Electron Beam Quality Limitations and Beam Conditioning in Free Electron Lasers*

Phillip Sprangle[a)], B. Hafizi[b)], Glenn Joyce[a)] and Philip Serafim[c)]

a) Plasma Physics Division, Naval Research Laboratory, Wash., DC 20375-5346
b) Icarus Research, 7113 Exfair Road, Bethesda, MD 20814
c) Northeastern University, Boston, MA 02115

Abstract

The operation of free electron lasers can be severely limited by the axial velocity spread of the beam electrons. We propose a method for reducing the axial velocity spread in electron beams by redistributing the electron energy via interaction with an axially symmetric, slow, TM waveguide mode. In this method, the energy redistribution is correlated with the electrons' betatron amplitude. Reductions of more than a factor of 40 in the rms axial velocity spread have been obtained in simulations.

Many coherent radiation generation mechanisms are based on the longitudinal bunching of electron beams. These sources include traveling wave tubes and free electron lasers (FELs) [1]. The degree to which an electron beam can be bunched is a strong function of the beam quality. The two independent contributions to the electron beam quality are the intrinsic energy spread and emittance, both of which lead to a spread in the axial electron velocity and limit the operating wavelength, gain and efficiency of the device [1]. A method for conditioning, i.e., reducing the axial beam velocity spread, was recently proposed in which the beam was propagated through a periodic array of focusing, drift, defocusing, drift channels and microwave cavities [2].

Here, we propose an alternative conditioning method which redistributes the electrons' energy according to their betatron amplitude by using the electric field of an axially symmetric, slow, TM waveguide mode [3].

In the FEL mechanism, the resonance condition is $\omega - v_z\,(k + k_w) \simeq 0$, where $\omega = ck$ is the frequency, v_z is the axial electron velocity, $k_w = 2\pi/\lambda_w$ and λ_w is the wiggler wavelength. This condition implies that the beam's axial velocity spread, δv_z, should satisfy $\delta\beta_z << \lambda/2L$ where $\delta\beta_z = \delta v_z/c$, λ is the radiation wavelength and L is the interaction length (e-folding length) of the radiation field in the low gain (high gain) regime. The axial velocity spread can be written as

$$\delta\beta_z = \left((1 + a_w^2/2)\delta\gamma/\gamma - \varepsilon_n^2/2r_b^2\right)/\gamma^2$$

where $\gamma = 1 + E/m_oc^2$ is the relativistic factor, E is the beam energy, $\delta\gamma/\gamma$ is the fractional intrinsic beam energy spread, ε_n is the normalized emittance, $a_w = |e|B_w/(k_w m_o c^2)$ is the wiggler strength parameter, B_w is the wiggler field amplitude and r_b is the radius of the matched electron beam. In many cases, electron beam quality is limited by the emittance contribution and not the energy spread term, i.e., $\delta\gamma/\gamma << (1/2)(\varepsilon_n/r_b)^2$. In any case, it is clear that electron beam quality, in particular, $\delta\beta_z$, limits the operation of FELs.

To analyze our conditioning method we consider the electron trajectories in a planar wiggler with parabolic pole faces. These orbits consist of rapidly varying (wiggler period scale length) and slowly varying (betatron period scale length) terms [4].

In the highly relativistic limit, the axial particle velocity normalized to the speed of light is given by $\beta_z \simeq 1 - 1/2\gamma^2 - (\beta_x^2 + \beta_y^2)/2$, where $\beta_{x,y} = v_{x,y}/c$ is the ratio of the transverse velocity components to the speed of light. The square of the perpendicular velocity, averaged over the wiggler period, is independent of z. Substituting the fast and slow orbits into the expression for β_z and setting $\gamma = \gamma_o + \delta\gamma$, where $\delta\gamma$ is the electron's energy deviation term, we find that $\beta_z = \beta_{oz} + \delta\beta_z$, with $\beta_{oz} = 1 - (1 + a_w^2/2)/(2\gamma_o^2)$, and

$$\delta\beta_z = \left(1 + a_w^2/2\right)\delta\gamma/\gamma_o^3 - k_\beta^2\, r_o^2/2. \qquad (1)$$

