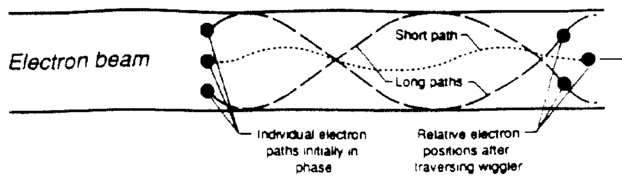

Figure 1. Matched electron beam in wiggler

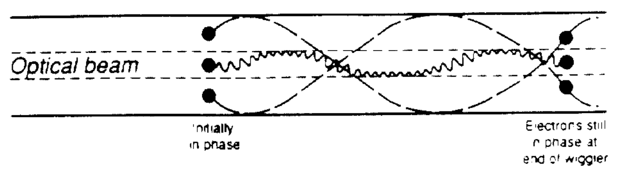

Figure 2. As Figure 1 but with addition of intense optical beam at core

The matching can be calculated in terms of $\gamma_{\parallel}$

$$\lambda_s = \frac{\lambda_w}{2\gamma_{\parallel}^2} \text{ where } \gamma_{\parallel}^2 = \gamma_0^2 / \left(1 + K^2 + \ldots\right)$$

and where λ_s is the FEL wavelength, λ_w is the wiggler wavelength, and $\gamma_{\parallel}$ is the standard relativistic factor but projected onto the z (propagation) axis. Typically terms other than K^2 are ignored. K is a function of offset from the axis; finite emittance requires a radial profile to the electron density. For a linear wiggler with infinite planes oriented with the field in the $\hat{y}$ direction

$$\gamma_{\parallel}^2 = \gamma_0^2 / \left(1 + \frac{K^2}{2}\left(1 + \frac{k_w^2 y^2}{2}\right)\right).$$

The result of the finite emittance of the electron beam is a variation on the order of 0.1% to 1% in the effective K^2 over the beam radius leading eventually to a phase mismatch across the beam. The phase slip is

$$\frac{dv}{dz} = k_w - \frac{k_s}{2\gamma^2}\left(1 + K^2 - 2\alpha_w \alpha_s \cos\Psi + \gamma^2 \beta_\perp^2 + a_{s1}^2\right)$$

where the new term a_{s1}^2 represents the addition of a new optical wave $a_{s1}^2 = \frac{eE_{s1}}{mc^2 k_{s1}}$. As an engineering formula for Gaussian beams at focus $a_{s1}^2 = 1.4 \times 10^{-15} \lambda_{s1} P_{s1} / R_{L1}$, where

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P_{s1} is the power of the additional laser and R_{L1} its Rayleigh range. We will attempt to decrease a_{s1}^2 off axis to compensate K^2 increase. It is important to note at this point that this effect is occurring in a non-resonant way, i.e., there is no particular restriction on the frequency relationship between the lasing wavelength and the new wave. In practice we will want $\omega \gg \omega_{s1}$, but such that many oscillators occur in a betatron period.

For resonant electrons at $\Psi = 0$ (a constant wiggler),

$$\Delta \upsilon = 2\pi N\left[2\frac{\Delta\gamma}{\gamma} - \frac{K^2 k_w^2 \gamma^2 \theta^2 + a_{s1}^2}{1+K^2}\right] \tag{1}$$

$$f(r) = \frac{e^{-r^2/2\bar{r}^2}}{2\pi\bar{r}^2} \qquad f(\Theta) = \frac{e^{-\theta^2/2\bar{\theta}^2}}{2\pi\bar{\theta}^2}$$

$$\Delta \upsilon_{\text{beam}} = 2\pi N\left[2\frac{\Delta\gamma}{\gamma} - \frac{K^2 k_w^2 \bar{r}^2 + 2\gamma^2\bar{\theta}^2 + a_{s1}^2}{1+K^2}\right] \tag{2}$$

For a matched beam, the first two terms in the numerator of the second term are equal and

$$r_m = \left[\frac{\sqrt{2}\varepsilon}{k_\beta}\right]^{\frac{1}{2}}.$$

In this case the phase slip refers to an average over the beam profile. Imagine now the effect of a_{s1} in Eq. (1) remembering that both K and a_{s1} are functions of r. K increases off-axis and a_{s1} decreases off-axis. With a proper choice of radial profile the laser beam can tend to compensate for the increase of wiggler field off-axis so that the resonant field is maintained over a larger volume. It is helpful at this point to consider an example: the proposed CEBAF IR FEL where $r_m = 0.34$ mm, $\lambda = 6$ cm, and $K = 1.76$. The emittance-driven two terms are 4×10^{-3} total. They represent an equivalent energy spread of 10^{-3}. If we introduce a 10^{13} W, 1 μm laser on axis with $R_{L1} = 1$ m, then $a_{s1}^2 = 1.4\times10^{-3}$ on axis and could therefore have a significant canceling effect.