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Here γ_o is the gamma associated with the reference electron, traveling along the z-axis without a betatron oscillation, $r_o^2 = x_o^2 + y_o^2$, where x_o, y_o are the amplitudes of the slow components of the displacement from the axis and terms varying on the wiggler wavelength scale have been neglected. The normalized beam emittance, for a matched beam in the focusing fields of the wiggler, is $\varepsilon_n = \gamma_o k_\beta r_b^2$, where $k_\beta = a_w k_w / 2\gamma$ is the betatron wavenumber. Note that the emittance contribution to the velocity spread in (1), i.e., $k_\beta^2 r_o^2/2$ is independent of propagation distance. It will be assumed that the axial velocity spread due to emittance initially dominates the velocity spread caused by the intrinsic energy spread.

The proposed conditioning field is an axially symmetric, slow, TM waveguide mode with axial electric field

$$E_z = - E_o I_o(k_\perp r) \cos \psi,$$

together with the associated transverse electric and magnetic fields, where E_o is the maximum electric field amplitude on axis, $k_\perp$ is the transverse wavenumber, k is the axial wavenumber, $\omega = c(k^2 - k_\perp^2)^{1/2}$ is the frequency, $\psi = kz - \omega t$ is the phase and I_n is the modified Bessel function of order n. The axial phase velocity of the traveling wave is matched to the axial beam velocity, $\beta_{ph} = \omega/ck = \beta_o$, where $\beta_o = (1 - 1/\gamma_o^2)^{1/2}$ is the normalized axial velocity of the reference electron. To maintain synchronism between the conditioning field and the electrons, the axial and transverse wavenumbers must satisfy $k = (\omega/c)\gamma_o(\gamma_o^2 - 1)^{-1/2} = (\omega/c)/\beta_o$ and $k_\perp = (\omega/c)(\gamma_o^2 - 1)^{-1/2} = (\omega/c)/(\gamma_o\beta_o)$, respectively.

In our conditioning method the beam electrons are given an energy increment which cancels out the emittance contribution to the axial velocity spread. To reduce the velocity spread to zero, the conditioning field must give all the individual electrons a different fractional energy increment, $\delta\gamma_c/\gamma_o$, given by

$$\frac{\delta\gamma_c}{\gamma_o} = \frac{\gamma_o^2 k_\beta^2 r_o^2}{2\left(1 + a_w^2/2\right)} = \frac{\varepsilon_n^2 r_o^2}{2\left(1 + a_w^2/2\right) r_b^4}.$$

The energy increment is proportional to the square of the betatron amplitude and the electron pulse length remains approximately constant. Our results show that the degree of beam conditioning can be significantly improved by removing the accelerating component of the TM field. It can be shown that complete conditioning of the beam is achieved at $k_\beta z = n\pi$, (n = 1,2,3,..) provided the normalized strength of the waveguide field, $a_o = |e|E_o/(m_o c\omega)$, is given by

$$a_o = \frac{4\gamma_o^5 (k_\beta/k)^3}{n\pi(1+a_w^2/2)} = \frac{\gamma_o^2}{2\pi n} \frac{a_w^3}{(1 + a_w^2/2)} \left(\frac{\lambda}{\lambda_w}\right)^2. \tag{2}$$

The conditioning method is illustrated with full scale particle simulation of two examples, a 10 MeV and a 1 MeV electron beam, see Table I. For the 10 MeV example, the axial velocity spread of the beam in the conditioning fields will reach a minimum at $z \simeq \pi/k_\beta = 375$ cm, beyond which it increases to its original value. To maintain the minimum spread the conditioning field is adiabatically removed at $z \lesssim \pi/k_\beta$. Figure 1 shows the evolution of the fractional axial energy spread for 100 randomly selected electrons as a function of distance along the waveguide. The convergence of the trajectories in Fig. 1 with propagation distance indicates that the spread in axial velocity of the electrons is significantly reduced by the conditioning field. Figure 2 shows the root mean square (rms) beam axial energy spread, $\gamma_o^2(\delta\beta_z)_{rms}$, as a function of distance. In this illustration the spread is reduced by a factor of ~ 40. For the 1 MeV example, the rms spread in the axial velocity is observed to be reduced by a factor of approximately 30. In both examples, the required value of the conditioning field is in excellent agreement with the analytical prediction in Eq. (2). For a waveguide diameter of 1 cm the power in the conditioning field is ~ 10 MW and ~ 4 kW for examples 1 and 2, respectively.