II. MODELING

1. Original 1-D Model

The following is a brief introduction about the 1-D model for FEL modeling when the external laser is not applied.

First, we assume that the wiggler field B_w and the laser field (E_s, B_s) have the following forms

$$B_w = B_0 \cos(k_w z)\hat{y}, \tag{3}$$

$$E_s = E_{s0} \cos\Psi\hat{x}, \tag{4.1}$$

$$B_s = B_{s0} \cos\Psi_s\hat{y}, \tag{4.2}$$

where $k_w = 2\pi/\lambda_w$ is the wiggler wavenumber, B_0 the peak on-axis wiggler magnetic field, $\Psi_s = (k_s z - w_s t + \phi)$ the phase of the optical field, and w_s and k_s are the angular frequency and wavenumber of the laser field, respectively.

Then the one-dimensional equations describing the interaction between the electrons and the optical fields with a linearly polarized wiggler configuration can be written in mks units as

$$\frac{d\gamma_i}{dz} = -k_s a_s a_w F_\xi \frac{\sin\Psi_i}{\gamma_i}, \tag{5.1}$$

$$\frac{d\Psi_i}{dz} = k_w - \frac{k_s}{2\gamma_i^2}(1 + a_w^2 + a_s^2 - 2a_s a_w \cos\Psi_i) + \frac{d\phi}{dz}, \tag{5.2}$$

$$\frac{d\phi}{dz} = \frac{\omega_p^2}{2c^2 k_s}\frac{a_w F_\xi}{a_s}\left\langle\frac{\cos\Psi}{\gamma}\right\rangle, \tag{5.3}$$

$$\frac{da_s}{dz} = \frac{\omega_p^2}{2c^2 k_s} a_w F_\xi \left\langle\frac{\sin\Psi}{\gamma}\right\rangle - \alpha a_s, \tag{5.4}$$

where

$a_w = eB_0/\sqrt{2}m_0 c k_w$ is the rms wiggler parameter;
$a_s = eE_s/\sqrt{2}m_0 c^2 k_s$ the normalized optical electrical field strength;
$\omega_p^2 = (e/m_0)Z_0|J|$ the electron plasma angular frequency;
e the electron charge amount;
c the speed of light;
m_0 the rest mass of an electron;
Z_0 the wave impedance of free space;
$J = I_p/\Sigma_0$ the electron current density;
I_p the electron micropulse current;
Σ_0 the average optical mode area;
α the attenuation coefficient of the optical field in the sense $a_s(z) = a_{s0}e^{-\alpha z}$;
$\langle\ldots\rangle$ averaging over sample electrons;
γ_i the relativistic factor of the ith electron;
$\Psi_i = (k_w + k_s)z - w_s t + \phi$ the phase of the ith electron in the ponderomotive potential well;
ϕ the slowly varying phase of the laser field; and
$F_\xi = J_0(\xi) - J_1(\xi)$ the coupling constant resulting from the linear wiggler configuration with $\xi = 0.5a_w^2/(1+a_w^2)$.

2. Modified 1-D Model

When the external laser is added, the original 1-D FEL model is modified as follows

$$\frac{d\gamma_i}{dz} = -k_s a_s a_w(y) F_\xi \frac{\sin\Psi_i}{\gamma_i}, \tag{6.1}$$

$$\frac{d\gamma_i}{dz} = k_w - \frac{k_s}{2\gamma_i^2}\Big(1 + a_w^2(0) + (a_w(0)k_w y)^2 + a_{s1}^2(y) + a_s^2 - 2a_s a_w(y)\cos\Psi_i\Big) + \frac{d\phi}{dz}, \tag{6.2}$$

$$\frac{d\phi}{dz} = \frac{w_p^2}{2c^2 k_s}\frac{a_w(y) F_\xi}{a_s}\left\langle\frac{\cos\Psi}{\gamma}\right\rangle, \tag{6.3}$$

$$\frac{da_s}{dz} = \frac{w_p^2}{2c^2 k_s} a_w(y) F_\xi \left\langle\frac{\sin\Psi}{\gamma}\right\rangle - \alpha a_s, \tag{6.4}$$

where $a_{s1} = eE_{s1} / \sqrt{2}m_0c^2k_{s1}$ is the normalized strength parameter of the external laser.