In conclusion, in this paper a method is proposed for dramatically reducing the electron axial velocity spread in FELs. The beam conditioning field is that of an axially symmetric, slow, TM waveguide mode. A reduction in the veloc'ty spread by a factor of 40 was obtained.

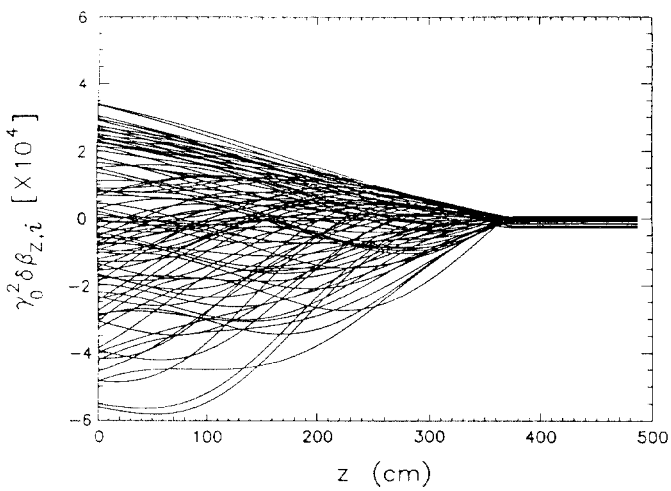

Fig. 1. Fractional axial energy spread, $\gamma_0^2 \delta\beta_{z,i}$, where i = 1, 100, versus distance along the conditioning waveguide. The curves represent 100 particles chosen randomly from a distribution of 10^3 particles. In this figure, the conditioning field is adiabatically turned off at ≃ 375 cm.

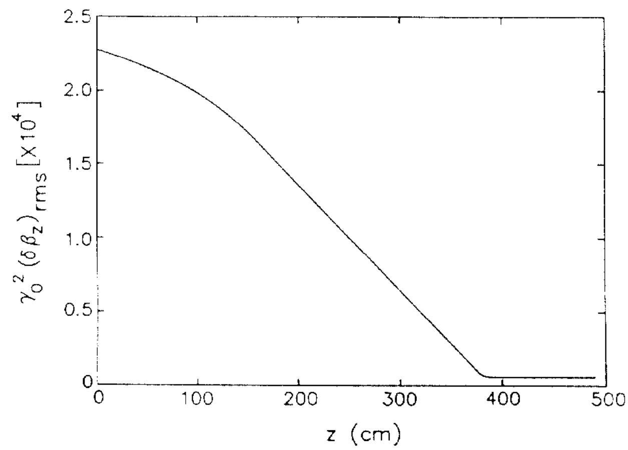

Fig. 2 Root mean square (rms) fractional axial energy spread versus distance. The spread is reduced by a factor of ~ 40.

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Table I

Electron Beam	Example #1	Example #2
Energy, E	10 MeV	1 MeV
Emittance, ε_n	3.5×10^{-3} cm-rad	3.5×10^{-3} cm-rad
RMS Radius, r_b	0.14 cm	0.14 cm
Initial Axial Energy Spread	2.3×10^{-4}	2.4×10^{-4}
Final (Min.) Axial Energy Spread	5.8×10^{-6}	7.9×10^{-6}
Wiggler		
Strength Parameter, a_w	0.175	0.175
Period, λ_w	3.14 cm	3.14 cm
Betatron Period, λ_β	754 cm	104 cm
Conditioning Field		
Wavelength, λ	2 cm	2 cm
Strength Parameter, a_o	0.1	2×10^{-3}
Electric Field, E_o	160 kV/cm	3.2 kV/cm
Interaction Length, ~ $\lambda_\beta/2$	375 cm	53 cm

Table I. Simulation parameters for conditioning a 10 MeV (Example #1) and 1 MeV (Example #2) electron beam.