It is noted that the main difference between eqs. (5.1)–(5.4) and eqs. (6.1)–(6.4) is that two terms, $(a_w(0)k_w y)^2$ and $a_{s1}^2(y)$, are added in eq. (6.2). In addition, all the fast-varying terms resulting from the external laser are neglected.

4. Canceling Effect or Compensation?

Let's look at the term

$$(a_w(0)k_w y)^2 + a_{s1}^2(y), \tag{7}$$

in eq. (6.2).

If we expand

$$a_{s1}^2(y) = a_{s1}^2(0)\left(1-(y/\sigma_{s1})^2\right), \tag{8}$$

and write expression (7) into

$$(a_w(0)k_w y)^2 + a_{s1}^2(0) - a_s^2(0)(y/\sigma_{s1})^2, \tag{9}$$

and then if we consider

$$(a_w(0)k_w y)^2 - a_{s1}^2(0)(y/\sigma_{s1})^2 = 0, \tag{10}$$

or

$$a_{s1}(0)/\sigma_{s1} = a_w(0)k_w, \tag{11}$$

this is a canceling effect.

If we consider

$$a_{s1}^2(0) \gg (a_w(0)k_w y)^2, \tag{12}$$

it is a compensation, since the term $a_{s1}^2(0)$ predominates in the phase evolution.

We may probably have the compensation effect only, since eq. (11) sets $a_{s1}(0)$ to a very high value that cannot be easily accomplished by using an E-M wave (the magnetostatic wiggler is the only way to get a field strength parameter like $a_w \geq 1$, and this is why few E-M wave wigglers or electrostatic wigglers are used for FELs). In addition, one principle should be that when the external laser is added, the resonance wavelength should not change too much.

III. SIMULATION RESULTS

Figure 3 shows the impact of applying the additional laser field. As an example, we are applying this idea to the CEBAF IR FEL. The parameters for the additional laser are: λ_{s1} (wavelength) = 1 μm, σ_{s1} (spot size) = 0.4 mm, R_L (Rayleigh range) = 1 m. The different field strength parameters are chosen for the additional laser, where a_{s1} = 0.0 and 0.02, respectively, for the curves #1 and #2, a_{s1} = 0.088 for curve #3, and a_{s1} = 0.2 for curve #4. The FEL power gain is calculated versus the position of the electrons in the y-direction. Note that when there is no additional laser, the gain drops to -8% at y_0 = 1.5 mm. But if the additional laser is applied with the parameters chosen in the calculation, we still can have a gain of 18% at y_0 = 1.5 mm.

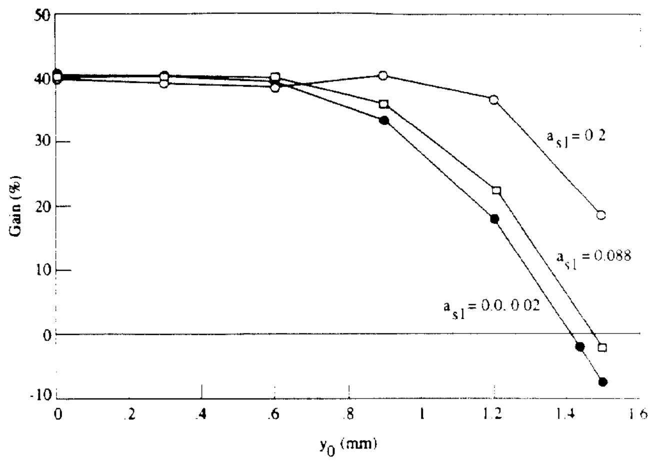

Figure 3. FEL gain for four different optical field strengths

It is pointed out that the above results were obtained with the 1-D model. Although we believe some of the most important aspects of this idea can be reflected with the 1-D model, as for FELs with no additional laser, the real 3-D calculations are necessary to confirm our estimations. Some further calculations will be presented in the near future.

III. REFERENCES

[1]A. M. Sessler, D. H. Whittum, and Li-Hua Yu, *Phys. Rev. Lett.* **68**, 309 (1992).